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# APPLICATION OF A MAPPING FUNCTION TECHNIQUE TO THE DESIGN OF DAMPED DETUNED STRUCTURES AND TO THE RAPID CALCULATION OF THEIR WAKEFIELDS

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#### Abstract

In order to reduce the dipole wake encountered by the first few bunches accelerated in a multi-bunch NLC scenario the DDS (damped detuned structure) was re-designed such that a much improved Gaussian fall-off occurs in the initial wake-function. From the 9 parameterised model of DDS1 we use a mapping function to allow DDS 3 & 4 to be modeled and hence avoid additional and prohibitively time consuming MAFIA runs. The equivalent circuit parameters and geometrical parameters are treated as functions of the synchronous frequency and are readily mapped onto the new synchronous frequencies. The new geometrical parameters form a family where each is associated with the iris diameter.

#### **1. Introduction**

The first ever manifold DDS was designed such that the geometrical parameters (iris radius and cavity radius) of the cells were inverse functions of error functions, Erf. Further, the mode density function (the reciprocal of the derivative of the uncoupled frequencies with respect to mode number: dn/df) was prescribed to be Gaussian. However, the short range dipole wake function, is given by twice the inverse transform of the dipole kick-factor (K) weighted density function and under a Gaussian prescription of dn/df, 2Kdn/df is markedly asymmetric. The consequence of the asymmetry is a poor definition of the minima in the short range wake function. DDS 3 & 4 have been re-designed under a Gaussian 2Kdn/df prescription with a bandwidth of 4.71 units of sigma (with sigma 2.125% of the central frequency of the Gaussian) and this leads to a significantly improved short-range wake function.

The inverse Fourier transform of the spectral function [1] allows the global wake-function to be evaluated. However, in order obtain the new spectral function, all nine parameters of the structure must be obtained for 206 cells. This is a substantial computational task in running the MAFIA code required for the spectral function and in the careful fitting procedure required for all the new functions. However, the method used herein obviates this excessive computational work and requires that we fit all the 9 circuit model parameters together with the beam kick-factor to ten functions which all depend on the synchronous frequency only. The new set of new synchronous frequencies, dictated in our case by the requirement that 2Kdn/df be Gaussian, allows the 206 x 10 new characteristic parameters to be

calculated. Similarly the 5 parameters, which define the geometry of the structure (the iris radius a, cavity radius b, iris thickness t, the radial distance of the edge of the manifold from the center of of a cell, H and the height of the manifold L) are also functionally dependent on the synchronous frequency of the beam and, under a new set of frequencies, 5 x 206 new dimensions are calculated for the DDS. Thus, in order to obtain the wake function and new geometrical parameters for fabrication all that is necessary is to obtain 15 functions.

However, under this new mapping one might express some concern as to whether the properties of the fundamental (i.e. accelerating) mode have been adversely affected and so with this in mind we conducted an intensive investigation as to the deviation of the cell dimensions, parameterised by the cavity diameter 2b, from their values designed in DDS 2 (all dimensions form an invariant family parameterised by the cavity diameter b). This is detailed in section 4 and successive sections.

# 2. The Mapping Function

In our design of DDS 2 we chose eleven representative sections to obtain frequency-phase pairs from detailed MAFIA simulations and hence obtain ten model parameters (nine circuit parameters plus the cell kick-factors) for each of the eleven sections. Parameters for all sections are subsequently obtained by error function fits and A similar procedure may be followed to interpolation. determine the five geometric parameters (i.e., cell and manifold dimensions) for all the sections from those for the original eleven. This is a substantial task for each structure design. However, as we now have all fifteen parameters as a function of synchronous frequency, we can take advantage of this functional dependence to explore new design distributions and to obtain the set of section dimensions which would be needed to realize them.

Based on our fit parameters we prescribe a smooth uncoupled spectral function,  $S_0(f_s)\lambda$  and impose the condition that:  $2K(f_s)dn/df = S_0(f_s)\lambda$ , where K is the uncoupled kick factor,  $f_s$  the synchronous frequency and,  $\lambda$  is a scale factor to be determined. The upper and lower truncation bounds on the synchronous frequencies are imposed,  $f_{s1}$  and  $f_{sN}$  and the normalisation condition is obtained:

$$\lambda = N / \int_{f_{s1}}^{f_{sN}} (\frac{1}{2}S_0 / K) df_s$$
 (2.1)

Then the new synchronous frequencies are determined according to:

$$\int_{f_{sn}}^{f_{sn+1}} \frac{1}{2K} \lambda S_0 df_s = 1$$
(2.2)

This enables all the cell synchronous frequencies to be determined and hence the new ten parameters are determined.

This procedure is implemented in the following section to calculate the spectral function and associated wake function for DDS 3.

#### 3. Calculation of the Wake function

In the revised design for DDS 3 we chose a truncated Gaussian distribution for the uncoupled 2Kdn/df<sub>syn</sub> distribution, with a bandwidth of 4.71 units of sigma, (a bandwidth of 10.159% of the central frequency and sigma is 2.125% of the central frequency) and this provides a basis for the determination of the 206 synchronous frequencies. The kick factor weighted density function for DDS 3 and DDS 1 are shown in Fig 1.



Figure 1: Twice the kick factor weighted density function for DDS 2 (shown dashed) and the corresponding function for the re-designed DDS3 and 4

It is evident that DDS 2 is markedly asymmetric and this adversely affects the sharpness and depth of the minima for the short range wake

In order to calculate the wake function we first are required to calculate the spectral function associated with the 9 mapped parameters. This spectral function calculated for DDS 3, and shown in Fig 2, maintains the Gaussian characteristics imposed upon it from the synchronous frequency distribution but modulated with oscillations of large amplitude (resulting in a large part from reflections occurring in the higher order mode couplers in the manifold).

Also, the spectral function exhibits the underlying damped mode structure as mentioned previously for DDS 2 [1] and is shifted with respect to the 2Kdn/df curves, as given in Fig. 1. The difference between the respective curves in Fig. 1 and Fig. 2 becomes more pronounced for higher

frequencies becoming more highly perturbed as one progresses down towards the higher energy end of the structure where the coupling to the manifold has been designed to be largest. These coupled synchronous frequencies will be discussed in a future publication [2]



Figure 2: Spectral function for DDS 2 and DDS 3

It is important to note that the sharp, rather precipitous, falloff in the spectral function in DDS 2 at approximately 15.8 GHz has detrimental affects on the short range wake function. In DDS 3 the spectral function falls off smoothly and gradually and this has beneficial effects on the range wake function in that it enables a faster fall-off to occur. Indeed for a perfectly smooth termination, which we refer to as our idealised case [1], it is possible to achieve more than an order of magnitude weaker wake function at the 90 bunch point.



Figure 3: Long range wake function for DDS 2 (shown dashed) and DDS 3. The points are at the location of each of the bunches, of which there are ninety.

It is interesting to note that the maxima of DDS 3 is a little larger than that of its counterpart DDS 2. However, the area under the curve (bounded by the upper & lower synchronous frequencies for each structure) corresponding to DDS 3 is slightly smaller than DDS 2. This reduction in the area of origin (since the wake function is given by the inverse transform of the spectral function) and this is in itself a consequence of the larger iris dimension in DDS 3.

# 4. Geometrical Parameters

Each of the five geometrical parameters are fitted with an interpolation function, the independent variable in each case being the synchronous frequency. Thence, armed with these new synchronous frequencies the new 206 x 5 parameters are readily obtained. Both a new and mapped cell parameter is shown in Fig. 4 and a manifold parameter in Fig. 5. The end points of DDS 2 and DDS 3 are identical by design so that no extrapolation is required in the determination of the DDS 3 parameters. Thus, in determining all parameters only third order interpolation between cell points has been employed, with a view to minimising any error in the generation of the new points.

It is evident from the curves that in the downstream end (or low energy end) of the structure the parameters are very close to that of the DDS 2 design whereas in the upstream end both the iris and the cavity diameter are increased significantly. This is a consequence of the asymmetry in the original design in which the kick factor weighted distribution reached too low a level in the upstream location of the structure and this has been corrected for in DDS 3. The thickness of the irises however, is reduced with respect to DDS 2, but this reduction is sufficiently small that the structure still maintains its mechanical integrity.

The manifold is tapered as one goes down the structure to enhance the coupling. This increased coupling is necessary because the modal composition of TE/TM is reduced as one moves towards the upstream end of the structure and to achieve a Q value in the neighborhood of a 1000 or so, increased coupling is required. There is a reverse in the taper towards the end cells in the upstream end and this is instituted in order to lower the cut-off frequency of the HOM (higher order mode) coupler and hence improve the match of the mitered bend of the HOMs at the lower frequency end of the band.

It is necessary to have well-matched HOM loads because the wake function is very sensitive to the power reflected back into the accelerator [1]. However, under the mapping the middle cell has effectively shifted forward by 11 cells and hence the upward taper (which occurs in cells 182 to 202 for DDS 2) maps the up-taper into a region where there are no manifold cells. Thus, in DDS 3 we change the mapped taper in this region by increasing the gradient of the taper and withdrawing its to cell 202. This will adversely affect the wake function but we are confident that its deleterious effect will minimal.



Figure 4 Cell geometrical parameters: iris thickness, t.



Figure 5: Manifold geometrical parameters: radial distance of the edge of the manifold the from center of a cell.

# 5. Conclusions

We have developed a method to rapidly design new DDSs based upon a mapping procedure. This method enables both the wake function and the new geometry of the structure to be evaluated. Indeed, we have applied this method to calculate the short range wake function for DDS 3 and we find that, on average, the wake function is reduced by a factor of 5 or more. The new geometrical parameters form an invariant family which are functionally dependent on the cavity iris diameter, 2 and the deviation of the new family of parameters from the old provides an indication as to the accelerating mode's phase advance of the new structure.

#### 6. Acknowledgments

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### 7. References

[1] R.M. Jones, et al. A Spectral Function Method Applied to the Calculation of the SLAC Damped Detuned Structure. Proc. Intl. Linac Conf. Geneva Switzerland, 1996 (and SLAC-PUB 7287)

[2] N.M. kroll, The SLAC Damped Detuned Structure, Concept and Design, SLAC-PUB7589, PAC97