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Near Horizon Superspace

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ABSTRACT

The $adS_{p+2} \times S^{d-p-2}$ geometry of the near horizon branes is promoted to a supergeometry: the solution of the supergravity constraints for the vielbein, connection and form superfields are found. This supergeometry can be used for the construction of new superconformal theories. We also discuss the Green-Schwarz action for a type IIB string on $adS_5 \times S_5$.

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There has been great interest recently in type IIB theory on $adS_5 \times S_5$ [1, 2, 3], due to its possible relation to N=4,d=4 Yang-Mills theory. This vacuum is the near horizon limit of a D3-brane geometry. Related to this, we also have the $adS_7 \times S_4$and $adS_4 \times S_7$ vacua of M-theory, which are the near horizon geometries of a M5-brane and an M2-brane respectively.

We would like to formulate a description of these vacua in superspace. This is of great use, since the knowledge of the supervielbeins enables us to write down worldvolume actions of branes in these backgrounds, in particular, the Green-Schwarz action for a type IIB string on $adS_5 \times S_5$.

A first step was taken in [4], where the torsion and curvature superfields were found in the framework of the standard supergravity superspace [5, 6, 7]. This knowledge enables one to prove that these vacua are exact solutions. In this paper, we will find in addition the vielbeins, connections and form superfields.

In the $adS_5 \times S^5$ case, this problem was attacked by Metsaev and Tseytlin [8] using the group manifold approach. We shall extend their construction to the M-theory vacua, and find closed form expressions for the supervielbeins and superconnections. This in particular enables one to directly write down a closed form expression for the Green-Schwarz action for a type IIB on $dS_5 \times S_5$, using [9].

We define the forms, following [10] where supergravity was defined in a group manifold approach:

$$
\begin{aligned}
L^a(x,\theta) &= E^a{}_\mu(x,\theta)dx^\mu + E^a{}_{\underline{\alpha}}(x,\theta)d\theta^{\underline{\alpha}} \\
L^\alpha(x,\theta) &= E^\alpha{}_\mu(x,\theta)dx^\mu + E^\alpha{}_{\underline{\alpha}}(x,\theta)d\theta^{\underline{\alpha}} \\
\omega^{ab}(x,\theta) &= \omega^{ab}{}_\mu(x,\theta)dx^\mu + \omega^{ab}{}_{\underline{\alpha}}(x,\theta)d\theta^{\underline{\alpha}}
\end{aligned} \tag{1}
$$

where L^a (L^α) corresponds to the bosonic(fermionic) components of the vielbein (E^a, E^α in [5]) and ω^{ab} is the connection superfield. The lowest ($\theta = 0$) components of E^a_μ and ω^{ab}_μ are the space-time vielbein and connection.

The space-time form-fields of supergravity $A_{\mu_1,\ldots\mu_{p+1}}(x)$ are also introduced in superspace with the help of the superspace forms. In terms of the vielbein $L^{\hat{A}} =$

(L^a, L^α) the form fields are

$$A = L^{\hat{A}_1} \dots L^{\hat{A}_{p+1}} A_{\hat{A}_1 \dots \hat{A}_{p+1}}(x,\theta), \qquad F = dA, \qquad dF = 0 \tag{2}$$

In the group-manifold approach, the curvature and torsions are defined somewhat differently. In effect, non-zero value of the curvature and torsion are absorbed by redefining them in such a way that the new curvature and torsion are zero. For example, the standard definition of adS geometry has a non-zero curvature

$$R_{ab}{}^{cd} = -\delta_{[a}{}^c \delta_{b]}{}^d \tag{3}$$

which translates in form language to

$$R^{cd} \equiv (d\omega + \omega \wedge \omega)^{cd} = -L^c \wedge L^d. \tag{4}$$

Instead, in the group manifold approach one uses the anti-de-Sitter algebra

$$[P_a, P_b] = J_{ab}\,, \qquad [P_a, J_{bc}] = \eta_{ab} P_c - \eta_{ac} P_b\,, \qquad [J_{ab}, J_{cd}] = \eta_{bc} J_{ad} + 3 \text{ terms} \tag{5}$$

and introduces the 1-form differential operator

$$\mathcal{D} = d + \omega^{ab} J_{ab} + L^a P_a. \tag{6}$$

The adS curvatures are now defined by

$$\mathcal{D}^2 \equiv \mathcal{R}^{ab} J_{ab} + \mathcal{R}^a P_a \tag{7}$$

and it is easily verified that

$$\mathcal{R}^{ab} = R^{ab} + L^a \wedge L^b \tag{8}$$

$$\mathcal{R}^a = T^a \tag{9}$$

which vanish for this geometry. Hence, the adS geometry can be defined by requiring vanishing generalized adS curvatures.

For the analogous description in superspace we must generalize the differential operator $\mathcal{D}$ to include all the generators of the appropriate superconformal algebra: $OSp(8|4)$ for the $adS_4 \times S^7$ vacuum of M theory, $OSp(6,2|4)$ for the $adS_7 \times S^4$ vacuum of M theory and $SU(2,2|4)$ for the $adS_5 \times S^5$ vacuum of IIB string theory. In general, the superconformal algebra is of the form

$$\begin{aligned} [B_A, B_B] &= f^C_{AB} B_C \\ [F_\alpha, B_B] &= f^\gamma_{\alpha B} F_\gamma \\ \{F_\alpha, F_\beta\} &= f^C_{\alpha\beta} B_C \end{aligned} \tag{10}$$

where B_A and F_α are the bosonic and fermionic generators, and the f are the structure constants. The differential operator is then

$$\mathcal{D} = d + L^A B_A + L^\alpha F_\alpha. \tag{11}$$

L^A and L^α are the left-invariant Cartan forms. The equation $\mathcal{D}^2 = 0$ leads then to the Maurer-Cartan equations

$$dL^A + L^B \wedge L^C f^A_{BC} - L^\alpha \wedge L^\beta f^A_{\alpha\beta} = 0 \tag{12}$$

$$dL^\alpha + L^A \wedge L^\beta f^\alpha_{A\beta} = 0. \tag{13}$$

We will now solve these equations using the standard method which was also employed in [8]. Let G be an element of the superconformal group, then

$$G^{-1} dG = L^A B_A + L^\alpha F_\alpha. \tag{14}$$

Writing $G = g(x) e^{\theta F}$, we find that

$$G^{-1} dG = e^{-\theta F} D e^{\theta F}, \tag{15}$$

where

$$D = d + L_0^A B_A \tag{16}$$

with $L_0^A = L^A(x, \theta = 0)$. It is now useful to decompose L as

$$L = L_0(x) + \tilde{L}(x, \theta). \tag{17}$$

To find explicitly $\tilde{L}^A$ and $\tilde{L}^\alpha$ one then replaces $\theta \to t\theta$ and introduces a t-dependence into the vielbein components (which will be eventually be removed by setting $t = 1$), giving rise to the modified relation

$$e^{-t\theta F} d e^{t\theta F} = \tilde{L}_t^A B_A + \tilde{L}_t^\alpha F_\alpha. \tag{18}$$

Differentiating both sides and utilizing the superalgebra (10) one finds the differential equations:

$$\partial_t \tilde{L}_t^A = \theta^\alpha f_{\alpha\beta}^A \tilde{L}_t^\beta \tag{19}$$

$$\partial_t \tilde{L}_t^\alpha = D\theta^\alpha - \theta^\gamma f_{\gamma A}^\alpha \tilde{L}_t^A. \tag{20}$$

which have the structure of coupled harmonic oscillators. Together with the initial conditions

$$\tilde{L}_{t=0}^A = \tilde{L}_{t=0}^\alpha = 0 \tag{21}$$

and (17), we can easily write down the explicit solution for the supervielbein (setting $t = 1$) in closed form. One finds that

$$L^\alpha = \left(\frac{\sinh \mathcal{M}}{\mathcal{M}}\right)^\alpha_{\ \beta} (D\theta)^\beta \tag{22}$$

and

$$L^A = L_0^A + 2\theta^\alpha f_{\alpha\beta}^A \left(\frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\right)^\beta_{\ \gamma} (D\theta)^\gamma, \tag{23}$$

where

$$\left(\mathcal{M}^2\right)^\alpha_{\ \beta} = -\theta^\gamma f_{\gamma A}^\alpha \theta^\delta f_{\delta\beta}^A. \tag{24}$$

Note that $\mathcal{M}^2$ is quadratic in θ and that all higher order terms in θ in eqs. (22), (23) are given by even powers of $\mathcal{M}$, i.e. by powers of $\mathcal{M}^2$ up to $(\mathcal{M}^2)^{16}$.

If we choose θ to be the standard fermionic coordinates of superspace, the above solution gives the superspace geometry in the Wess-Zumino gauge. This can be readily verified by noting that

$$(D\theta)^\alpha = d\theta^\alpha + (L_0^A B_A \theta)^\alpha, \tag{25}$$

hence $L_{\underline{\beta}}^\alpha(x, \theta = 0) = \delta_{\underline{\beta}}^\alpha$.

However, it turns out that there is an often more convenient gauge which simplifies the geometry considerably. Consider space-time dependent θ^α of the form

$$\theta^\alpha(x) = e_{\underline{\alpha}}^\alpha(x)\theta^{\underline{\alpha}} \tag{26}$$

so that

$$(D\theta)^\alpha = e_{\underline{\alpha}}^\alpha(x) d\theta^{\underline{\alpha}} + \left((d + L_0^A B_A)^\alpha_\beta e_{\underline{\alpha}}^\beta(x)\right)\theta^{\underline{\alpha}} \tag{27}$$

The second term drops if we choose θ to be the Killing spinors of the background! As is well-known, those are precisely defined by the equation

$$(d + L_0^A B_A)^\alpha_\beta \epsilon^\beta(x)_{Kill} = 0 \tag{28}$$

and the solution is of the form [11]

$$\epsilon^\alpha(x)_{Kill} = e^\alpha_{\underline{\alpha}}(x)\epsilon^{\underline{\alpha}}_{\text{const}}. \tag{29}$$

where $\epsilon^{\underline{\alpha}}_{\text{const}}$ is a constant spinor. Hence, choosing $\theta = \epsilon_{Kill}(x)$ in (15) leads to

$$(D\theta)^\alpha = e^\alpha_{\underline{\alpha}}(x) d\theta^{\underline{\alpha}}. \tag{30}$$

With the explicit Killing spinors, constructed for all relevant $adS_* \times S^*$ spaces in [11], the superspace structure simplifies as follows:

$$L^\alpha = e^\alpha_{\underline{\alpha}}(x,\theta)d\theta^{\underline{\alpha}} \qquad L^A = e^A_M(x,\theta=0)dx^M + e^A_{\underline{\alpha}}(x,\theta)d\theta^{\underline{\alpha}}, \tag{31}$$

where

$$e^\alpha_{\underline{\alpha}}(x,\theta) = \left(\frac{\sinh \mathcal{M}}{\mathcal{M}}\right)^\alpha_\beta e^\beta_{\underline{\alpha}}(x) \ , \tag{32}$$

and

$$e^A_{\underline{\alpha}}(x,\theta) = \theta^\alpha(x) f^A_{\alpha\beta} \left(\frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\right)^\beta_\gamma e^\gamma_{\underline{\alpha}}(x) \tag{33}$$

where $\mathcal{M}$ now contains the space-time dependent spinors $\theta^\alpha(x)$

$$(\mathcal{M}^2)^\alpha_\beta = -\theta^\delta(x) f^\alpha_{\delta A} \theta^\gamma(x) f^A_{\gamma\beta}. \tag{34}$$

Instead of the standard Wess-Zumino gauge with $e^\alpha_{\underline{\alpha}}(x,\theta=0) = \delta^\alpha_{\underline{\alpha}}$ we have a Killing spinor gauge since the fermion-fermion part of the vielbein at $\theta = 0$ is 'curved' with

$$E^\alpha_{\underline{\alpha}}(x,\theta=0) = e^\alpha_{\underline{\alpha}}(x). \tag{35}$$

Like in the flat superspace, there are no fermions, the gravitino vielbein component ${\psi^\alpha}_\mu(x,\theta)dx^\mu$ is not generated at all levels of θ. This also explains why the boson-boson component of the vielbein $e^A_M(x,\theta=0)$ is θ-independent.

After developing the general formalism let us now turn to a specific example, the type IIB theory on $adS_5 \times S^5$. This vacuum is a special case of the IIB superspace [7]. It was presented in [4] in eqs. (46-51). The expressions for the constant Lorentz curvature and torsion of the string vacuum can be reinterpreted via the superconformal group manifold approach. Following [8] we consider a coset superspace $\frac{SU(2,2|4)}{SO(4,1)\times SO(5)}$ with the even part $adS_5 \times S^5 = \frac{SO(4,2)}{SO(4,1)} \times \frac{SO(6)}{SO(5)}$. The even generators are then two pairs of translations and rotations, (P_a, J_{ab}) with $a = 0,1,2,3,4$ for adS_5 and $(P_{a'}, J_{a'b'}), a' = 5,6,7,8,9$ for S^5 and the odd generators are the two $D = 10$ Majorana-Weyl spinors $Q_{\alpha\alpha' I}$. The differential operator related to $SU(2,2|4)$ superalgebra is given by

$$\mathcal{D} = d + G^{-1}dG = d + L^a P_a + L^{ab} J_{ab} + L^a P_a + L^{ab} J_{ab} + L^{\alpha\alpha' I} Q_{\alpha\alpha' I} \tag{36}$$

satisfying $\mathcal{D}^2 = 0$. Taking care of the change of notations with respect to the different representation for the fermions and splitting the ten-dimensional $\hat{a}$ into two five-dimensional ones a, a' one can verify that the supergravity constraint for the vector part of the torsion can be brought to the form

$$\begin{aligned} \mathcal{R}^a &= dL^a + L^b \wedge L^{ba} - i\bar{L}^I \gamma^a \wedge L^I = 0 \\ \mathcal{R}^{a'} &= dL^{a'} + L^{b'} \wedge L^{b'a'} - i\bar{L}^I \gamma^{a'} \wedge L^I = 0 \end{aligned} \tag{37}$$

where we follow the conventions of [8]. For the spinorial part of the torsion one finds the following form (upon changing the spinorial representation)

$$\mathcal{R}^{\alpha\alpha' I} = dL^{\alpha\alpha' I} + (\frac{i}{2}\gamma^a \epsilon^{IJ} L^J \wedge L^a - \frac{1}{4}\gamma^{ab} L^I \wedge L^{ab})^{\alpha\alpha'} + \ldots = 0 \tag{38}$$

The Lorentz curvature form of supergravity has a non-vanishing fermion-fermion component as well as the boson-boson component. It can easily be shown that these equations are equivalent to the constraints of the $adS_5 \times S^5$ discussed in [4]. Thus the results for the Lorentz-valued curvatures and torsions can be rewritten as $\mathcal{D}^2 = 0$ where $\mathcal{D}$ is based on superconformal algebra. In addition, the supergravity $adS_5 \times S^5$ vacuum is defined by the presence of the 3-form and 5-form.

$$F_{(3)} = -iL^{\hat{a}} \wedge L^{\gamma} (\sigma^{\hat{a}})_{\gamma\delta} \wedge L^{\delta} + c.c. \tag{39}$$

$$
\begin{aligned}
G_{(5)} &= L^a \wedge L^b \wedge L^c \wedge L^d \wedge L^e \epsilon_{abcde} + L^{a'} \wedge L^{b'} \wedge L^{c'} \wedge L^{d'} \wedge L^{e'} \epsilon_{a'b'c'd'e'} \\
&+ L^{\hat{a}} \wedge L^{\hat{b}} \wedge L^{\hat{c}} \wedge L^{\gamma} (\sigma_{\hat{a}\hat{b}\hat{c}})_{\gamma\delta} \wedge L^{\delta} \qquad (40)
\end{aligned}
$$

Here we use the notation of [7] but for convenience we split the ten-dimensional vector index $\hat{a} = a, a'$.

We now turn to the supergeometry of this space following our general procedure and using the conventions of [8]. The complete superspace forms in the Killing gauge specified for the $adS_5 \times S^5$ are

$$
L^{\alpha\alpha' I} = e^{\alpha\alpha' I}_{\underline{\alpha\alpha' I}}(x,\theta) d\theta^{\underline{\alpha\alpha' I}} \qquad (41)
$$

$$
\begin{aligned}
L^{\hat{a}} &= e^{\hat{a}}_m(x,\theta=0)dx^m + e^{\hat{a}}_{\underline{\alpha\alpha' I}}(x,\theta) d\theta^{\underline{\alpha\alpha' I}} \qquad (42) \\
L^{\hat{a}\hat{b}} &= \omega^{\hat{a}\hat{b}}_m(x,\theta=0)dx^m + e^{\hat{a}\hat{b}}_{\underline{\alpha\alpha' I}}(x,\theta) d\theta^{\underline{\alpha\alpha' I}} \qquad (43)
\end{aligned}
$$

Here

$$
e^{\alpha\alpha' I}_{\underline{\alpha\alpha' I}}(x,\theta) = \left(\frac{\sinh \mathcal{M}}{\mathcal{M}}\right)^{\alpha\alpha' I}_{\beta\beta' J} e^{\beta\beta' J}_{\underline{\alpha\alpha' I}}(x,\theta=0) \ , \qquad (44)
$$

$$
e^{\hat{a}}_{\underline{\alpha\alpha' I}}(x,\theta) = -2i(\bar{\theta}^I(x)\gamma^{\hat{a}})_{\alpha\alpha'} \left(\frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\right)^{\alpha\alpha' I}_{\beta\beta' J} e^{\beta\beta' J}_{\underline{\alpha\alpha' I}}(x,\theta=0) \qquad (45)
$$

$$
e^{\hat{a}b}_{\underline{\alpha\alpha' I}}(x,\theta) = 2\epsilon^{IJ}(\bar{\theta}^I(x)\gamma^{\hat{a}\hat{b}})_{\alpha\alpha'} \left(\frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\right)^{\alpha\alpha' J}_{\beta\beta' K} e^{\beta\beta' K}_{\underline{\alpha\alpha' I}}(x,\theta=0) \qquad (46)
$$

where

$$
\begin{aligned}
(\mathcal{M}^2)^I_L &= [\epsilon^{IJ}(-\gamma^a \theta^J(x)\bar{\theta}^L(x)\gamma^a + \gamma^{a'}\theta^J(x)\bar{\theta}^L(x)\gamma^{a'}) \\
&+ \frac{1}{2}\epsilon^{KL}(\gamma^{ab}\theta^I(x)\bar{\theta}^K(x)\gamma^{ab} + \gamma^{a'b'}\theta^I(x)\bar{\theta}^K(x)\gamma^{a'b'})] \qquad (47)
\end{aligned}
$$

We use the simplifying notation $\theta^{\alpha\alpha' I}(x) = \theta^{\underline{\alpha\alpha' I}} e^{\alpha\alpha' I}_{\underline{\alpha\alpha' I}}(x)$. We expect that this form of the superspace will be most suitable for the construction of the superconformal D3 brane action.

It may also be useful to work within the Wess-Zumino gauge in the superspace. The solution is than given by

$$L^I = \left[\left(\frac{\sinh \mathcal{M}}{\mathcal{M}} \right) D\theta \right]^I \tag{48}$$

and

$$L^{\hat{a}} \;\; = \;\; e_m^{\hat{a}}(x) dx^m - 4i\bar{\theta}^I \gamma^{\hat{a}} \left(\frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2} D\theta \right)^I \tag{49}$$

where $\mathcal{M}^2$ is given by eq. (47) with constant θ. Here,

$$(D\theta)^I = \left(d + \frac{1}{4}(\omega^{ab}\gamma_{ab} + \omega^{a'b'}\gamma_{a'b'}) \right) \theta^I - \frac{1}{2} i\epsilon^{IJ}(e^a\gamma_a + ie^{a'}\gamma_{a'})\theta^J \tag{50}$$

The complete form of the GS type IIB κ-symmetric string action in the generic supergravity background was presented in [9]:

$$S = -\frac{1}{2}\int_{\partial M_3} d^2\sigma \, \sqrt{g}\, g^{ij} L_i^{\hat{a}} L_j^{\hat{a}} + \mathrm{i}\int_{M_3} s^{IJ} L^{\hat{a}} \wedge \bar{L}^I \gamma^{\hat{a}} \wedge L^J \ , \tag{51}$$

where S^{IJ} has non-vanishing elements $S^{11} = -S^{22} = 1$. This action in $adS_5 \times S^5$ background was given in [8] up to terms of the order θ^4. The complete action in this background is the one in eq. (51) where the pull-back of the space-time forms to the world-sheet $L_i^{\hat{a}}, L_i^I$ is obtained from eqs. (48), (49) with $d\theta \equiv d\sigma^i \partial_i \theta$ and $dx \equiv d\sigma^i \partial_i x$.

In conclusion, we have solved the supergravity superspace constraints for $adS_{p+2} \times S^{d-p-2}$ backgrounds. We have found here the complete expressions for the superfield values of the vielbeins, connections and forms in two different gauges. One gauge that utilizes the space-time Killing spinors of the background leads to a vanishing gravitino superfield $E_\mu^\alpha(x,\theta) = 0$, and a curved fermion-fermion vielbein. In this gauge we observe a dramatic simplification of the superspace structure. The constraints can also be solved in the standard Wess-Zumino gauge. Here, the gravitino superfield picks up terms of linear and higher order in θ, and the fermion-fermion vielbein is flat at $\theta = 0$. For various brane actions both of these gauge choices may be useful.

Thus we have made here a necessary step for constructing the brane actions in $adS_{p+2} \times S^{d-p-2}$ backgrounds. As explained in [12] the classical actions of M2, M5, D3 and some other branes in their near horizon superspace geometry will have two types of symmetries: global superconformal symmetry due to the isometry of the background, and local reparametrization and κ-symmetry. Upon gauge-fixing, the local symmetries of the gauge-fixed theory will have some form of non-linearly realized superconformal symmetry. It will be a challenge to construct such superconformal actions as well as to find the gauges in which the actions take the simplest form.

We have also made progress towards understanding of the GS type IIB string theory in $adS_5 \times S^5$ background. We extended the results of [8], where the action was explicitly given up to θ^4 terms, by solving the constraints exactly which enabled us to give a complete and closed form of the string action in this background.

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