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Vacuum Energy Cancellation in a Non-Supersymmetric String

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We present a nonsupersymmetric orbifold of type II string theory and show that it has vanishing cosmological constant at the one and two loop level. We argue heuristically that the cancellation persists at higher loops.

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1. Introduction

One of the most intriguing and puzzling pieces of data is the (near-)vanishing of the cosmological constant Λ [1]. Unbroken supersymmetry would ensure that perturbative quantum corrections to the vacuum energy vanish (in the absence of a U(1) D-term) due to cancellations between bosonic and fermionic degrees of freedom. However, although both bosons and fermions appear in the low-energy spectrum, they are not related by supersymmetry and this mechanism for cancelling Λ is not realized.

Because string theory (M-theory) is a consistent quantum theory which incorporates gravity, it is interesting (and necessary) to see how string theory copes with the cosmological constant. In a perturbative string framework, because the string coupling g_{st} (the dilaton) is dynamical, the quantum vacuum energy constitutes a potential for it. So the issue of turning on a nontrivial string coupling is related to the form of the vacuum energy in string theory.

In this paper we present a class of perturbative string models in which supersymmetry is broken at the string scale but perturbative quantum corrections to the cosmological constant cancel. We begin with a simple mechanism that ensures the (trivial) vanishing of the 1-loop vacuum energy (as well as certain tadpoles and mass renormalizations). We then compute the (spin-structure-dependent part of the) 2-loop partition function and demonstrate that it vanishes. This requires some analysis of worldsheet gauge-fixing conditions, modular transformations, and contributions from the boundaries of moduli space. Examination of the general form of higher-loop amplitudes suggests that they similarly cancel and we next present this argument. We are unable to rigorously generalize our 2-loop calculation to higher loops at this point because of the complications of higher-genus moduli space. We hope to be able to make the higher-genus result more precise by using an operator formalism as will become clearer in the text, though we leave that for future work.

In addition we discuss how this model may fit into the framework [2] relating conformal fixed lines/points in quantum field theory to vanishing dilaton potentials/isolated minima of the dilaton potential in string theory. This provides hints as to where to look for more general models with vanishing Λ. In particular we will be interested in models without the tree level bose-fermi degeneracy that we have here, as well as models in which the dilaton is stabilized. We should note in this regard that instead of working in 4d perturbative string theory as we do here, we could consider the same class of models in 3d string theory and consider the limit of large g_{st}. If the appropriate D-brane bound states exist in this theory to provide Kaluza-Klein modes of an M-theoretic fourth dimension, one could obtain in this way 4d M-theory vacua with vanishing cosmological constant and no dilaton (in this way similar to the scenario of [3], but here without the need for 3d supersymmetry).

We understand that a complementary set of models has been found in the free fermionic description [4]. We would like to thank Zurab Kakushadze for pointing out (and fixing) an error in our original model as presented at Strings '98.

2. Nonabelian Orbifolds and the 1-loop Cosmological Constant

Consider the worldsheet path integral formulation of orbifold compactifications [5]. In general one mods out by a discrete symmetry group of the 10-dimensional string theory. This group involves rotations of the left and right-moving worldsheet scalars $X^{\mu}_{L,R}$ and fermions $\psi^{\mu}_{L,R}$ as well as shifts of the scalars $X^{\mu}_{L,R}$. Here $\mu = 1, \ldots, 10$ is a spacetime $SO(9,1)$ vector index. The worldsheet path integral at a given loop order h splits up into a sum over different twist structures, in which the fields are twisted by orbifold group elements in going around the various cycles of the genus-h Riemann surface Σ_h. These twists must respect the homology relation

$$\prod_{i=1}^{h} a_i b_i a_i^{-1} b_i^{-1} = 1 \tag{2.1}$$

where a_i and b_i are the canonical 1-cycles on Σ_h. In particular, at genus 1, one sums over pairs (g, h) of *commuting* orbifold space group elements g and h.

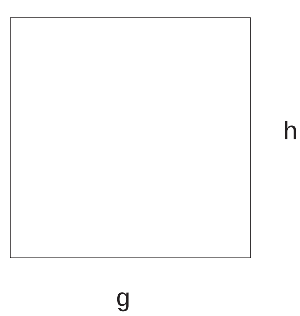

Figure 1: Torus twisted by elements (g, h)

In considering nonsupersymmetric orbifolds, this suggests an interesting class of models. Consider orbifolds in which no commuting pair of group elements breaks all the supersymmetry (i.e. projects out all of the gravitinos), but in which the full group does break all the supersymmetry. At the one-loop level, each contribution to the path integral then effectively preserves some supersymmetry and therefore vanishes. This is a formal way of encoding the fact that the spectrum for this type of model will have bose-fermi degeneracy at all mass levels (though no supersymmetry). So the one-loop partition function, as well as appropriate tadpoles, mass renormalizations, and three-point functions, are uncorrected.

We will discuss the following specific model.[1] Let us start with type II string theory compactified on a square torus $T^6 \sim (S^1)^6$ at the self-dual radius $R = l_s/\sqrt{2}$ (where $l_s = \sqrt{\alpha'}$ is the string length scale). Consider the asymmetric orbifold generated by the elements f and g:

S^1	f	g
1	$(-1, s)$	$(s, -1)$
2	$(-1, s)$	$(s, -1)$
3	$(-1, s)$	$(s, -1)$
4	$(-1, s)$	$(s, -1)$
5	$(s^2, 0)$	(s, s)
6	(s, s)	$(0, s^2)$
	$(-1)^{F_R}$	$(-1)^{F_L}$

We have indicated here how each element acts on the left and right moving RNS degrees of freedom of the superstring. Here s refers to a shift by $R/2 = l_s/2\sqrt{2}$. So for example f reflects the left-moving fields $X_L^{1\ldots4}$, $\psi_L^{1\ldots4}$ and shifts $X_R^{1\ldots4}$ by $R/2$, X_L^5 by R, and $X^6 = \frac{1}{2}(X_L^6 + X_R^6)$ by $R/2$. In addition it includes an action of $(-1)^{F_R}$ which acts with a (-1) on all spacetime spinors coming from right-moving worldsheet degrees of freedom. This can be thought of as discrete torsion [7]: in the right-moving Ramond sector the f-projection has the opposite sign from what it would have without the $(-1)^{F_R}$ action. Similarly the above table indicates the action of the generator g on the worldsheet fields. This orbifold satisfies level-matching and the necessary conditions derived in [7,8] for higher-loop modular invariance (we do not know if these conditions are sufficient).

There are several features to note about the spectrum of this model. First, it is not supersymmetric. In particular, f projects out all the gravitinos with spacetime spinor

[1] Other similar models can be constructed, some of which do not actually require the group to be nonabelian to get 1-loop cancellation [6].

quantum numbers coming from the right-movers. Similarly g projects out the gravitinos with left-moving spacetime spinor quantum numbers. Because of the shifts included in our orbifold action, there are no massless states in twisted sectors, so in particular no supersymmetry returns in twisted sectors. Second, the model is nonetheless bose-fermi degenerate. In particular the massless spectrum has 32 bosonic and 32 fermionic physical states.

In addition to the spectrum of perturbative string states there is a D-brane spectrum in this theory which one can analyze along the lines of [9].This will be of interest in placing this example in a more general context in the final section.

Our orbifold group elements satisfy the following algebraic relations:

$$fg = gfT_L^{-1}T_R \quad fT_L^q = T_L^{-q}f \quad gT_R^q = T_R^{-q}g \tag{2.2}$$

where T_L denotes a shift by R on $X_L^{1\cdots4}$ and T_R denotes a shift by R on $X_R^{1\cdots4}$. Clearly also f commutes with T_R and g commutes with T_L.

The first relation in (2.2) tells us that f and g do not commute in the orbifold space group. Therefore at the one loop level they never both appear as twists (f,g) in the partition function (i.e. we cannot twist by f on the a-cycle and by g on the b-cycle). Furthermore we can check that no commuting pair of elements break all the supersymmetry. In order to break the supersymmetry we would need pairs of the form $(fT_L^aT_R^b, gT_L^cT_R^d)$ or $(fT_L^{\tilde a}T_R^{\tilde b}, fgT_L^{\tilde c}T_R^{\tilde d})$, for arbitrary integers $a,b,c,d,\tilde a,\tilde b,\tilde c,\tilde d$. (We could also have the latter form with f interchanged with g but these are isomorphic.) By using the relations (2.2) we see that neither pair of elements commutes:

$$(fT_L^aT_R^b)(gT_L^cT_R^d) = (gT_L^cT_R^d)(fT_L^aT_R^b)T_L^{2c+1}T_R^{1-2b} \tag{2.3}$$

So there is no choice of integers a,b,c,d for which the two elements commute in the space group of the orbifold. Similarly

$$(fT_L^{\tilde a}T_R^{\tilde b})(fgT_L^{\tilde c}T_R^{\tilde d}) = (fgT_L^{\tilde c}T_R^{\tilde d})(fT_L^{\tilde a}T_R^{\tilde b})T_L^{2\tilde c-2\tilde a-1}T_R^{1-2\tilde b} \tag{2.4}$$

So at the one loop level, there will not be any contribution to the partition function.

3. The 2-loop vacuum energy

At two loops the orbifold algebra itself does not automatically ensure the cancellation of the partition function. Let us denote the canonical basis of 1-cycles by $2h$-dimensional vectors $(a_1, \ldots, a_h; b_1, \ldots, b_h)$. At genus two, we run into twist structures like $(1, 1; f, g)$ around the canonical cycles:

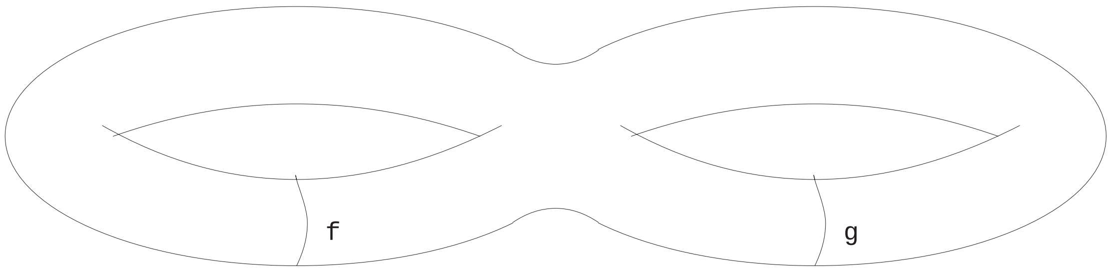

Figure 2: Basic twist structure at genus 2.

In the figure we indicate the *cuts* in the diagram in a given twist structure–here the fields are twisted in going around the b-cycles, as in doing so they pass through the indicated cuts. In particular this diagram involves both f and g twists, and therefore has the information about the full supersymmetry breaking of the model. Is there reason to believe the vacuum energy might nonetheless cancel? Heuristically, the following argument suggests that we should indeed expect a cancellation. Consider evaluating the diagram of Figure 2 near the factorization limit in which the diagram looks like a propagator tube connecting two tori. Because of the homology relations, in this twist structure the intermediate state in this propagator is untwisted. The diagram thus becomes a sum over products of tadpoles of untwisted propagating states (weighted by e^{-mT} where m is the mass of the state and T gives the length of the tube). Each term is a tadpole of the untwisted state in the g-twisted theory times a tadpole of the untwisted state in the f-twisted theory. The contour deformation arguments of [10] imply that these tadpoles vanish. In order to make this rigorous one needs to see explicitly that unphysical states decouple properly (which only has to happen after summing over all twist structures). In what follows we will provide an explicit computation of the 2-loop contribution and verify that it vanishes.

3.1. Back to 1-loop.

In order to appreciate the relevant mechanism, it is worth returning momentarily to the 1-loop (supersymmetric) contribution $(1, f)$.

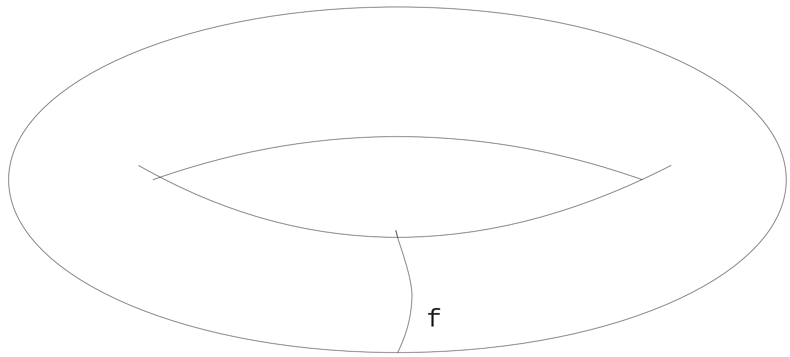

Figure 3: One-loop diagram with an f twist on the b cycle.

This contribution must vanish by supersymmetry, but it is instructive to observe how the spin structure sum works in this case before going on to our 2-loop diagram. The amplitude is

$$\mathcal{A}_1 = \int \frac{d^2\tau}{Im\tau} Tr(q^{L_0}\bar{q}^{\bar{L}_0} f) \tag{3.1}$$

where $q = e^{2\pi i\tau}$ and L_0 and $\bar{L}_0$ are the usual Virasoro zero mode generators. Let us consider the spin-structure dependent piece of this amplitude. As explained in [11], the determinants for the worldsheet Dirac operators acting on the RNS fermions are proportional to theta functions. The θ-function is defined (for general genus h) by

$$\theta[\alpha,\beta](z|\tau) = \sum_n e^{[\pi i(n+\alpha)^t\tau(n+\alpha)+2\pi i(n+\alpha)(z+\beta)]} \tag{3.2}$$

Here $z \in \mathbb{C}^{\mathbf{h}}/(\mathbb{Z}^{\mathbf{h}} + \tau\mathbb{Z}^{\mathbf{h}})$ and τ is the period matrix of the Riemann surface, defined in terms of the canonical basis of holomorphic 1-forms ω_i by $\oint_{a_j}\omega_i = \delta_{ij}$ and $\oint_{b_j}\omega_i = \tau_{ij}$. The characteristics α, β encode the spin structure [12], i.e. the boundary conditions of the fermions around the a and b cycles respectively of the Riemann surface. So for example if $\alpha_1 = 1/2$ (resp. 0), the corresponding fermion has *periodic* (resp. *antiperiodic*) boundary conditions around the a_1 cycle.

The integrand of the 1-loop amplitude (3.1) is proportional to

$$\mathcal{A}_1 \propto \sum_{\alpha,\beta} \eta_{\alpha,\beta}\theta^2[\alpha,\beta](0|\tau)\theta^2[\alpha,\beta+\frac{1}{2}](0|\tau) \tag{3.3}$$

where $\eta_{\alpha,\beta}$ are the phases encoding the GSO projection. The first θ^2 factor comes from the left-moving RNS fermions $\psi_L^{1\cdots 4}$ and the second θ^2 factor comes from the other four transverse left-moving fermions $\psi_L^{5\cdots 8}$. The symmetry between these two factors will play an important role for us. Let us consider first the terms in the sum (3.3) with $\alpha = 1/2$. This describes left-moving Ramond-sector states propagating in the loop, as the left-moving fermions ψ_L are periodic around the a-cycle. Because we have an f-twist around the b-cycle, half the ψ_L^μ are periodic around the b-cycle and half are antiperiodic around the b-cycle for each value of β in the sum. Thus in each $\alpha = 1/2$ term half the RNS fermions have zero modes, so these terms identically vanish.

Let us now consider the terms with $\alpha = 0$, which describe left-moving Neveu-Schwarz states propagating in the loop. These give

$$\sum_{\beta=0,1/2} \eta_{0,\beta}\theta^2[0,\beta](0|\tau)\theta^2[0,\beta+1/2](0|\tau). \tag{3.4}$$

Note that both terms in this sum have the same functional form $(\theta^2[0,1/2](0|\tau)\theta^2[0,0](0|\tau))$. The only issue left is then the relative phase between them. The sum over β is simply the GSO projection on the states propagating around the b-cycle. Let us normalize $\eta_{0,0}$ to 1. Then $\eta_{0,1/2} = -1$. This follows from the fact that *in the NS sector* the GSO projection operator is $1-(-1)^F$. This encodes the fact that we must project onto odd fermion number in the superstring in order to project out the tachyon which would otherwise come from the vacuum at the $-1/2$ mass level. So our integrand is

$$(1-1)\theta^2[0,0]\theta^2[0,1/2] = 0. \tag{3.5}$$

3.2. (Non-)Superstring Perturbation Theory

This mechanism will carry over essentially to the 2-loop diagram of Figure 2. But first we must consider various subtleties arising in string loop computations for strings with worldsheet supersymmetry. (See for example [13,14,15] for reviews with some references.) Let us begin by briefly reviewing some of the issues. We will work in the RNS formulation; for discussion of the supersymmetric case in Green-Schwarz language see for example [16].

In performing the Polyakov path integral at genus h, we must integrate over all the worldsheet fields including the worldsheet metric $\hat{h}$ and gravitino χ. This infinite dimensional space is reduced to a finite dimensional space of (super-)moduli by dividing out the diffeomorphisms and local supersymmetry transformations. There are $3h-3$ complex bosonic moduli τ and $2h-2$ complex supermoduli ζ. At genus $h=2$ we can take the gravitino to have delta-function support on the worldsheet for even spin structures [17].

(For odd spin structures the amplitude vanishes due to zero modes.) The partition function, after taking into account the Jacobian for the change of variables from $\hat{h}$ and χ to τ and ζ and integrating over the odd supermoduli ζ, takes the form [18,13]

$$\sum_{\alpha,\beta,twists}\int |d\tau|^{6h-6}[dX][dB][dC]e^{-S}(\hat{\eta},b)^{6h-6}\xi(x_0)\prod_{a=1}^{2h-1}:e^{\phi}T_F(z_a):\prod_{a=2h-2}^{4h-4}:e^{\bar{\phi}}\bar{T}_F(z_a): \tag{3.6}$$

Here $\hat{\eta}$ is a Beltrami differential and b,c are the spin-(2,-1) conformal ghosts. The superconformal ghosts $\beta=\partial\xi e^{-\phi}$, $\gamma=\eta e^{\phi}$ are defined in terms of spin-0 and spin -1 fermions ξ,η and a scalar ϕ [10]. The spin-0 fermion ξ has a zero mode on the surface which is absorbed by the insertion of $\xi(x_0)$ in (3.6). There is an anomaly in the ghost number $U(1)$ current which requires insertions of operators with total ghost number $2h-2$ to get a nonvanishing result. The insertions of $T_F(z_a)$ arise due to the coupling of the gravitino χ to the worldsheet supercurrent T_F. As explained in [10,18,13] the quantum operators $:e^{\phi}T_F(z_a):$ are given by picture-changing operators $c\partial\xi+e^{\phi}\psi^{\mu}\partial X^{\mu}-\frac{1}{4}\partial\eta e^{2\phi}b-\frac{1}{4}\partial(\eta e^{2\phi}b)$. In writing (3.6), we have already assumed a gauge choice for the gravitino which satisfies $\partial\bar{\chi}/\partial\tau=0$ [13]. We will make such a gauge choice below. As noted in [18,19,13], changing the positions z_a of these insertions changes the amplitude by a total derivative on the moduli space. We will consider the relevant potential boundary contributions later.

In [11,20,18,21,22,23] the relevant determinants and correlation functions were computed and assembled into an explicit expression for the 2-loop partition function. We are interested in the spin-structure-dependent factors in each term of this expression. In addition we must adjust the theta function characteristics and phases to reflect the f and g twists we have on our diagram. Let us consider first the terms in $\langle :e^{\phi}T_F(z_1)::e^{\phi}T_F(z_2):\rangle$ which come from the matter part $\psi_{\mu}\partial X^{\mu}$ of the supercurrent T_F. There are two types of terms here depending on whether we are contracting components $\mu=1\ldots4$ or $\mu=5\ldots10$ in the correlation function. In the first case we get

$$\sum_{\alpha,\beta}\eta_{\alpha,\beta}\frac{\theta^3[\alpha,\beta](0|\tau)\theta[\alpha,\beta+(1/2,0)](0|\tau)\theta[\alpha,\beta+(1/2,0)](z_1-z_2|\tau)}{\theta[\alpha,\beta](z_1+z_2-2\Delta_\delta|\tau)} \tag{3.7}$$

where Δ_δ is the divisor class of a reference spin structure δ (whose choice does not affect the final answer [20]). Here we have used the standard notation in which we write the argument of the theta function in terms of a degree-zero divisor class $\sum_k p_k-\sum_k q_k$ which maps to an element of the Jacobian torus $\mathbb{C}^{\mathbf{h}}/(\mathbb{Z}^{\mathbf{h}}+\tau\mathbb{Z}^{\mathbf{h}})$ via the map $\sum_k p_k-$

$\sum_k q_k \to \sum_k \int_{q_k}^{p_k} \omega_i$. The first factor of θ^3 comes from the determinant for the RNS fermions ψ_L^{5-10} and the remaining factors in the numerator come from the RNS fermions ψ_L^{1-4}. The denominator comes from the superconformal ghost determinant. Similarly the terms involving correlators of the form $<: e^\phi \psi^{5-10} \partial X^{5-10} :: e^\phi \bar\psi^{5-10} \partial \bar X^{5-10} :>$ are

$$\sum_{\alpha,\beta} \eta_{\alpha,\beta} \frac{\theta^2[\alpha,\beta+(1/2,0)](0|\tau)\theta^2[\alpha,\beta](0|\tau)\theta[\alpha,\beta](z_1-z_2|\tau)}{\theta[\alpha,\beta](z_1+z_2-2\Delta_\delta|\tau)} \tag{3.8}$$

We will now fix the gauge for the gravitinos by making a definite choice of points $z_{1,2}$. As explained in [13], the choice of points must be taken in such a way that the gauge slice chosen is transverse to the gauge transformations. It must also respect modular invariance of the amplitude [18,13]. Let us choose δ to correspond to an odd spin structure and take

$$z_1 = z_2 = \Delta_\delta. \tag{3.9}$$

As explained in [13], this choice (which amounts to putting the insertions at one of the branch points in a hyperelliptic description of the surface) satisfies transversality. It was argued in [23,24] that despite earlier worries [13], this choice is also consistent with modular invariance. The modular invariance is not manifest in the description in terms of θ-functions, as the calculation of correlation functions on the Riemann surface [20] involve a choice of reference spin structure δ. Having to choose a spin structure naively appears to violate modular invariance. Had we chosen a different reference spin structure δ', we would have shifted the arguments of our theta functions by elements $n + m\tau$ of the Jacobian lattice. Such a shift introduces a τ-dependent phase multiplying the θ-function–the θ-functions transform as sections of line bundles over the Jacobian torus. These phases must cancel out of the properly defined integrand, and in [23] this was demonstrated explicitly for certain (nonvanishing) 2-loop contributions.

In an orbifold model, one can consider separately different twist structures, and analyze the fundamental domain of the modular group that preserves a given twist structure. In general there are an infinite number of contributions coming from different choices of bosonic shifts. In §4 we will analyze the twist structure of Figure 2 (with no additional bosonic shifts) and see that the resulting modular group acts freely on τ. In this situation, the choice of a branch point for $z_{1,2}$ is manifestly modular invariant; the possible obstruction to modular invariance discussed in [18,13] does not arise, as there are no orbifold points in the moduli space. We also analyze in §4 the boundary contributions and see that they vanish. Instead of explicitly analyzing the contributions with arbitrary additional bosonic shifts, we will simply use the results of [23,24] to argue that the choice (3.9) always satisfies modular invariance once we sum over all twist structures.

Making the choice (3.9), we obtain for both types of contribution above

$$\sum \eta_{\alpha\beta}\theta^2[\alpha,\beta](0|\tau)\theta^2[\alpha,\beta+(1/2,0)](0|\tau) \tag{3.10}$$

Let us consider first the terms in the spin structure sum with $\alpha_1 = 1/2$. This describes left-moving Ramond-sector states propagating around handle 1 of the diagram. For all such terms there is some set of left-moving fermions with a zero mode. This is easiest to see by using the fact that $\theta[\alpha,\beta](z|\tau)$ is even or odd in z if $4\alpha\cdot\beta$ is even or odd, respectively. Here since

$$4\alpha\cdot\beta - 4\alpha\cdot[\beta+(1/2,0)]\Big|_{\alpha_1=1/2} = -1 \tag{3.11}$$

we see that either $\theta[\alpha,\beta]$ or $\theta[\alpha,\beta+(1/2,0)]$ is odd for every $\alpha_1 = 1/2$ term in the sum. This means these terms vanish due to a fermion zero mode.

Now let us consider the terms with $\alpha_1 = 0$. These describe NS states propagating around handle 1 in the diagram. Let us fix α_2, β_2 and sum over β_1. This amounts to performing the GSO projection on handle 1. This gives us two terms of the same functional form:

$$\sum_{\alpha_2,\beta_2} \eta_{(0,\alpha_2;0,\beta_2)}(1-1)\theta^2[0,\alpha_2;0,\beta_2]\theta^2[0,\alpha_2;1/2,\beta_2] = 0 \tag{3.12}$$

The relative phase of (-1) here again comes from the fact that in the NS sector, the GSO projection operator is $(1-(-1)^F)$.

We must now consider the terms in the correlator of picture-changing operators arising from the ghost part of the worldsheet supercurrent. These were computed in [18,22,23]. We must adjust their expressions to take into account our particular orbifold twist.

These terms in the partition function are given by determinants of matter fermions times the ghost piece

$$\langle \xi(x_0)c\partial\xi(z_1)\big(-\frac{1}{4}\partial\eta e^{2\phi}b - \frac{1}{4}\partial(\eta e^{2\phi}b)\big)(z_2)\rangle \tag{3.13}$$

As mentioned before, the factor $\xi(x_0)$ soaks up the ξ zero mode. Therefore we should find that the amplitude is independent of x_0. This we will see explicitly following the analysis of [23]. In general we need the correlator [20]

$$\langle \prod_{i=0}^{n}\xi(x_i)\prod_{j=1}^{n}\eta(y_j)\prod_k e^{q_k\phi(w_k)}\rangle = \frac{\prod_{j=1}^{n} Z_{\alpha,\beta;3/2}(-y_j+\sum x - \sum y + \sum q_k w_k)}{\prod_{i=0}^{n} Z_{\alpha,\beta;3/2}(-x_i+\sum x - \sum y + \sum q_k w_k)} \tag{3.14}$$

where

$$Z_{\alpha,\beta;3/2}(\sum A_r z_r) = Z_1^{\frac{1}{2}}\theta[\alpha,\beta](\sum A_r z_r - 2\Delta)\prod_{i<j} E(z_i,z_j)^{A_i A_j}\prod_i \sigma(z_i)^{2A}. \tag{3.15}$$

Here Z_1 is the partition function of a chiral scalar, $E(z_i, z_j)$ is the prime form [25], and $\sigma(z)$ is a holomorphic $h/2$-differential defined in [20]. Also Δ is the Riemann class [20], a divisor for the spin structure $[0,0]$ (2Δ is the canonical class, as for the divisor class of any spin bundle–we will go back to writing our expressions in terms of the reference odd spin structure δ below).

Let us first consider the first term in (3.13). The spin-structure dependent piece of our amplitude is

$$\theta^3[\alpha,\beta](0)\theta^2[\alpha,\beta+(1/2,0)](0)\frac{\partial}{\partial y}\left(\frac{\partial}{\partial z_1}\langle\xi(x_0)\xi(z_1)\eta(y)e^{2\phi(z_2)}\rangle\right)\bigg|_{y=z_2} \tag{3.16}$$

The first two factors come from the determinants from the RNS fermions. This in turn is equal to (using (3.14))

$$\theta^3[\alpha,\beta](0)\theta^2[\alpha,\beta+(1/2,0)](0)\frac{\partial}{\partial y}\left(\left[\frac{\partial}{\partial z_1}\frac{Z_{\alpha,\beta,3/2}(-2y+x_0+z_1+2z_2)}{Z_{\alpha,\beta,3/2}(z_1-y+2z_2)}\right]\frac{1}{Z_{\alpha,\beta,3/2}(x_0-y+2z_2)}\right) \tag{3.17}$$

Following [23] we can now use a corollary of the trisecant identity (eq (D.4) of [23])

$$\partial_d\frac{Z_{\alpha,\beta;3/2}(d-b+D)}{Z_{\alpha,\beta;3/2}(d-a+D)} = \frac{Z_{\alpha,\beta;3/2}(D)Z_{\alpha,\beta;3/2}(2d-a-b+D)}{Z^2_{\alpha,\beta;3/2}(d-a+D)} \tag{3.18}$$

for any divisor D of degree 2.

Using this identity we can rewrite our amplitude (3.17) as

$$\theta^3[\alpha,\beta](0)\theta^2[\alpha,\beta+(1/2,0)](0)\frac{\partial}{\partial y}\left(\frac{Z_{\alpha,\beta;3/2}(x_0+2z_2-y)Z_{\alpha,\beta;3/2}(2z_1-2y+2z_2)}{Z^2_{\alpha,\beta;3/2}(z_1+2z_2-y)Z_{\alpha,\beta;3/2}(x_0-y+2z_2)}\right) \tag{3.19}$$

Now we see, as in the supersymmetric case, that the x_0-dependent pieces cancel and the contribution is proportional to

$$\theta^3[\alpha,\beta](0)\theta^2[\alpha,\beta+(1/2,0)](0)\frac{\partial}{\partial y}\left(\frac{Z_{\alpha,\beta;3/2}(2z_1-2y+2z_2)}{Z^2_{\alpha,\beta;3/2}(z_1+2z_2-y)}\right) \tag{3.20}$$

Depending on whether the derivative $\frac{\partial}{\partial y}$ acts on the theta functions or on the spin-structure-independent factors in (3.15), we have terms of the form

$$\theta^3[\alpha,\beta](0)\theta^2[\alpha,\beta+(1/2,0)](0)\frac{\theta[\alpha,\beta](2z_1-2y+2z_2-2\Delta_\delta)}{\theta^2[\alpha,\beta](z_1+2z_2-y-2\Delta_\delta)} \tag{3.21}$$

and terms of the form

$$\theta^3[\alpha,\beta](0)\theta^2[\alpha,\beta+(1/2,0)](0)\frac{\partial}{\partial y}\frac{\theta[\alpha,\beta](2x_1-2y+2z_2-2\Delta_\delta)}{\theta^2[\alpha,\beta](x_1+2z_2-y-2\Delta_\delta)} \tag{3.22}$$

Now making our gauge choice (3.9) $z_1=y=z_2=\Delta_\delta$, the contributions (3.21) become

$$\theta^2[\alpha,\beta](0)\theta^2[\alpha,\beta+(1/2,0)](0). \tag{3.23}$$

This is exactly the form we had for the matter correlators (3.10) and the sum vanishes in the same way. The contributions (3.22)become

$$\theta[\alpha,\beta](0)(\omega_i\partial_i\theta[\alpha,\beta](0))\theta^2[\alpha,\beta+(1/2,0)](0). \tag{3.24}$$

These contributions simplify as follows. If the spin structure $[\alpha,\beta]$ is odd, then the product of theta functions multiplying the derivative vanishes. If the spin structure $[\alpha,\beta]$ is even on the other hand, then the derivative $\partial_i\Big|_0 \theta[\alpha,\beta]$ vanishes.

The second term in (3.13) leads in the same way to the same two types of contributions. (In particular, there is always a derivative acting on $\xi(z_1)$ and that leads through (3.18) to the removal of the x_0-dependence as above; the remaining derivative either acts on the theta function, giving the result (3.24), or it acts on the spin-structure-independent factors, giving the result (3.23)).

So we see that despite the fact that this diagram involves twists by elements which together break all the supersymmetry, its contribution to the vacuum energy cancels. We will see in §5 that we can find a modular transformation that turns any other (supersymmetry breaking) twist structure into this one up to shifts on bosonic elements, which do not affect the spin-structure-dependent sums. In the next section, we address the question of the boundaries of the relevant modular domain of integration.

4. Boundary Contributions

In the previous section, we studied the two loop diagram with twists by f and g going around the $b_{1,2}$ cycles, i.e. with twist structure $(1,1,f,g)$. We saw that the computation yields a *vanishing integrand* if we make a very specific choice of insertion points for the picture-changing operators $:e^{\phi}T_F:$. Since the answer should be independent of the choice of these insertion points, this seems to imply that the two loop vacuum energy vanishes.

However, under a change of the choice of insertion points, it can be shown that the computation changes by a total derivative [18,19]

$$\int_{\mathcal{F}} \partial\omega \tag{4.1}$$

where $\mathcal{F}$ is the appropriate fundamental domain of integration for the computation. Therefore, one must worry about contributions arising at the boundary of $\mathcal{F}$ [13].

4.1. The Fundamental Domain

What is the fundamental domain $\mathcal{F}$ for this computation? At genus two, the Teichmuller space is given very explicitly in terms of the Siegel upper half space of 2×2 matrices:

$$\mathcal{H}_2 = \{\tau_{2\times 2} : \quad \tau^{tr} \ = \ \tau, \ \ \mathrm{Im}\ \tau \ > \ 0\}$$

τ is the period matrix of the genus two surface. The modular group at genus two is $G = Sp(4, \mathbb{Z})$. The moduli space can then be constructed by taking the quotient of $\mathcal{H}_2$ by G. One must also remove the modular orbit of the diagonal matrices.

For our computation, on the other hand, we have twists $(1, 1, f, g)$ about the (a_1, a_2, b_1, b_2) cycles of the surface. Therefore, we need to integrate the correlator of the picture changing operators over $\mathcal{F} = \mathcal{H}_2/\tilde{G}$, where $\tilde{G}$ is the subgroup of $Sp(4, \mathbb{Z})$ which preserves the twist structure $(1, 1, f, g)$.

It is easy to see that the allowed matrices are the ones that act on the homology (a_1, a_2, b_1, b_2) like

$$\begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ x & y & 1 & 0 \\ z & w & 0 & 1 \end{pmatrix} \tag{4.2}$$

Denoting the 2×2 blocks as $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ we must impose

$$A^{tr}C = C^{tr}A, \quad B^{tr}D = D^{tr}B, \quad A^{tr}D - C^{tr}B = 1 \tag{4.3}$$

which is just the requirement that (4.2) is in $Sp(4, \mathbb{Z})$. This further restricts the allowed matrices (4.2) to be of the form

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ x & y & 1 & 0 \\ y & w & 0 & 1 \end{pmatrix} \tag{4.4}$$

Now, if $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ acts on the homology, then the action on the period matrix τ is given by $\begin{pmatrix} D & C \\ B & A \end{pmatrix}$ – in other words,

$$\tau \rightarrow (D\tau \ + \ C)\ (B\tau \ + \ A)^{-1} \tag{4.5}$$

So from the allowed actions on the homology (4.4), we see that the identifications to be made on the period matrices are

$$\begin{pmatrix} \tau_1 & \tau_{12} \\ \tau_{12} & \tau_2 \end{pmatrix} \rightarrow \begin{pmatrix} \tau_1 + x & \tau_{12} + y \\ \tau_{12} + y & \tau_2 + w \end{pmatrix} \tag{4.6}$$

In addition, positivity of Im τ requires that

$$\text{Im } \tau_{1,2} > 0, \quad (\text{Im } \tau_{12})^2 < \text{Im } \tau_1 \text{ Im } \tau_2 \tag{4.7}$$

The constraints (4.6) and (4.7) together yield the correct fundamental domain $\mathcal{F} \subset \mathcal{H}_2$ for our computation. $\tau_{1,2}$ live on strips with real part between $(-1/2, 1/2)$ and positive imaginary part, while τ_{12} has real part between $(-1/2, 1/2)$ and imaginary part bounded above and below by the second inequality in (4.7). Also, we must recall that in describing the moduli space of Riemann surfaces in terms of $\mathcal{H}_2$, we had to delete the modular orbit of diagonal matrices, yielding an additional boundary at $\tau_{12} \ \rightarrow \ 0$.

4.2. The Boundaries

Now that we have determined $\mathcal{F}$, we can look for boundaries where the total derivative (4.1) might give a contribution after integration by parts. There are in fact three boundaries in $\mathcal{F}$. We will examine each of these boundaries in turn, and argue that no boundary contribution exists.

1) τ_1 or $\tau_2 \ \rightarrow \ i\infty$

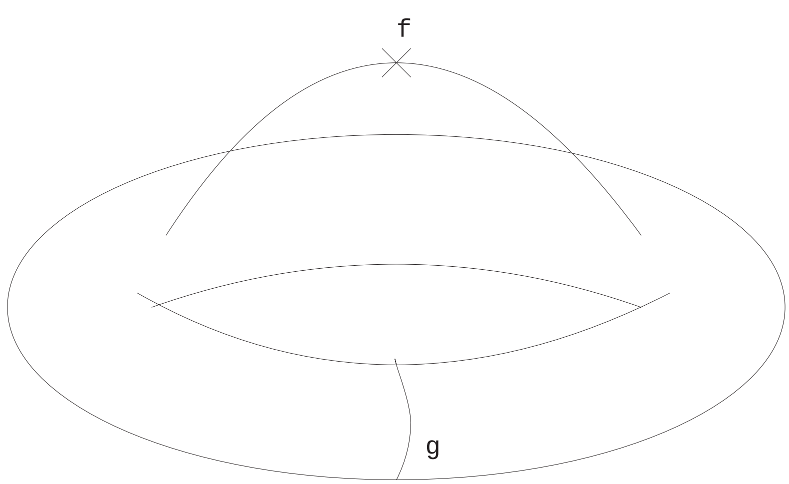

Figure 4: Picture of boundary 1).

In this limit, one of the handles degenerates to a semi-circle glued on to the “fat” handle at two points (i.e. a homology cycle collapses). It was argued in [13] that in such a limit, no boundary contribution exists in theories without physical tachyons. Our theory has no physical tachyons, so we will receive no contribution from this boundary.

2) $\tau_{12} \ \rightarrow \ 0$

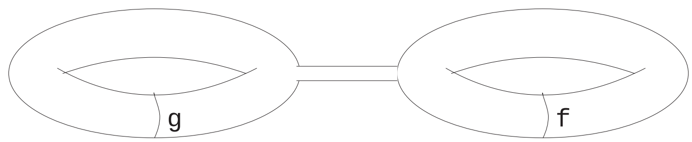

Figure 5: Picture of boundary 2).

In this limit, the genus two surface degenerates into two tori connected by a very long, thin tube. Only massless physical states propagate in this tube [13], and in this limit the genus two vacuum amplitude is related to a sum of products of one loop tadpoles for the massless states.

The relevant one loop tadpoles are computed on tori with twists $(1,f)$ or $(1,g)$ around the (a,b) cycles. Now, the f and g twist alone preserve $d=4$, $\mathcal{N}=2$ supersymmetry. So, there are no one loop tadpoles for states in the f or g twisted theory. This implies that the genus two diagram vanishes in this limit.

3) Im $\tau_{1,2} \ \rightarrow \ 0$ or $(\text{Im } \tau_{12})^2 \ \rightarrow \ (\text{Im } \tau_1)\ (\text{Im } \tau_2)$

To see the vanishing in this limit, we recall that the integrand for the vacuum amplitude contains a factor of $e^{-S(X)}$, i.e. the action for map from the genus two surface to spacetime. The relevant maps (given the f and g twists about the b cycles of the surface) wind around the X_5 and X_6 directions of spacetime. This yields a contribution to the action which goes like

$$S \simeq \frac{R^2}{\alpha'} \{ \frac{\text{Im } \tau_1 + \text{Im } \tau_2}{\text{Im } \tau_1 \ \text{Im } \tau_2 \ - \ (\text{Im } \tau_{12})^2} \} \tag{4.8}$$

where R is the radius of the X_5 and X_6 circles [26,27]. Now, positivity of Im τ comes to the rescue:

• If Im $\tau_1 \ \rightarrow \ 0$ at fixed Im τ_2, then the second inequality in (4.7) implies that $S \rightarrow \infty$.

• If Im $\tau_{1,2} \ \rightarrow \ 0$, one can prove that the denominator in (4.8) vanishes as the square of the numerator (once again using positivity of Im τ), so $S \rightarrow \infty$.

• If Im $\tau_{1,2}$ are fixed and $(\text{Im } \tau_{12})^2$ approaches Im τ_1 Im τ_2, it is obvious that the action diverges.

The upshot is that the $e^{-S(X)}$ in the integrand vanishes quickly enough at this boundary to rule out any contributions.

4.3. Cases with Shifts

In additional to the amplitude $(1,1,f,g)$ which knows about the supersymmetry breaking at genus two, there are other genus 2 amplitudes with $(1,1,f,g)$ twists on the worldsheet fermions around the (a_1,a_2,b_1,b_2) cycles but with additional shifts acting on the bosonic fields. In fact we show in §5 that this is (up to modular transformations) the full set of supersymmetry breaking diagrams that we need to consider at genus two.

If we wanted to analyze, case by case, the various consistent choices of additional shifts at genus two, then we would need to change the analysis of the fundamental domain and possible boundary contributions in each case. This would be rather cumbersome. Instead, we use the results of [23,24] as follows. They argue that we can make the choice of a branch point for the insertion of the picture-changing operators even if we need to mod out by the full modular group $Sp(4,\mathbb{Z})$. Thus, for each choice of shifts we can make this choice of $z_{1,2}$.

Having done this, we will again find that the spin-structure dependent part of the vacuum amplitude vanishes. This will leave the issue of possible boundary contributions. On the other hand, after summing over the various twist structures, we know the genus two vacuum energy can be written as an integral over $\mathcal{M}_2$, the moduli space of genus two Riemann surfaces. The possible boundary contributions (after we compactify $\mathcal{M}_2$) will come from boundaries of type 1) and 2) in §4.2 (where a handle collapses or the surface degenerates into two surfaces of lower genus connected by a long, thin tube). Hence, if we can argue that with arbitrary twist and shift structures on the a,b cycles the vacuum amplitude vanishes at boundaries of type 1) and 2), we will be done.

As we will discuss in §5, up to additional shifts on various cycles the possible structures (which break all of the supersymmetry) are basically $(1,1,f,g)$, (f,g,g,f) and (f,fg,fg,f) (up to possible exchanges of the role of f and g). Since we could use modular transformation to relate these to $(1,1,f,g)$ twist structure on the fermions, the spin-structure dependent piece of the amplitude vanishes in each of these cases. In addition, each of these vanishes at boundaries of type 1) because there are no physical tachyons. This leaves the analysis of boundary 2).

Any amplitude with $(1,1,f,g)$ twists on the fermions, regardless of additional shifts, vanishes at boundary 2) because it can be written as a product of tadpoles in the $\mathcal{N}=2$ supersymmetric f and g orbifolds (as in §4.2). On the other hand, the amplitude with twist (f,g,g,f) would naively yield a product of one loop tadpoles in a nonsupersymmetric theory. However, it turns out that the state propagating on the tube between the first and second handle must be a massive state because it must be twisted to be emitted from the "subtorus" with (f,g) twist on its (a,b) cycles. Since only massive states can run in the tube, there is no contribution at the boundary of moduli space (where the tube becomes infinitely long). A similar discussion applies to the (f,fg,fg,f) twist structure with arbitrary shifts.

5. Twists at Genus $h \geq 2$

A priori on a genus h Riemann surface, one needs to consider any combination of twists on the various cycles a_i, b_i for $i = 1, \cdots, h$ consistent with the relation

$$a_1 b_1 a_1^{-1} b_1^{-1} \cdots a_h b_h a_h^{-1} b_h^{-1} \;=\; 1 \tag{5.1}$$

In this section, we will argue that in fact using modular transformations one can greatly reduce the kinds of twist structures that one needs to consider.

For our considerations, we do not need to worry about twists that preserve some of the spacetime supersymmetry at genus h (for instance, twists only by f around various cycles). The real concern will be sets of twists around different cycles which break the full spacetime supersymmetry. We will now show that, up to inducing shifts on the worldsheet bosons around some cycles, one *only* has to consider f and g twists on the b_{h-1} and b_h cycles with no twists on any other cycles. Any twist which breaks all of the spacetime supersymmetry can be brought to this canonical $(1, \cdots 1, f, g)$ form by modular transformations.

Since in this section we will be ignoring the possible shifts on bosons around various cycles (we're only interested in the f,g action on fermions), we can use relations like

$$f^2 \;=\; g^2 \;=\; 1, \quad fg = gf \tag{5.2}$$

which are true for the action on fermions (but only true in the full model up to shifts in the space group).

5.1. Genus $h = 2$

We will show that all twists of interest can be taken to the $(1, \cdots, f, g)$ form in several steps. First, consider genus two surfaces. The modular group $Sp(4, \mathbb{Z})$ is generated by

$$D_{a_1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad D_{a_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \tag{5.3}$$

$$D_{b_1} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad D_{b_2} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5.4}$$

$$D_{a_1^{-1}a_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{pmatrix} \tag{5.5}$$

which are simply the Dehn twists about the various cycles of the genus two surface, acting on the homology (a_1, a_2, b_1, b_2).[2]

We now consider genus $h = 2$ twists which are not of the canonical $(1, 1, f, g)$ form but which break all the supersymmetry:

1) First, take the cases where no "subtorus" has twists which break the full supersymmetry (i.e., no f, g twists on dual (a, b) cycles). Then, by using $SL(2, \mathbb{Z}) \subset Sp(4, \mathbb{Z})$ transformations which act on the (a_1, b_1) and (a_2, b_2) cycles, one can arrange to have twists only on the b cycles, so the twist structure is $(1, 1, *, *)$. Then the only cases we need to worry about are $(1, 1, f, fg)$ and $(1, 1, g, fg)$. One can easily see that $(1, 1, f, fg)$ is mapped by D_{b_1} to $(f, 1, f, fg)$ and then by $D_{a_1^{-1}a_2}$ to $(f, 1, 1, g)$. That in turn is $SL(2, \mathbb{Z})$ equivalent to $(1, 1, f, g)$. A similar manipulation works for the $(1, 1, g, fg)$ case.

2) Second, consider the case where there *are* twists on some "subtorus" that break the full supersymmetry. Examples are (f, g, g, f) and (f, fg, fg, f). Now, for instance, (f, g, g, f) can be mapped by $D_{a_1^{-1}a_2}$ to (f, g, f, g) which is equivalent (using $SL(2, \mathbb{Z})$ transformations on both subtori) to $(1, 1, f, g)$. One can similarly reduce (f, fg, fg, f) and other analogous

[2] There are also inhomogeneous terms that shift the characteristics of the theta functions coming from the fermion determinants under such a modular transformation; these lead to a change of spin structure but do not change the orbifold twist structure.

structures to the canonical form. (Recall that in this discussion we are ignoring extra bosonic shifts that make the twist structures considered here consistent with (5.1)).

So, we find that *all* supersymmetry breaking twists at genus $h = 2$ can be mapped by the modular group to $(1, 1, f, g)$ (up to shifts on worldsheet bosons). This is important because our vanishing at $h = 2$ was for the spin structure dependent part of precisely this twist structure, and is independent of any shifts on worldsheet bosons.

5.2. Genus $h > 2$

We now argue that at arbitrary genus, one can reduce all supersymmetry breaking twist structures to $(1, \cdots, 1, f, g)$ using modular transformations. We will need to use three important facts:

1) Among the elements of $Sp(2h, \mathbb{Z})$ there are matrices that allow one to permute the different "subtori" (sets of conjugate a, b cycles) of the genus h surface.

2) In order to satisfy (5.1), there must exist an *even* number of "subtori" with twists on the (a_i, b_i) cycles that break all the supersymmetry.

3) Using $Sp(4, \mathbb{Z}) \subset Sp(2h, \mathbb{Z})$ one can map

$$(1, 1, f, f) \;\rightarrow\; (1, 1, f, 1) \tag{5.6}$$

i.e. one can group like twists on neighboring b cycles onto a single b cycle.

Putting together our $h = 2$ result with facts 1)-3) above, we see that at genus $h > 2$ the only twist structure we need to consider is $(1, 1, \cdots, 1, f, g)$. To prove this, we simply work on genus 2 subsurfaces (using $Sp(4, \mathbb{Z})$ subgroups of the modular group) to reduce everything to f or g twists on b cycles, and then use 1) and 3) to simplify to a single f and g twist.

6. Comments on Higher Loop Vanishing

Once we have put the twists on our genus h surface Σ_h into the canonical $(1, \cdots, 1, f, g)$ form, we can provide rough physical arguments for the vanishing. This section is very heuristic; it would be nice to make these arguments more precise.

The first argument involves supersymmetry. One can think of Σ_h in terms of a genus $h-1$ surface Σ_{h-1} (with a g projection on one cycle) connected to an extra handle (holding the f projection) by a nondegenerate tube (on which massive or massless string states may propagate).

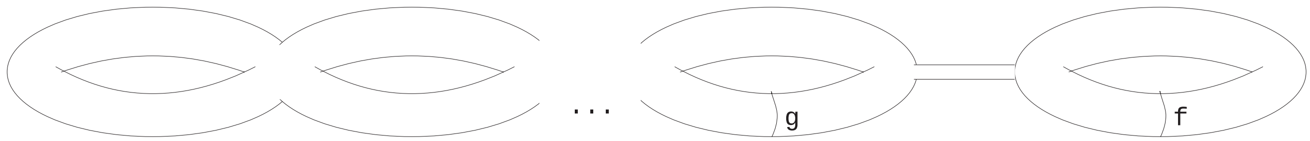

Figure 6: One sums over states in the channel between the f and g twisted handles.

This suggests that one rewrite the diagram as

$$\Sigma_{\rm s}\ (\text{s tadpole on } \Sigma_{\rm h-1})\ \times e^{-M_s T}\ \times (\text{s tadpole on f} - \text{projected handle}) \qquad (6.1)$$

where the sum runs over possible intermediate physical states s of mass M_s, and the tube has length T. In this way of thinking about it, the diagram vanishes because even for massive string states, the tadpoles at genus $h-1$ in the g projected theory and at genus one in the f projected theory should vanish (as those theories are both 4d $\mathcal{N}=2$ supersymmetric).

Another way of thinking about the same diagram suggests a cancellation not by using supersymmetry of lower-genus subdiagrams (as above), but by using the symmetry of the g projected theory under f. Consider the f projected loop as a loop coming off of a diagram in the g projected theory. The loop can be replaced by a trace over two point functions of the states propagating in the loop, multiplied by their f quantum number. Now the f quantum number is $+1$ for f-invariant and -1 for f anti-invariant states, so one replaces the h loop graph with

$$\Sigma_s\ f_s \times \langle V_s(z)V_s(z')\rangle_{h-1} \qquad (6.2)$$

where s runs over states propagating in the loop and V_s is the appropriate vertex operator; the correlation function is evaluated in the g orbifold at genus $h-1$. Now it turns out that there are an *equal* number of $f=\pm1$ physical states (among bosons and fermions separately) at all mass levels in the g orbifold. Thus, one can hope that they give equal and opposite contributions to the sum over two-point functions (6.2). In fact, one can study this explicitly for e.g. massless physical states in the g projected theory (whose OPEs have this symmetry). In this way of looking at things, higher genus vanishing might be a consequence of the very symmetric nature of the spectrum of the theory.

6.1. Toward a (perturbative) symmetry argument

The worldsheet arguments for the perturbative vanishing of the cosmological constant in supersymmetric string theories used contour deformations of the spacetime supercurrents crucially [28,18,14]. Thus one could see the constraint of spacetime supersymmetry through worldsheet current algebra.

In our theory, of course, the spacetime supercurrents are projected out, which is to say that they have monodromy around the twist fields in the orbifold. On the Riemann surface with a given twist structure (as in Figure 6), these operators pick up phases upon traversing the cuts in the diagram.

Let us consider the argument of [28] in this context. One splits open one handle of the surface (say the handle with the f-cut), and rewrites the propagating state V as $\oint_{a_f} S_{+-}V'$ for some operator V'. (So in particular V' describes a boson if the original state was fermionic in spacetime and vice versa). Here S_{+-} refers to a would-be spacetime supercurrent with eigenvalues $+1$ under f and -1 under g. The cycle a_f is the a-cycle on the handle with the f cut. Without the g cut, one can deform the contour integral around the rest of the Riemann surface and turn the fermion loop into a boson loop (or vice versa) with a cancelling sign. With the g-cut, however, one is left with a remainder contribution of the form

$$2\oint_{a_f} dx \oint_{a_g} dy \; \langle S_{+-}(x)S_{+-}(y)\rangle \tag{6.3}$$

The direct calculation of this contribution could be done at genus 2 in a similar way to the partition function calculation we presented in the previous sections (using the correlation functions of [20]). As before it would be hard to then generalize this computation to higher genus precisely, due to our lack of explicit understanding of the moduli space (and the problem of choosing a consistent gauge slice for the worldsheet gravitino). It would be nice to understand if there is some simple topological reason that this remainder must vanish at arbitrary genus.

7. Relation to AdS/CFT Correspondence

There has been a great deal of recent work on the fascinating conjectures relating conformal field theories in various dimensions to string theory in Anti de Sitter backgrounds

[29,30].[3] It was argued in [2] that certain nonsupersymmetric instances of this correspondence could lead to the discovery of nonsupersymmetric string backgrounds with vanishing cosmological constant. The predictions of new fixed lines at the level of the leading large-N theory based on the correspondence [2] were verified directly through remarkable cancellations in perturbative diagrams [31] in a host of models that could be constructed quite systematically [32]. Here, we review and elaborate on the idea of extending predictions of the correspondence to finite-N fixed lines, and explain how the class of orbifolds we have discussed in this paper may be realizations.

The correspondences between 4d CFTs and string backgrounds have (in the 't Hooft limit)

$$\frac{\alpha'}{R^2} \text{ expansion } \rightarrow \text{ expansion in } (g_{YM}^2 N)^{-\frac{1}{2}} \tag{7.1}$$

$$g_{st} \text{ expansion} \rightarrow \text{ expansion in } g_{YM}^2 \;=\; \frac{1}{N} \tag{7.2}$$

In cases where one has a *nonsupersymmetric* fixed line (to all orders in $\frac{1}{N}$ as well as $g_{YM}^2 N$) realized on branes in string theory, we would obtain by this correspondence a stable nonsupersymmetric string vacuum which exists at arbitrary values of the coupling g_{st}.

The equation of motion for the dilaton $g_{st} = e^{\phi}$ is

$$(-g)^{\frac{1}{2}} \partial_\mu (\sqrt{(-g)} g^{\mu\nu} \partial_\nu \phi) \;=\; -\frac{\partial V}{\partial \phi} \tag{7.3}$$

where $V(\phi)$ is the dilaton potential and g is the AdS metric. So, stability of the spacetime background for arbitrary dilaton VEV $\langle\phi\rangle$ (which is implied by the existence of a nonsusy fixed line as above) would imply that there is no g_{st} dependent vacuum energy. This line of thinking also suggests that one should be able to find correspondences between isolated nonsupersymmetric conformal fixed points and stable isolated nonsupersymmetric string backgrounds.

Non-supersymmetric theories with vanishing β-function at leading order in $1/N$ but nonzero β-functions at subleading orders [2,32,31] should provide a concrete testing ground for ideas relating the holographic principle [33] to the cosmological constant [34]. In particular, one would generically expect perturbative contributions to the cosmological constant (dilaton tadpole), which are related (via a possibly nontrivial map) to beta functions in the

[3] Indeed, the subject has been as popular as the Chicago Bulls.

boundary field theory. In perturbative string theory these contributions come in generically at the order of the supersymmetry breaking scale, which is the string scale in these models. In general, the AdS/CFT correspondence relates perturbative string corrections to $1/N$ corrections in the boundary QFT. Therefore the perturbative string corrections should be encoded in the boundary theory in a way consistent with the holographic reduction in the number of degrees of freedom.

Dualities between field theory fixed lines (which exist to all orders in $1/N$) and non-supersymmetric string backgrounds would have different consequences in the different AdS_d dualities. In the orbifolds $AdS_5 \times (S^5/,\ )$, the duality would imply that the effective $ten-dimensional$ cosmological constant vanishes. In the large $g_{YM}^2 N$ limit, we expect from (7.1) that the AdS and orbifolded sphere also each become flat. However, in this limit the spacetime theory regains supersymmetry away from the fixed loci of , , so this does not yield a nonsupersymmetric theory in the bulk of spacetime.

However, there are cases where the large N limit *could* yield a nonsupersymmetric theory in bulk with vanishing cosmological constant. Consider for instance dualities between IIB strings on $AdS_2 \times S^2 \times (T^6/,\ )$ and conformally invariant quantum mechanical systems (some supersymmetric instances of such dualities were conjectured in [29]). In these cases, going through the analogous arguments we would be talking about the effective $four-dimensional$ cosmological constant. In the large N limit, $AdS_2 \times S^2 \ \rightarrow\ \mathbb{R}^4$ while the size of the $T^6/,$ remains *fixed* (it does not decompactify). Thus, if we break supersymmetry on the internal space we might be able to find examples of the AdS/CFT correspondence which predict vanishing 4d cosmological constant in a bulk nonsupersymmetric theory. This provides a strong motivation for understanding conformally invariant quantum mechanical systems with "fixed lines" (corresponding to the spacetime g_{st}). In particular, at least naively a quantum mechanical model which is classically conformal will not develop a β-function since there are no ultraviolet divergences (though one may need to worry about IR problems).

We have two comments about trying to find models in this way via the AdS/CFT correspondence:

1) The $AdS_2 \times S^2$ geometries of interest arise as the near horizon limits of Reissner-Nordstrom black holes. In nonsupersymmetric situations, where π_1 of the compactification is typically small, it can be very difficult to find stable black holes of this sort by wrapping branes. In part this is because one often finds a 4d effective Lagrangian of the form

$$\mathcal{L} \;=\; \int d^4x \; \phi F_{\mu\nu}F^{\mu\nu} \;+\; \partial^\mu\phi\partial_\mu\phi \;+\cdots \tag{7.4}$$

where F is the field strength for the $U(1)$ gauge field under which the Reissner-Nordstrom black hole carries charge, and ϕ is some scalar field. Then, the equation of motion for ϕ becomes

$$\partial^\mu\partial_\mu\phi \;\sim\; F_{\mu\nu}F^{\mu\nu} \tag{7.5}$$

and this forces ϕ to have a nontrivial profile in the black hole solution which breaks the AdS isometry.

In order to get around problems of stability and of the existence of scalars with linear couplings to F^2, it is useful to start with models containing very few scalars. Asymmetric orbifolds are one natural source of such models. Starting with configurations of wrapped branes invariant under the orbifold group, one can obtain Reissner-Nordstrom black holes in asymmetric orbifolds such as the one we have studied here. One can then predict vanishing cosmological constant based on the conformally invariant family of quantum mechanical systems living on the boundary of the near-horizon geometry, as in the argument above. It is intriguing that this rather indirect argument relates the problem of fixing moduli to the cosmological constant problem. It would be nice to understand the constraints more systematically.

2) The orbifold we've been discussing not only has

$$\Lambda_{1-loop} \;=\; \Lambda_{2-loop} \;=\; \cdots \;=\; 0 \tag{7.6}$$

but also has

$$\Lambda_{1-loop} \;=\; \int d^2m_i \; 0, \;\; \Lambda_{2-loop} \;=\; \int d^6m_i \; 0, \;\cdots \tag{7.7}$$

That is, the vacuum amplitudes vanish *point by point* on the moduli space of Riemann surfaces. This vanishing integrand reflects the exceptionally simple spectrum of our theories (bose-fermi degeneracy, etc.).

In more general examples that might come out of nonsupersymmetric versions of the AdS/CFT correspondence as above, we would expect (7.6) to hold (since the conformal quantum mechanics has a fixed line, and the dilaton VEV is arbitrary). However, there is no reason to expect (7.7) to hold in general examples. It would be nice to find an example where e.g. Λ_{1-loop} vanishes but not point-by-point (as in Atkin-Lehner symmetry [35]).

It would be very interesting to find similar models with a more realistic low energy spectrum. In addition, we could potentially find non-supersymmetric models in 4d with no dilaton by finding 3d string models satisfying our conditions and taking $g_{st} \to \infty$.

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