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# QCD Phenomena and The Light-Cone Wavefunctions of Hadrons \*

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# QCD PHENOMENA AND THE LIGHT-CONE WAVEFUNCTIONS OF HADRONS

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Light-cone Fock-state wavefunctions encode the properties of a hadron in terms of its fundamental quark and gluon degrees of freedom. A recent experiment at Fermilab, E791, demonstrates that the color coherence and shape of light-cone wavefunctions in longitudinal momentum fraction can be directly measured by the high energy diffractive jet dissociation of hadrons on nuclei. Given the proton's light-cone wavefunctions, one can compute not only the quark and gluon distributions measured in deep inelastic lepton-proton scattering, but also the multi-parton correlations which control the distribution of particles in the proton fragmentation region and dynamical higher twist effects. First-principle predictions can be made for structure functions at small and large light-cone momentum fraction  $x$ . Light-cone wavefunctions also provide a systematic framework for evaluating exclusive hadronic matrix elements, including timelike heavy hadron decay amplitudes and form factors. In principle, light-cone wavefunctions can be computed in nonperturbative QCD by diagonalizing the light-cone Hamiltonian using the DLCQ method, as in dimensionally reduced collinear QCD.

## 1 Introduction

In a relativistic collision, the incident hadron projectile presents itself as an ensemble of coherent states containing various numbers of quark and gluon quanta. Thus when a laser beam crosses a proton at fixed “light-cone” time  $\tau = t + z/c = x^0 + x^z$ , it encounters a baryonic state with a given number of quarks, anti-quarks, and gluons in flight with  $n_q - n_{\bar{q}} = 3$ . The natural formalism for describing these hadronic components in QCD is the light-cone Fock representation obtained by quantizing the theory at fixed  $\tau$ .<sup>1</sup> For example, the proton state has the Fock expansion

$$\begin{aligned} |p\rangle &= \sum_n \langle n | p \rangle |n\rangle \\ &= \psi_{3q/p}^{(\Lambda)}(x_i, \vec{k}_{\perp i}, \lambda_i) |uud\rangle \\ &\quad + \psi_{3qg/p}^{(\Lambda)}(x_i, \vec{k}_{\perp i}, \lambda_i) |uudg\rangle + \dots \end{aligned} \tag{1}$$

representing the expansion of the exact QCD eigenstate on a non-interacting quark and gluon basis. The probability amplitude for each such  $n$ -particle state

of on-mass shell quarks and gluons in a hadron is given by a light-cone Fock state wavefunction  $\psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$ , where the constituents have longitudinal light-cone momentum fractions

$$x_i = \frac{k_i^+}{p^+} = \frac{k^0 + k_i^z}{p^0 + p^z}, \quad \sum_{i=1}^n x_i = 1, \quad (2)$$

relative transverse momentum

$$\vec{k}_{\perp i}, \quad \sum_{i=1}^n \vec{k}_{\perp i} = \vec{0}_{\perp}, \quad (3)$$

and helicities  $\lambda_i$ . The effective lifetime of each configuration in the laboratory frame is  $\frac{2P_{\text{lab}}}{\mathcal{M}_n^2 - M_p^2}$  where

$$\mathcal{M}_n^2 = \sum_{i=1}^n \frac{k_{\perp i}^2 + m^2}{x} < \Lambda^2 \quad (4)$$

is the off-shell invariant mass and  $\Lambda$  is a global ultraviolet regulator. The form of the  $\psi_{n/H}^{(\Lambda)}(x_i, \vec{k}_{\perp i}, \Lambda_c)$  is invariant under longitudinal boosts; *i.e.*, the light-cone wavefunctions expressed in the relative coordinates  $x_i$  and  $k_{\perp i}$  are independent of the total momentum  $P^+$ ,  $\vec{P}_{\perp}$  of the hadron.

Thus the interactions of the proton reflects an average over the interactions of its fluctuating states. For example, a valence state with small impact separation, and thus a small color dipole moment, would be expected to interact weakly in a hadronic or nuclear target reflecting its color transparency. The nucleus thus filters differentially different hadron components.<sup>2,3</sup> The ensemble  $\{\psi_{n/H}\}$  of such light-cone Fock wavefunctions is a key concept for hadronic physics, providing a conceptual basis for representing physical hadrons (and also nuclei) in terms of their fundamental quark and gluon degrees of freedom. Given the  $\psi_{n/H}^{(\Lambda)}$ , we can construct any spacelike electromagnetic or electroweak form factor from the diagonal overlap of the LC wavefunctions.<sup>4</sup> Similarly, the matrix elements of the currents that define quark and gluon structure functions can be computed from the integrated squares of the LC wavefunctions.<sup>5,6</sup>

It is thus important to not only compute the spectrum of hadrons and gluonic states, but also to determine the wavefunction of each QCD bound state in terms of its fundamental quark and gluon degrees of freedom. If we could obtain such nonperturbative solutions of QCD, then we could compute the quark and gluon structure functions and distribution amplitudes which control hard-scattering inclusive and exclusive reactions as well as calculate

the matrix elements of currents which underlie electroweak form factors and the weak decay amplitudes of the light and heavy hadrons. The light-cone wavefunctions also determine the multi-parton correlations which control the distribution of particles in the proton fragmentation region as well as dynamical higher twist effects. Thus one can analyze not only the deep inelastic structure functions but also the fragmentation of the spectator system. Knowledge of hadron wavefunctions would also open a window to a deeper understanding of the physics of QCD at the amplitude level, illuminating exotic effects of the theory such as color transparency, intrinsic heavy quark effects, hidden color, diffractive processes, and the QCD van der Waals interactions.

Solving a quantum field theory such as QCD is clearly not easy. However, highly nontrivial, one-space one-time relativistic quantum field theories which mimic many of the features of QCD, have already been completely solved using light-cone Hamiltonian methods.<sup>1</sup> Virtually any (1+1) quantum field theory can be solved using the method of Discretized Light-Cone-Quantization (DLCQ).<sup>7,8</sup> In DLCQ, the Hamiltonian  $H_{LC}$ , which can be constructed from the Lagrangian using light-cone time quantization, is completely diagonalized, in analogy to Heisenberg's solution of the eigenvalue problem in quantum mechanics. The quantum field theory problem is rendered discrete by imposing periodic or anti-periodic boundary conditions. The eigenvalues and eigensolutions of collinear QCD then give the complete spectrum of hadrons, nuclei, and gluonium and their respective light-cone wavefunctions. A beautiful example is "collinear" QCD: a variant of  $QCD(3+1)$  defined by dropping all of interaction terms in  $H_{LC}^{QCD}$  involving transverse momenta.<sup>9</sup> Even though this theory is effectively two-dimensional, the transversely-polarized degrees of freedom of the gluon field are retained as two scalar fields. Antonuccio and Dalley<sup>10</sup> have used DLCQ to solve this theory. The diagonalization of  $H_{LC}$  provides not only the complete bound and continuum spectrum of the collinear theory, including the gluonium states, but it also yields the complete ensemble of light-cone Fock state wavefunctions needed to construct quark and gluon structure functions for each bound state. Although the collinear theory is a drastic approximation to physical  $QCD(3+1)$ , the phenomenology of its DLCQ solutions demonstrate general gauge theory features, such as the peaking of the wavefunctions at minimal invariant mass, color coherence and the helicity retention of leading partons in the polarized structure functions at  $x \rightarrow 1$ . The solutions of the quantum field theory can be obtained for arbitrary coupling strength, flavors, and colors.

The light-cone Fock formalism is defined in the following way: one first constructs the light-cone time evolution operator  $P^- = P^0 - P^z$  and the invariant mass operator  $H_{LC} = P^- P^+ - P_\perp^2$  in light-cone gauge  $A^+ = 0$  from

the QCD Lagrangian. The total longitudinal momentum  $P^+ = P^0 + P^z$  and transverse momenta  $\vec{P}_\perp$  are conserved, *i.e.* are independent of the interactions. The matrix elements of  $H_{LC}$  on the complete orthonormal basis  $\{|n\rangle\}$  of the free theory  $H_{LC}^0 = H_{LC}(g=0)$  can then be constructed. The matrix elements  $\langle n|H_{LC}|m\rangle$  connect Fock states differing by 0, 1, or 2 quark or gluon quanta, and they include the instantaneous quark and gluon contributions imposed by eliminating dependent degrees of freedom in light-cone gauge. In the discretized light-cone method (DLCQ), the matrix elements  $\langle n|H_{LC}^A|m\rangle$ , are made discrete in momentum space by imposing periodic or anti-periodic boundary conditions in  $x^- = x^0 - x^z$  and  $\vec{x}_\perp$ . Upon diagonalization of  $H_{LC}$ , the eigenvalues provide the invariant mass of the bound states and eigenstates of the continuum.

In practice it is essential to introduce an ultraviolet regulator in order to limit the total range of  $\langle n|H_{LC}|m\rangle$ , such as the “global” cutoff in the invariant mass of the free Fock state. One can also introduce a “local” cutoff to limit the change in invariant mass  $|\mathcal{M}_n^2 - \mathcal{M}_m^2| < \Lambda_{\text{local}}^2$  which provides spectator-independent regularization of the sub-divergences associated with mass and coupling renormalization. Recently, Hiller, McCartor, and I have shown<sup>11</sup> that the Pauli-Villars method has advantages for regulating light-cone quantized Hamiltonian theory. We show that Pauli-Villars fields satisfying three spectral conditions will regulate the interactions in the ultraviolet, while at same time avoiding spectator-dependent renormalization and preserving chiral symmetry.

The natural renormalization scheme for the QCD coupling is  $\alpha_V(Q)$ , the effective charge defined from the scattering of two infinitely-heavy quark test charges. The renormalization scale can then be determined from the virtuality of the exchanged momentum, as in the BLM and commensurate scale methods.<sup>12,13,14,15</sup>

In principle, we could also construct the wavefunctions of QCD(3+1) starting with collinear QCD(1+1) solutions by systematic perturbation theory in  $\Delta H$ , where  $\Delta H$  contains the terms which produce particles at non-zero  $k_\perp$ , including the terms linear and quadratic in the transverse momenta  $\vec{k}_\perp$  which are neglected in the Hamilton  $H_0$  of collinear QCD. We can write the exact eigensolution of the full Hamiltonian as

$$\psi_{(3+1)} = \psi_{(1+1)} + \frac{1}{M^2 - H + i\epsilon} \Delta H \psi_{(1+1)} ,$$

where

$$\frac{1}{M^2 - H + i\epsilon} = \frac{1}{M^2 - H_0 + i\epsilon} + \frac{1}{M^2 - H + i\epsilon} \Delta H \frac{1}{M^2 - H_0 + i\epsilon}$$

can be represented as the continued iteration of the Lippmann Schwinger resolvent. Note that the matrix  $(M^2 - H_0)^{-1}$  is known to any desired precision from the DLCQ solution of collinear QCD.

## 2 Electroweak Matrix Elements

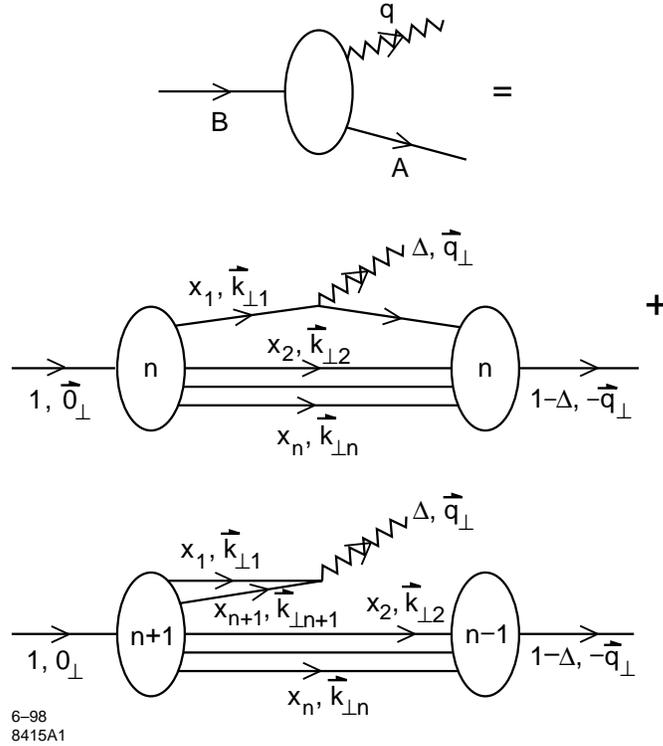


Figure 1: Exact representation of electroweak decays and time-like form factors in the light-cone Fock representation.

Dae Sung Hwang and I have recently shown that exclusive semileptonic  $B$ -decay amplitudes, such as  $B \rightarrow A \ell \bar{\nu}$  can be evaluated exactly in the light-cone formalism.<sup>16</sup> These timelike decay matrix elements require the computation of the diagonal matrix element  $n \rightarrow n$  where parton number is conserved,

and the off-diagonal  $n + 1 \rightarrow n - 1$  convolution where the current operator annihilates a  $q\bar{q}'$  pair in the initial  $B$  wavefunction. See Fig. 1. This term is a consequence of the fact that the time-like decay  $q^2 = (p_\ell + p_{\bar{q}})^2 > 0$  requires a positive light-cone momentum fraction  $q^+ > 0$ . Conversely for space-like currents, one can choose  $q^+ = 0$ , as in the Drell-Yan-West representation of the space-like electromagnetic form factors. However, as can be seen from the explicit analysis of the form factor in a perturbation model, the off-diagonal convolution can yield a nonzero  $q^+/q^+$  limiting form as  $q^+ \rightarrow 0$ . This extra term appears specifically in the case of “bad” currents such as  $J^-$  in which the coupling to  $q\bar{q}$  fluctuations in the light-cone wavefunctions are favored. In effect, the  $q^+ \rightarrow 0$  limit generates  $\delta(x)$  contributions as residues of the  $n + 1 \rightarrow n - 1$  contributions. The necessity for this zero mode  $\delta(x)$  terms has been noted by Chang, Root and Yan,<sup>17</sup> and Burkardt.<sup>18</sup>

The off-diagonal  $n + 1 \rightarrow n - 1$  contributions provide a new perspective for the physics of  $B$ -decays. A semileptonic decay involves not only matrix elements where a quark changes flavor, but also a contribution where the leptonic pair is created from the annihilation of a  $q\bar{q}'$  pair within the Fock states of the initial  $B$  wavefunction. The semileptonic decay thus can occur from the annihilation of a nonvalence quark-antiquark pair in the initial hadron. This feature will carry over to exclusive hadronic  $B$ -decays, such as  $B^0 \rightarrow \pi^- D^+$ . In this case the pion can be produced from the coalescence of a  $d\bar{u}$  pair emerging from the initial higher particle number Fock wavefunction of the  $B$ . The  $D$  meson is then formed from the remaining quarks after the internal exchange of a  $W$  boson.

In principle, a precise evaluation of the hadronic matrix elements needed for  $B$ -decays and other exclusive electroweak decay amplitudes requires knowledge of all of the light-cone Fock wavefunctions of the initial and final state hadrons. In the case of model gauge theories such as QCD(1+1)<sup>19</sup> or collinear QCD<sup>10</sup> in one-space and one-time dimensions, the complete evaluation of the light-cone wavefunction is possible for each baryon or meson bound-state using the DLCQ method. It would be interesting to use such solutions as a model for physical  $B$ -decays.

The existence of an exact formalism provides a basis for systematic approximations and a control over neglected terms. For example, one can analyze exclusive semileptonic  $B$ -decays which involve hard internal momentum transfer using a perturbative QCD formalism patterned after the analysis of form factors at large momentum transfer.<sup>5</sup> The hard-scattering analysis proceeds by writing each hadronic wavefunction as a sum of soft and hard contributions

$$\psi_n = \psi_n^{\text{soft}}(\mathcal{M}_n^2 < \Lambda^2) + \psi_n^{\text{hard}}(\mathcal{M}_n^2 > \Lambda^2), \quad (5)$$

where  $\mathcal{M}_n^2$  is the invariant mass of the partons in the  $n$ -particle Fock state and  $\Lambda$  is the separation scale. The high internal momentum contributions to the wavefunction  $\psi_n^{\text{hard}}$  can be calculated systematically from QCD perturbation theory by iterating the gluon exchange kernel. The contributions from high momentum transfer exchange to the  $B$ -decay amplitude can then be written as a convolution of a hard scattering quark-gluon scattering amplitude  $T_H$  with the distribution amplitudes  $\phi(x_i, \Lambda)$ , the valence wavefunctions obtained by integrating the constituent momenta up to the separation scale  $\mathcal{M}_n < \Lambda < Q$ . This is the basis for the perturbative hard scattering analyses.<sup>20,21,22,23</sup> In the exact analysis, one can identify the hard PQCD contribution as well as the soft contribution from the convolution of the light-cone wavefunctions. Furthermore, the hard scattering contribution can be systematically improved. For example, off-shell effects can be retained in the evaluation of  $T_H$  by utilizing the exact light-cone energy denominators.

More generally, hard exclusive hadronic amplitudes such as quarkonium decay, heavy hadron decay, and scattering amplitudes where the hadrons are scattered with momentum transfer can be factorized as the convolution of the light-cone Fock state wavefunctions with quark-gluon matrix elements<sup>5</sup>

$$\begin{aligned} \mathcal{M}_{\text{Hadron}} = & \prod_H \sum_n \int \prod_{i=1}^n d^2 k_{\perp} \prod_{i=1}^n dx \delta \left( 1 - \sum_{i=1}^n x_i \right) \delta \left( \sum_{i=1}^n \vec{k}_{\perp i} \right) \\ & \times \psi_{n/H}^{(\Lambda)}(x_i, \vec{k}_{\perp i}, \Lambda_i) T_H^{(\Lambda)}. \end{aligned} \quad (6)$$

Here  $T_H^{(\Lambda)}$  is the underlying quark-gluon subprocess scattering amplitude, where the (incident or final) hadrons are replaced by quarks and gluons with momenta  $x_i p^+$ ,  $x_i \vec{p}_{\perp} + \vec{k}_{\perp i}$  and invariant mass above the separation scale  $\mathcal{M}_n^2 > \Lambda^2$ . The essential part of the wavefunction is the hadronic distribution amplitudes,<sup>5</sup> defined as the integral over transverse momenta of the valence (lowest particle number) Fock wavefunction; *e.g.* for the pion

$$\phi_{\pi}(x_i, Q) \equiv \int d^2 k_{\perp} \psi_{q\bar{q}/\pi}^{(Q)}(x_i, \vec{k}_{\perp i}, \lambda) \quad (7)$$

where the global cutoff  $\Lambda$  is identified with the resolution  $Q$ . The distribution amplitude controls leading-twist exclusive amplitudes at high momentum transfer, and it can be related to the gauge-invariant Bethe-Salpeter wavefunction at equal light-cone time  $\tau = x^+$ . The  $\log Q$  evolution of the hadron distribution amplitudes  $\phi_H(x_i, Q)$  can be derived from the perturbatively-computable tail of the valence light-cone wavefunction in the high transverse

momentum regime.<sup>5</sup> In general the LC ultraviolet regulators provide a factorization scheme for elastic and inelastic scattering, separating the hard dynamical contributions with invariant mass squared  $\mathcal{M}^2 > \Lambda_{\text{global}}^2$  from the soft physics with  $\mathcal{M}^2 \leq \Lambda_{\text{global}}^2$  which is incorporated in the nonperturbative LC wavefunctions. The DGLAP evolution of quark and gluon distributions can also be derived by computing the variation of the Fock expansion with respect to  $\Lambda^2$ .<sup>5</sup>

Given the solution for the hadronic wavefunctions  $\psi_n^{(\Lambda)}$  with  $\mathcal{M}_n^2 < \Lambda^2$ , one can construct the wavefunction in the hard regime with  $\mathcal{M}_n^2 > \Lambda^2$  using projection operator techniques.<sup>5</sup> The construction can be done perturbatively in QCD since only high invariant mass, far off-shell matrix elements are involved. One can use this method to derive the physical properties of the LC wavefunctions and their matrix elements at high invariant mass. Since  $\mathcal{M}_n^2 = \sum_{i=1}^n \left( \frac{k_{\perp i}^2 + m^2}{x} \right)_i$ , this method also allows the derivation of the asymptotic behavior of light-cone wavefunctions at large  $k_{\perp}$ , which in turn leads to predictions for the fall-off of form factors and other exclusive matrix elements at large momentum transfer, such as the quark counting rules for predicting the nominal power-law fall-off of two-body scattering amplitudes at fixed  $\theta_{cm}$ .<sup>6</sup> The phenomenological successes of these rules can be understood within QCD if the coupling  $\alpha_V(Q)$  freezes in a range of relatively small momentum transfer.<sup>15</sup>

### 3 Measurement of Light-cone Wavefunctions via Diffractive Dissociation.

Diffractive multi-jet production in heavy nuclei provides a novel way to measure the shape of the LC Fock state wavefunctions and test color transparency. For example, consider the reaction  $^2,3 \pi A \rightarrow \text{Jet}_1 + \text{Jet}_2 + A'$  at high energy where the nucleus  $A'$  is left intact in its ground state. The transverse momenta of the jets have to balance so that  $\vec{k}_{\perp 1} + \vec{k}_{\perp 2} = \vec{q}_{\perp} < \mathcal{R}_A^{-1}$ , and the light-cone longitudinal momentum fractions have to add to  $x_1 + x_2 \sim 1$  so that  $\Delta p_L < R_A^{-1}$ . The process can then occur coherently in the nucleus. Because of color transparency, i.e., the cancelation of color interactions in a small-size color-singlet hadron, the valence wavefunction of the pion with small impact separation will penetrate the nucleus with minimal interactions, diffracting into jet pairs.<sup>2</sup> The  $x_1 = x, x_2 = 1 - x$  dependence of the di-jet distributions will thus reflect the shape of the pion distribution amplitude; the  $\vec{k}_{\perp 1} - \vec{k}_{\perp 2}$  relative transverse momenta of the jets also gives key information on the underlying shape of the valence pion wavefunction. The QCD analysis can be confirmed

by the observation that the diffractive nuclear amplitude extrapolated to  $t = 0$  is linear in nuclear number  $A$ , as predicted by QCD color transparency. The integrated diffractive rate should scale as  $A^2/R_A^2 \sim A^{4/3}$ . A diffractive dissociation experiment of this type, E791, is now in progress at Fermilab using 500 GeV incident pions on nuclear targets.<sup>24</sup> The preliminary results from E791 appear to be consistent with color transparency. The momentum fraction distribution of the jets is consistent with a valence light-cone wavefunction of the pion consistent with the shape of the asymptotic distribution amplitude. Data from CLEO for the  $\gamma\gamma^* \rightarrow \pi^0$  transition form factor also favor a form for the pion distribution amplitude close to the asymptotic solution<sup>5</sup>  $\phi_\pi^{\text{asympt}}(x) = \sqrt{3}f_\pi x(1-x)$  to the perturbative QCD evolution equation.<sup>25,26,15</sup> It will also be interesting to study diffractive tri-jet production using proton beams  $pA \rightarrow \text{Jet}_1 + \text{Jet}_2 + \text{Jet}_3 + A'$  to determine the fundamental shape of the 3-quark structure of the valence light-cone wavefunction of the nucleon at small transverse separation. Conversely, one can use incident real and virtual photons:  $\gamma^*A \rightarrow \text{Jet}_1 + \text{Jet}_2 + A'$  to confirm the shape of the calculable light-cone wavefunction for transversely-polarized and longitudinally-polarized virtual photons. Such experiments will open up a remarkable, direct window on the amplitude structure of hadrons at short distances.

#### 4 Other Applications of Light-Cone Quantization to QCD Phenomenology

*Diffractive vector meson photoproduction.* The light-cone Fock wavefunction representation of hadronic amplitudes allows a simple eikonal analysis of diffractive high energy processes, such as  $\gamma^*(Q^2)p \rightarrow \rho p$ , in terms of the virtual photon and the vector meson Fock state light-cone wavefunctions convoluted with the  $gp \rightarrow gp$  near-forward matrix element.<sup>27</sup> One can easily show that only small transverse size  $b_\perp \sim 1/Q$  of the vector meson distribution amplitude is involved. The hadronic interactions are minimal, and thus the  $\gamma^*(Q^2)N \rightarrow \rho N$  reaction can occur coherently throughout a nuclear target in reactions without absorption or shadowing. The  $\gamma^*A \rightarrow VA$  process thus provides a natural framework for testing QCD color transparency.<sup>28</sup> Evidence for color transparency in such reactions has been found by Fermilab experiment E665.

*Regge behavior of structure functions.* The light-cone wavefunctions  $\psi_{n/H}$  of a hadron are not independent of each other, but rather are coupled via the equations of motion. Antonuccio, Dalley and I<sup>29</sup> have used the constraint of finite “mechanical” kinetic energy to derive “ladder relations” which interrelate the light-cone wavefunctions of states differing by one or two gluons. We then use these relations to derive the Regge behavior of both the polarized and

unpolarized structure functions at  $x \rightarrow 0$ , extending Mueller's derivation of the BFKL hard QCD pomeron from the properties of heavy quarkonium light-cone wavefunctions at large  $N_C$  QCD.<sup>30</sup>

*Structure functions at large  $x_{bj}$ .* The behavior of structure functions where one quark has the entire momentum requires the knowledge of LC wavefunctions with  $x \rightarrow 1$  for the struck quark and  $x \rightarrow 0$  for the spectators. This is a highly off-shell configuration, and thus one can rigorously derive quark-counting and helicity-retention rules for the power-law behavior of the polarized and unpolarized quark and gluon distributions in the  $x \rightarrow 1$  endpoint domain. It is interesting to note that the evolution of structure functions is minimal in this domain because the struck quark is highly virtual as  $x \rightarrow 1$ ; *i.e.* the starting point  $Q_0^2$  for evolution cannot be held fixed, but must be larger than a scale of order  $(m^2 + k_\perp^2)/(1 - x)$ .<sup>5,6,31</sup>

*Intrinsic gluon and heavy quarks.* The main features of the heavy sea quark-pair contributions of the Fock state expansion of light hadrons can also be derived from perturbative QCD, since  $\mathcal{M}_n^2$  grows with  $m_Q^2$ . One identifies two contributions to the heavy quark sea, the "extrinsic" contributions which correspond to ordinary gluon splitting, and the "intrinsic" sea which is multi-connected via gluons to the valence quarks. The intrinsic sea is thus sensitive to the hadronic bound state structure.<sup>32</sup> The maximal contribution of the intrinsic heavy quark occurs at  $x_Q \simeq m_\perp Q / \sum_i m_\perp$  where  $m_\perp = \sqrt{m^2 + k_\perp^2}$ ; *i.e.* at large  $x_Q$ , since this minimizes the invariant mass  $\mathcal{M}_n^2$ . The measurements of the charm structure function by the EMC experiment are consistent with intrinsic charm at large  $x$  in the nucleon with a probability of order  $0.6 \pm 0.3\%$ .<sup>33</sup> Similarly, one can distinguish intrinsic gluons which are associated with multi-quark interactions and extrinsic gluon contributions associated with quark substructure.<sup>34</sup> One can also use this framework to isolate the physics of the anomaly contribution to the Ellis-Jaffe sum rule.

*Materialization of far-off-shell configurations.* In a high energy hadronic collisions, the highly-virtual states of a hadron can be materialized into physical hadrons simply by the soft interaction of any of the constituents.<sup>35</sup> Thus a proton state with intrinsic charm  $|uud\bar{c}c\rangle$  can be materialized, producing a  $J/\psi$  at large  $x_F$ , by the interaction of a light-quark in the target. The production occurs on the front-surface of a target nucleus, implying an  $A^{2/3}$   $J/\psi$  production cross section at large  $x_F$ , which is consistent with experiment, such as Fermilab experiments E772 and E866.

*Rearrangement mechanism in heavy quarkonium decay.* It is usually assumed that a heavy quarkonium state such as the  $J/\psi$  always decays to light hadrons via the annihilation of its heavy quark constituents to gluons. However, as Karliner and I<sup>36</sup> have recently shown, the transition  $J/\psi \rightarrow \rho\pi$  can

also occur by the rearrangement of the  $c\bar{c}$  from the  $J/\psi$  into the  $|q\bar{q}c\bar{c}\rangle$  intrinsic charm Fock state of the  $\rho$  or  $\pi$ . On the other hand, the overlap rearrangement integral in the decay  $\psi' \rightarrow \rho\pi$  will be suppressed since the intrinsic charm Fock state radial wavefunction of the light hadrons will evidently not have nodes in its radial wavefunction. This observation provides a natural explanation of the long-standing puzzle why the  $J/\psi$  decays prominently to two-body pseudoscalar-vector final states, whereas the  $\psi'$  does not.

*Asymmetry of intrinsic heavy quark sea.* As Burkardt and Wari<sup>37</sup> first noted, the higher Fock state of the proton  $|uuds\bar{s}\rangle$  should resemble a  $|K\Lambda\rangle$  intermediate state, since this minimizes its invariant mass  $\mathcal{M}$ . In such a state, the strange quark has a higher mean momentum fraction  $x$  than the  $\bar{s}$ .<sup>37,38,39</sup> Similarly, the helicity intrinsic strange quark in this configuration will be anti-aligned with the helicity of the nucleon.<sup>37,39</sup> This  $Q \leftrightarrow \bar{Q}$  asymmetry is a striking feature of the intrinsic heavy-quark sea.

*Comover phenomena.* Light-cone wavefunctions describe not only the partons that interact in a hard subprocess but also the associated partons freed from the projectile. The projectile partons which are comoving (*i.e.*, which have similar rapidity) with final state quarks and gluons can interact strongly producing (a) leading particle effects, such as those seen in open charm hadroproduction; (b) suppression of quarkonium<sup>40</sup> in favor of open heavy hadron production, as seen in the E772 experiment; (c) changes in color configurations and selection rules in quarkonium hadroproduction, as has been emphasized by Hoyer and Peigne.<sup>41</sup> All of these effects violate the usual ideas of factorization for inclusive reactions. Further, more than one parton from the projectile can enter the hard subprocess, producing dynamical higher twist contributions, as seen for example in Drell-Yan experiments.<sup>42,43</sup>

*Jet hadronization in light-cone QCD.* One of the goals of nonperturbative analysis in QCD is to compute jet hadronization from first principles. The DLCQ solutions provide a possible method to accomplish this. By inverting the DLCQ solutions, we can write the “bare” quark state of the free theory as  $|q_0\rangle = \sum |n\rangle \langle n|q_0\rangle$  where now  $\{|n\rangle\}$  are the exact DLCQ eigenstates of  $H_{LC}$ , and  $\langle n|q_0\rangle$  are the DLCQ projections of the eigensolutions. The expansion is automatically infrared and ultraviolet regulated if we impose global cutoffs on the DLCQ basis:  $\lambda^2 < \Delta\mathcal{M}_n^2 < \Lambda^2$  where  $\Delta\mathcal{M}_n^2 = \mathcal{M}_n^2 - (\Sigma\mathcal{M}_i)^2$ . It would be interesting to study jet hadronization at the amplitude level for the existing DLCQ solutions to QCD (1+1) and collinear QCD.

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## References

1. For a review of light-cone Hamiltonian methods and further references, see S. J. Brodsky, H. C. Pauli, and S. S. Pinsky, SLAC-PUB-7484, hep-ph/9705477 (to be published in *Physics Reports*). See also the lectures: S. Brodsky, SLAC-PUB-7645, hep-ph/9710288.
2. G. Bertsch, S. J. Brodsky, A. S. Goldhaber, and J. F. Gunion, *Phys. Rev. Lett.* **47**, 297 (1981).
3. L. Frankfurt, G. A. Miller, and M. Strikman, *Phys. Lett.* **B304**, 1 (1993), hep-ph/9305228.
4. S. J. Brodsky and S. D. Drell, *Phys. Rev.* **D22**, 2236 (1980).
5. G. P. Lepage and S. J. Brodsky, *Phys. Rev.* **D22**, 2157 (1980). For a recent PQCD analysis of the proton form factor, see B. Kundu, H. Li, J. Samuelsson and P. Jain, hep-ph/9806419.
6. S. J. Brodsky and G. P. Lepage, in *Perturbative Quantum Chromodynamics*, A. H. Mueller, Ed. (World Scientific, 1989).
7. H. C. Pauli and S. J. Brodsky, *Phys. Rev.* **D32**, 2001 (1985).
8. S. J. Brodsky and H. C. Pauli, SLAC-PUB-5558, published in Schlading 1991, Proceedings.
9. S. Dalley, and I. R. Klebanov, *Phys. Rev.* **D47**, 2517 (1993).
10. F. Antonuccio and S. Dalley, *Phys. Lett.* **B376**, 154, (1996), hep-ph/9512106, and references therein.
11. S. J. Brodsky, J. R. Hiller G. McCartor, SLAC-PUB-7745, hep-th/9802120
12. S. J. Brodsky, G. P. Lepage, and P. B. Mackenzie, *Phys. Rev.* **D28**, 228 (1983).
13. H. J. Lu and S. J. Brodsky, *Phys. Rev.* **D48**, 3310 (1993).
14. S. J. Brodsky, G. T. Gabadadze, A. L. Kataev and H. J. Lu, *Phys. Lett.* **372B**, 133, (1996).
15. S. J. Brodsky, C.-R. Ji, A. Pang, and D. G. Robertson, SLAC-PUB-7473, *Phys. Rev.* **D57** 245 (1998), hep-ph/9705221.
16. S. J. Brodsky and D. S. Hwang, SLAC-PUB-7839, hep-ph/9806358.

17. S. J. Chang, R.G. Root and T. M. Yan, *Phys. Rev.* **D7**, 1133 (1973).
18. M. Burkardt, *Nucl. Phys.* **A504**, 762 (1989); *Nucl. Phys.* **B373**, 613 (1992); *Phys. Rev.* **D52**, 3841 (1995).
19. K. Hornbostel, S. J. Brodsky, and H. C. Pauli, *Phys. Rev.* **D41** 3814 (1990).
20. A. Szczepaniak, E. M. Henley and S. J. Brodsky, *Phys. Lett.* **B243**, 287 (1990).
21. A. Szczepaniak, *Phys. Rev.* **D54**, 1167 (1996).
22. P. Ball, hep-ph/9802394 (1998); hep-ph/9803501 (1998).
23. P. Ball and V. M. Braun, hep-ph/9805422 (1998).
24. D. Ashery, *et al.*, Fermilab E791 Collaboration, to be published.
25. P. Kroll and M. Raulfs, *Phys. Lett.* **B387**, 848 (1996).
26. I. V. Musatov and A. V. Radyushkin, hep-ph/9702443.
27. S. J. Brodsky, L. Frankfurt, J. F. Gunion, A. H. Mueller, and M. Strikman, *Phys. Rev.* **D50**, 3134 (1994), hep-ph/9402283.
28. S. J. Brodsky and A.H. Mueller, *Phys. Lett.* **206B**, 685 (1988).
29. F. Antonuccio, S. J. Brodsky, and S. Dalley, SLAC-PUB-7472, *Phys. Lett.* **B412** 104 (1997), hep-ph/9705413.
30. A. H. Mueller, *Nucl. Phys.* **B415**, 373 (1994).
31. D. Mueller, SLAC-PUB-6496, May 1994, hep-ph/9406260.
32. S. J. Brodsky, P. Hoyer, C. Peterson, and N. Sakai, *Phys. Lett.* **93B**, 451 (1980).
33. B. W. Harris, J. Smith, and R. Vogt, *Nucl. Phys.* **B461**, 181 (1996), hep-ph/9508403.
34. S. J. Brodsky and I. A. Schmidt, *Phys. Lett.* **B234**, 144 (1990).
35. S. J. Brodsky, P. Hoyer, A. H. Mueller, W.-K. Tang, *Nucl. Phys.* **B369**, 519 (1992).
36. S. J. Brodsky and M. Karliner SLAC-PUB-7463, *Phys. Rev. Lett.* **78**, 4682 (1997), hep-ph/9704379.
37. M. Burkardt and Brian Warr, *Phys. Rev.* **D45**, 958 (1992).
38. A. I. Signal and A. W. Thomas, *Phys. Lett.* **191B**, 205 (1987).
39. S. J. Brodsky and B-Q Ma, *Phys. Lett.* **B381**, 317 (1996), hep-ph/9604393.
40. S. J. Brodsky and A. Mueller, *Phys. Lett.* **206B**, 685 (1988). R. Vogt, S. J. Brodsky, and P. Hoyer, SLAC-PUB-5421, *Nucl. Phys.* **B360**, 67 (1991); SLAC-PUB-5827, *Nucl. Phys.* **B383**, 643 (1992).
41. P. Hoyer and S. Peigne, hep-ph/9806424.
42. E. L. Berger and S. J. Brodsky, *Phys. Rev. Lett.* **42**, 940 (1979).
43. A. Brandenburg, S. J. Brodsky, V.V. Khoze, and D. Mueller, *Phys. Rev. Lett.* **73**, 939 (1994), hep-ph/9403361.