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Combining CP Asymmetries in $B \to K\pi$ Decays

Michael Gronau[1]
Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309

and

Jonathan L. Rosner
Enrico Fermi Institute and Department of Physics
University of Chicago, Chicago, IL 60637

ABSTRACT

We prove an approximate equality, to leading order in dominant terms, between CP-violating rate differences in $B^0/\overline{B}^0 \to K^\pm\pi^\mp$ and $B^\pm \to K^\pm\pi^0$. We propose several versions of averaged asymmetry measurements in these two processes, for which present data are already capable of yielding a statistically significant nonzero result in the most favorable case of weak and strong phases.

[1]Permanent Address: Physics Department, Technion – Israel Institute of Technology, 32000 Haifa, Israel.

Up to now, CP violation has only been observed in the mixing of neutral K meson states [1]. Thus, it remains to be confirmed that CP violation in the kaon system arises from phases in the Cabibbo-Kobayashi-Maskawa matrix [2] describing weak charge-changing transitions of quarks. Such evidence can be provided by B meson decays, in which the standard model predicts sizable CP asymmetries between partial rates of B mesons and their corresponding antiparticles [3]. Model-dependent calculations of CP asymmetries in B decays to a pair of charmless pseudoscalar mesons have been carried out by a large number of authors [4].

In the present Letter we study relations between direct CP asymmetries in $B \to K\pi$ decays. Observation of three of these decays, $B^0 \to K^+\pi^-$, $B^+ \to K^+\pi^0$ and $B^+ \to K^0\pi^+$, combining processes with their charge conjugates, was reported recently by CLEO [5, 6]. The number of events in these modes is 43, 38, and 12, respectively. We shall show in this note that while each individual measurement is unlikely to provide a statistically significant nonzero asymmetry measurement, the rate asymmetries in the first two processes are expected to be approximately equal. With present statistics and with an estimate of the maximum asymmetry (44%) possible in the standard model, the combined sample of $K^\pm\pi^\mp$ and $K^\pm\pi^0$ events is sufficiently large to display an averaged asymmetry of up to four standard deviations.

In order to study $B \to K\pi$ decays, we will employ a diagrammatic approach based on flavor SU(3) [7]. Since we concentrate on strangeness-changing processes, the major part of our analysis will only require isospin symmetry. SU(3) symmetry and SU(3) breaking effects [8] will be introduced when relating these processes to corresponding strangeness-conserving $B \to \pi\pi$ decays. The decomposition of decay amplitudes in terms of flavor flow topologies is [9]

$$\begin{aligned}
-A(B^0 \to K^+\pi^-) &= (P + \frac{2}{3}P^c_{EW}) + (T) = B_{1/2} - A_{1/2} - A_{3/2} \ , \\
-\sqrt{2}A(B^+ \to K^+\pi^0) &= (P + P_{EW} + \frac{2}{3}P^c_{EW}) + (T + C + A) = B_{1/2} + A_{1/2} - 2A_{3/2} \ , \\
A(B^+ \to K^0\pi^+) &= (P - \frac{1}{3}P^c_{EW}) + (A) = B_{1/2} + A_{1/2} + A_{3/2} \ , \\
\sqrt{2}A(B^0 \to K^0\pi^0) &= (P - P_{EW} - \frac{1}{3}P^c_{EW}) - (C) = B_{1/2} - A_{1/2} + 2A_{3/2} \ . \qquad (1)
\end{aligned}$$

On the right-hand-sides we also include an equivalent decomposition in terms of isospin amplitudes [10], where A and B are $\Delta I = 1$ and $\Delta I = 0$ amplitudes and subscripts denote the isospin of $K\pi$. This equivalence is implied by the relations

$$\begin{aligned}
B_{1/2} &= (P + \frac{1}{6}P^c_{EW}) + \frac{1}{2}(T + A) \ , \\
A_{1/2} &= (\frac{1}{3}P_{EW} - \frac{1}{6}P^c_{EW}) + (-\frac{1}{6}T + \frac{1}{3}C + \frac{1}{2}A) \ , \\
A_{3/2} &= -\frac{1}{3}(P_{EW} + P^c_{EW}) - \frac{1}{3}(T + C) \ . \qquad (2)
\end{aligned}$$

The terms in the first parenthesis of Eqs. (1) and (2), a QCD penguin (P), an electroweak penguin (P_{EW}) and a color-suppressed electroweak penguin (P^c_{EW}) amplitude, carry each a weak phase $\mathrm{Arg}(V^*_{tb}V_{ts}) = -\pi$. The other three terms, tree (T), color-suppressed (C) and annihilation (A) amplitudes, carry a different weak phase $\mathrm{Arg}(V_{ub}^* V_{us}) = \gamma$.

We will assume a hierarchy among amplitudes carrying the same weak phase [9]

$$|P| \gg |P_{EW}| \gg |P^c_{EW}| \ , \tag{3}$$

$$|T| \gg |C| \gg |A| \ , \tag{4}$$

where a roughly common hierarchy factor of about 0.2 describes the ratio of sequential amplitudes. The hierarchy between penguin amplitudes is based on QCD and electroweak loop factors and is supported by model calculations of short distance operator matrix elements [11]. The hierarchy between C and T is taken from short distance QCD corrections and phenomenology of $B \to \overline{D}\pi$ decays [12]. The measured rates of color-suppressed processes, such as $B^0 \to \overline{D}^0\pi^0$ [13], show that rescattering effects do not enhance C to the level of T. This will also be assumed to be the case for $B \to K\pi$. Finally, the hierarchy between A and C follows essentially from a f_B/m_B factor in A relative to T [14]. Several authors [15, 16] have noted recently that the last assumption, $|A| \ll |C|$, can be spoiled by rescattering effects (from intermediate states mediated by T) through soft annihilation or up-quark penguin topologies. We will therefore leave open the possibility that $|A| \sim |C|$. The case $A| \sim |T|$, utilized in some model-dependent calculations [17], will be excluded. We consider it unlikely in view of existing limits on rescattering in $B^0 \to K^+K^-$ [15].

Interference between amplitudes carrying different weak phases and different strong phases leads to CP rate differences between the processes in Eqs. (1) and their charge conjugates. Such interference involves the product of the magnitudes of the amplitudes appearing in the first parenthesis with the amplitudes in the second parenthesis, a sine factor of their relative weak phase and a sine of the relative strong phase. Thus, all the contributions are proportional to $\sin\gamma$, whereas the strong phase difference is generally unknown and may depend on the product. We denote by $2\vec{P}\vec{T}$ the interference between P and T contributing to $\Delta(K^+\pi^-) \equiv \Gamma(B^0 \to K^+\pi^-) - \Gamma(\overline{B}^0 \to K^-\pi^+)$, and use similar notations for other interference terms and other CP rate differences. One then finds for the B-$\overline{B}$ rate differences the following expressions, where terms are written in decreasing order using Eqs. (3) and (4), and the smallest terms are neglected:

$$\begin{aligned}
\Delta(K^+\pi^-) &= 2\vec{P}\vec{T} + \frac{4}{3}\vec{P}^c_{EW}\vec{T} \ , \\
\Delta(K^+\pi^0) &= \vec{P}\vec{T} + \vec{P}_{EW}\vec{T} + \vec{P}\vec{C} + \vec{P}\vec{A} + \vec{P}_{EW}\vec{C} + \frac{2}{3}\vec{P}^c_{EW}\vec{T} + \ldots \ , \\
\Delta(K^0\pi^+) &= 2\vec{P}\vec{A} + \ldots \ , \\
\Delta(K^0\pi^0) &= -\vec{P}\vec{C} + \vec{P}_{EW}\vec{C} + \frac{1}{3}\vec{P}^c_{EW}\vec{C} \ .
\end{aligned} \tag{5}$$

We note that, in the absence of electroweak penguin amplitudes, one finds [18]

$$\Delta(K^+\pi^-) + \Delta(K^0\pi^+) = 2\Delta(K^+\pi^0) + 2\Delta(K^0\pi^0) \ . \tag{6}$$

However, this relation is spoiled by electroweak penguin contributions.

Comparing the four rate differences, we see that the dominant terms of the form $\vec{P}\vec{T}$ appear only in the first two rate differences, leading at this order to a very simple relation

$$\Delta(K^+\pi^-) \approx 2\Delta(K^+\pi^0) \ . \tag{7}$$

The next-to-leading terms correcting this relation are $\vec{P}_{EW}\vec{T}$ and $\vec{P}\vec{C}$. The first term can be shown to lead to a negligible rate difference. The argument is based on a property of the $A_{3/2}$ amplitude, which was shown recently [19] to consist of $T + C$ and electroweak penguin contributions with approximately equal strong phases. Using this property we conclude that

$$\vec{P}_{EW}\vec{T} + \vec{P}_{EW}\vec{C} + \vec{P}^c_{EW}\vec{T} + \vec{P}^c_{EW}\vec{C} \approx 0 \ , \tag{8}$$

or, to leading order, that $\vec{P}_{EW}\vec{T} \approx 0$. Since the term $\vec{P}\vec{C}$ is the only next-to-leading correction to Eq. (7), this equality is expected to hold to about 20%.

Using the hierarchy $|A| \ll |C|$, it has often been assumed that the rate difference $\Delta(K^0\pi^+)$ is extremely small. However, recently it was argued [15, 16] that rescattering effects may enhance A to the level of C, thus leading to a CP asymmetry in this process at a level of 10%. This would imply that the term $\vec{P}\vec{A}$ appearing in both $\Delta(K^0\pi^+)$ and $\Delta(K^+\pi^0)$ is next-to-leading and may be comparable to $\vec{P}\vec{C}$. In this case a better approximation than Eq. (7) becomes

$$\Delta(K^+\pi^-) + \Delta(K^0\pi^+) \approx 2\Delta(K^+\pi^0) \ . \tag{9}$$

A way of gauging the importance of the $\vec{P}\vec{A}$ term would be by measuring a nonzero value for $\Delta(K^0\pi^+)$. The dominant correction to the approximate relation (9) is the term $-2\vec{P}\vec{C}$ which is contributed by $2\Delta(K^0\pi^0)$ on the right-hand side of (6).

In order to estimate the CP asymmetries in $B^0/\overline{B}^0 \to K^\pm\pi^\mp$ and $B^\pm \to K^\pm\pi^0$, one must know the ratio $|T/P|$. Using previous data [5] we have shown [20] that this ratio is smaller than one, representing another hierarchy factor of about 0.2. Let us update information about P and T using more recent data [6]. We will quote squares of amplitudes in branching ratio units of 10^{-6}.

The most straightforward way of obtaining $|P|$ is from the observed CP-averaged branching ratio [6]

$$\mathcal{B}(B^\pm \to K^0/\overline{K}^0\pi^\pm) = (14 \pm 5 \pm 2) \times 10^{-6} \ , \tag{10}$$

since there are no first order corrections to P in these processes even when $|A|$ is as large as $|C|$. Thus $|P|^2 = 14.0 \pm 5.4$ or $|P| = 3.74 \pm 0.72$.

An estimate of $|T|$ is more uncertain at this time. While CLEO has quoted upper limits at 90% confidence level:

$$\mathcal{B}(B^0/\overline{B}^0 \to \pi^+\pi^-) < 8.4 \times 10^{-6} \ , \tag{11}$$

and

$$\mathcal{B}(B^\pm \to \pi^\pm\pi^0) < 16 \times 10^{-6} \ , \tag{12}$$

their data imply signals for these decays with significance of 2.9 and 2.3 standard deviations, respectively. Taking these signals seriously, we may obtain from the reported event rate and efficiency the branching ratios

$$\mathcal{B}(B^0/\overline{B}^0 \to \pi^+\pi^-) = (3.7^{+2.0}_{-1.7}) \times 10^{-6} \ , \tag{13}$$

$$\mathcal{B}(B^\pm \to \pi^\pm\pi^0) = (5.9^{+3.2}_{-2.7}) \times 10^{-6} \ . \tag{14}$$

While destructive interference between tree and penguin amplitudes in $B^0 \to \pi^+\pi^-$ and/or constructive interference between tree and color-suppressed or electroweak penguin amplitudes in $B^+ \to \pi^+\pi^0$ may lead to $\mathcal{B}(B^0 \to \pi^+\pi^-) < 2\mathcal{B}(B^+ \to \pi^+\pi^0)$ [7], we shall ignore such effects as in Ref. [20]. Thus, using an SU(3) relation between $B \to K\pi$ and $B \to \pi\pi$, and introducing SU(3) breaking through f_K/f_π [7, 8], we have the two independent estimates

$$|T|^2 = \left[\frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi}\right]^2 \frac{\mathcal{B}(B^+ \to \pi^+\pi^-)}{10^{-6}} \ , \tag{15}$$

$$|T|^2 = 2\left[\frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi}\right]^2 \frac{\mathcal{B}(B^+ \to \pi^+\pi^0)}{10^{-6}} \ , \tag{16}$$

whose average (assuming a 20% error from neglecting a penguin amplitude in $B^0 \to \pi^+\pi^-$ and a color-suppressed amplitude in $B^+ \to \pi^+\pi^0$) leads to $|T/P| = 0.152 \pm 0.053$, or $|T/P| < 0.22$ at 90% confidence level. A more precise determination of this ratio requires more statistics. We will assume its preliminary value. A slightly larger value of $|T + C|/|P| = 0.24 \pm 0.06$ was estimated in Refs. [19] and [21].

As we have shown, CP asymmetries in $B^0/\overline{B}^0 \to K^\pm\pi^\mp$ and $B^\pm \to K^\pm\pi^0$ are equal to each other, to leading order in $|T/P|$, $|P_{EW}/P|$ and $|C/T|$, and are given by $2|T/P|\sin\gamma\sin\delta$, where δ is the strong phase difference between P and T. This phase is generally unknown, but could be substantial. While the tree amplitudes is expected to factorize, thus showing little evidence for rescattering effects, the penguin amplitude obtains a large contribution from a so-called charming penguin term [22], involving long distance effects of rescattering from charm-anticharm intermediate states. It is therefore conceivable that δ could attain a large value, such that $\sin\delta \sim 1$. The values of γ allowed at present [23] include those around 90° obeying $\sin\gamma \sim 1$. We therefore conclude that an interesting range of asymmetry measurements includes the value $2|T/P|$ which we found to be 0.30 ± 0.11 to leading order, or less than 44% at 90% confidence level.

Since CP asymmetries in the processes $B^0/\overline{B}^0 \to K^\pm\pi^\mp$ and $B^\pm \to K^\pm\pi^0$ are approximately equal, we propose that they be averaged. To evaluate the statistical power of this procedure, we note that for an asymmetry $A = (N_+ - N_-)/(N_+ + N_-)$ based on N_+ events in one mode and N_- events in the charge-conjugate mode, the squared error $(\delta A)^2$ is

$$(\delta A)^2 = \left[\frac{\partial A}{\partial N_+}\delta N_+\right]^2 + \left[\frac{\partial A}{\partial N_-}\delta N_-\right]^2 = \left[\frac{2N_-}{N^2}\right]^2 N_+ + \left[\frac{-2N_+}{N^2}\right]^2 N_- = \frac{4N_+N_-}{N^3} \leq \frac{1}{N} \ , \tag{17}$$

where $N \equiv N_+ + N_-$ and we have used $4N_+N_- \leq (N_+ + N_-)^2 = N^2$. In the case of a small asymmetry one has $(\delta A)^2 \approx 1/N$. If we now average two asymmetries A_1 and A_2 based on two samples of $N_1 = N_{1+} + N_{1-}$ and $N_2 = N_{2+} + N_{2-}$ events, the result is

$$A_{\rm av} = \left[\frac{A_1}{(\delta A_1)^2} + \frac{A_2}{(\delta A_2)^2}\right] / \left[\frac{1}{(\delta A_1)^2} + \frac{1}{(\delta A_2)^2}\right] \simeq \frac{N_1 A_1 + N_2 A_2}{N_1 + N_2} \quad , \tag{18}$$

where the last relation holds for small asymmetries, with

$$\delta A_{\rm av} = \left\{1/\left[\frac{1}{(\delta A_1)^2} + \frac{1}{(\delta A_2)^2}\right]\right\}^{1/2} \leq \frac{1}{\sqrt{N_1 + N_2}} = \frac{1}{\sqrt{N_{\rm tot}}} \quad . \tag{19}$$

We have thus shown that if two processes are expected to have the same asymmetry A, the total number of events $N_{\rm tot} = N_1 + N_2$ required to observe this asymmetry at the n-standard-deviation level of significance does not exceed $N_{\rm tot} = (n/A)^2$. Thus, for $|A_{\rm max}| = 0.44$ and $N_{\rm tot} = 43 + 38 = 81$ events, one could see a signal as large as four standard deviations, whereas the maximum signals based on N_1 and N_2 separately would not be expected to exceed 3σ. Backgrounds will degrade these estimates somewhat.

The next step beyond averaging the two asymmetries mentioned above is to check the sum rule (9), whose validity would check our assumption of the negligible nature of the $\vec{P}\vec{C}$ interference term. The relation (6) contains the correction due to this term, but requires the tagging of the neutral B's flavor for verification.

The rates for the $B \to K\pi$ processes (and their charge-conjugates) are expected to satisfy the relations

$$\Gamma(B^+ \to K^0\pi^+) = \Gamma(B^0 \to K^+\pi^-) = 2\Gamma(B^+ \to K^+\pi^0) = 2\Gamma(B^0 \to K^0\pi^0) \tag{20}$$

to leading order, since they are all dominated by the (gluonic) penguin terms P in Eq. (1). As a result of the relation (7) and the predicted equality of decay rates to lowest order, there are a continuum of equivalent ways to combine the data on charged and neutral decays to estimate A, not all of which have the same statistical power. For example, A will also be given by the expression

$$A = \frac{\Gamma(B^+ \to K^+\pi^0) + \Gamma(B^0 \to K^+\pi^-) - \Gamma(B^- \to K^-\pi^0) - \Gamma(\bar{B}^0 \to K^-\pi^+)}{\Gamma(B^+ \to K^+\pi^0) + \Gamma(B^0 \to K^+\pi^-) + \Gamma(B^- \to K^-\pi^0) + \Gamma(\bar{B}^0 \to K^-\pi^+)} \quad . \tag{21}$$

(Note that one uses equal weights for charged and neutral B decays despite the factor of 2 ratio in their approximately predicted decay rates.) In this case one finds

$$(\delta A)^2 = \frac{4(N_{+0} + N_{+-})(N_{-0} + N_{-+})}{N_{\rm tot}^3} \leq \frac{1}{N_{\rm tot}} \quad , \tag{22}$$

where N_{ij} is the number of $B \to K^i\pi^j$ events, and $N_{\rm tot}$ is the total of $K^\pm\pi^0$ and $K^\pm\pi^\mp$ events. In the limit of small asymmetry this estimate has statistical power equivalent to simply averaging the asymmetries in charged and neutral decay modes.

To first order in small quantities, the $B \to K\pi$ rates satisfy the sum rule [24]

$$2\Gamma(B^+ \to K^+\pi^0) + 2\Gamma(B^0 \to K^0\pi^0) = \Gamma(B^+ \to K^0\pi^+) + \Gamma(B^0 \to K^+\pi^-) \quad , \tag{23}$$

which may be used to anticipate a small $B^0 \to K^0\pi^0$ rate. The difference between $\mathcal{B}(B^+ \to K^+\pi^0)$ and $\mathcal{B}(B^+ \to K^0\pi^+)/2$ can be used [19] to place model-independent bounds on the weak phase γ. Because of the rate difference that can arise between the two processes involving charged kaons, which is dominated by the interference of P and P_{EW}, our prescriptions for combining asymmetries are limited to lowest order in small quantities. A quantity less subject to this shortcoming is the CP-violating rate difference itself. Using Eq. (7), one can combine the rate differences in $K^+\pi^-$ and $K^+\pi^0$ decays to display an effect with greater statistical significance than in each individual channel.

To conclude, we have shown that to leading order in small quantities it makes sense to combine the CP-violating rate asymmetries in the decays $B^0 \to K^+\pi^-$ and $B^+ \to K^+\pi^0$. Whereas the identification of the flavor of charged secondaries in $B^0/\overline{B}^0 \to K^\pm\pi^\mp$ decays requires good particle identification in order to avoid a kinematic ambiguity involving $\pi \leftrightarrow K$ interchange, no such ambiguity afflicts the decays $B^\pm \to K^\pm\pi^0$. The averaged rate asymmetry can be large enough in the standard model that it would be detectable at present levels of sensitivity.

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