SLAC–PUB–7819
MPI H-V17-1998
May 1998

The Spin Structure of a Polarized Photon *

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Abstract

We show that the first moment of the spin-dependent structure function $g_1^\gamma(x, Q^2)$ of a real photon vanishes independent of the momentum transfer Q^2 it is probed with. This result is non-perturbative: it holds to all orders in perturbation theory in abelian and non-abelian gauge theory and at every twist.

*Work supported in part by the Department of Energy, contract DE–AC03–76SF00515 and by Fondecyt (Chile) under grant 1960536 and by a Cátedra Presidencial (Chile).

1 A sum-rule for g_1^γ

Polarized photon-photon collisions offer a new laboratory for studying QCD spin physics. In polarized deep inelastic scattering the spin-dependent structure function $g_1^\gamma(x, Q^2)$ of a polarized photon [1, 2, 3] is sensitive to the axial anomaly [3, 4] and thus to the realization of chiral symmetry in QCD [5, 6]. The spin-dependent parton distributions of the polarized photon could be measured in photoproduction studies with a polarized proton beam at HERA [7, 8]. Polarized real photon collisions could be studied with high-energy real photon beams at the NLC [9, 10].

A remarkable feature of polarized deep inelastic scattering for $(Q^2 \to \infty)$ on a real photon target is that the leading twist (=2) contribution to the first moment of g_1^γ vanishes[3]. This (deep inelastic) result is nonperturbative and follows directly from electromagnetic gauge invariance and the absence of any exactly massless Goldstone boson in the physical spectrum. In addition, it has recently been shown[11] that the first moment of the box graph contribution to polarized $\gamma\gamma$ fusion vanishes when one or both of the incident photons is real – independent of the virtuality of the second photon. In this paper we generalize these two results and show that the first moment of g_1^γ for a real photon vanishes to all orders and at every twist.

Consider polarized $\gamma - \gamma$ scattering where σ_A and σ_P denote the two cross-sections for the absorption of a transversely polarized photon with spin anti-parallel σ_A and parallel σ_P to the spin of the target photon. The photons in a lepton-lepton collider can be real or spacelike. We let q_μ and p_μ denote the momentum of the "incident" and "target" photons and define $Q^2 = -q^2$, $P^2 = -p^2$ and $\nu = p.q$. The spin-dependent part of the total $\gamma\gamma$ cross-section is given by

$$(\sigma_A - \sigma_P) = \frac{8\pi^2 \alpha}{\mathcal{F}} g_1^\gamma(Q^2, \nu, P^2). \tag{1}$$

Here g_1^γ is the target photon's spin-dependent structure function, and $\mathcal{F}$ is the flux factor for the incident photon. The flux factor is discussed in Eqs. (5-8) below. The structure function $g_1^\gamma(Q^2, \nu, P^2)$ is symmetric under the exchange of the incident and target photons $(p \leftrightarrow q)$. There is no g_2 contribution to $(\sigma_A - \sigma_P)$ [1, 2].

Now consider a real photon beam: $Q^2 = 0$. The Drell-Hearn-Gerasimov sum-rule [12] (— for a review see [13]) for spin-dependent photoproduction tells us that the integral $\int_0^\infty \frac{d\nu}{\nu}(\sigma_A - \sigma_P)$ is proportional to α times the square of the (photon) target's anomalous magnetic moment. The Drell-Hearn-Gerasimov sum-rule is derived from the dispersion relation for the spin-dependent part of the forward Compton amplitude.[†] It follows from the general principles of causality, unitarity, Lorentz and electromagnetic gauge invariance and the assumption that g_1^γ satisfies an unsubtracted dispersion relation. Modulo this no-subtraction hypothesis, the Drell-Hearn-Gerasimov sum-rule is valid for a target of arbitrary spin S, whether elementary or composite [15].

For a real incident photon the flux factor $\mathcal{F} = \nu$. Furry's theorem tells us that the photon has zero anomalous magnetic moment (both in QED and in QED coupled to QCD). It follows that

$$\int_0^\infty \frac{d\nu}{\nu}(\sigma_A - \sigma_P) = 8\pi^2\alpha \int_{\nu_{th}}^\infty \frac{d\nu}{\nu}\frac{g_1^\gamma}{\nu} = 0, \qquad (P^2 = Q^2 = 0). \tag{2}$$

Here ν_{th} is the threshold energy: $\nu_{th} = 2m_e^2$ in QED and $\nu_{th} = \frac{1}{2}m_\pi^2$ in QCD. Eq. (2) is a non-perturbative result. It holds to all orders in perturbation theory in both QED and QCD. If we replace the photon target by a $W^\pm$ boson target, then the Drell-Hearn-Gerasimov integral (2) is finite starting at $\mathcal{O}(\alpha^3)$ since the $W^\pm$ boson has a finite anomalous magnetic moment starting at $\mathcal{O}(\alpha)$ [16].

We now generalize this result to the case where one of the two photons becomes virtual: $Q^2 > 0$. Furry's theorem implies that the anomalous magnetic moment of a photon vanishes independently of whether the photon is real or virtual. Since g_1^γ and ν are each symmetric under the exchange of $(p \leftrightarrow q)$, we can treat the virtual photon as the target and the real photon as the beam, and then apply the Drell-Hearn-Gerasimov sum-rule to find

$$I^\gamma(Q^2) \equiv \int_{\nu_{th}}^\infty \frac{d\nu}{\nu}\frac{g_1^\gamma(\nu, Q^2, P^2)}{\nu} = 0 \tag{3}$$

independent of Q^2 provided that $P^2 = 0$. Changing the integration variable from ν

[†]The Drell-Hearn-Gerasimov sum-rule is derived for QED and QCD with a finite mass gap (massive fermions). The dispersion relation for the spin-dependent part $f_2(\nu)$ of the forward Compton amplitude relates the integral on the right hand side of Eq. (2) to the first derivative of the real part of $f_2(\nu)$ evaluated at $\nu \to 0$. Provided that there is a finite mass gap between the ground state and continuum contributions to forward Compton scattering, when we take the low energy limit that $\nu \to 0$ the leading term in $\mathrm{Re}f_2(\nu)$ is proportional to ν times the square of the target's anomalous magnetic moment [14, 15].

to Bjorken $x = \frac{Q^2}{2\nu}$, we can rewrite Eq. (3) as

$$I^\gamma(Q^2) = \frac{2}{Q^2}\int_0^{x_{max}} dx g_1^\gamma(x, Q^2, P^2 = 0) = 0 \qquad \forall Q^2. \tag{4}$$

The threshold factors in Eqs. (3) and (4) are $\nu_{th} = (Q^2 + 4m_e^2)/2$ and $x_{max} = Q^2/(Q^2 + 4m_e^2)$ in QED, and $\nu_{th} = (Q^2 + m_\pi^2)/2$ and $x_{max} = Q^2/(Q^2 + m_\pi^2)$ in QCD.

The function $I^\gamma(Q^2)$ interpolates between $Q^2 = 0$ and polarized deep inelastic scattering. The corresponding integral for a nucleon target was introduced previously by Anselmino, Ioffe and Leader in [17].

Equations (3) and (4) give our main result. A corollary is that g_1^γ must change sign at least once at a value $x = x^*(Q^2)$ since the first moment of g_1^γ vanishes. The crossing point x^* for the box graph contribution to polarized $\gamma\gamma$ fusion has been calculated in [11].

It is important to note that the new sum-rule (4) involves g_1^γ instead of $(\sigma_A - \sigma_P)$. For real incident photons the flux factor $\mathcal{F}$ is equal to $\nu = p.q$. For virtual incident photons the flux factor is convention dependent subject to the requirement that

$$\lim_{Q^2 \to 0} \mathcal{F} = \nu. \tag{5}$$

There are two popular choices due to Gilman [18] and Hand [19] which are used in virtual-photon nucleon collisions. Both of these conventions readily generalize to photon targets as follows:

$$\mathcal{F}_{\text{Gilman}} = \sqrt{\nu^2 + P^2 Q^2} \tag{6}$$

and

$$\mathcal{F}_{\text{Hand}} = \nu(1 - x). \tag{7}$$

In a recent paper [11], Brodsky and Schmidt have employed:

$$\mathcal{F}_{\text{BS}} = \nu = \frac{1}{2}(s + Q^2 + P^2). \tag{8}$$

The Gilman and the Brodsky-Schmidt conventions preserve the $(p \leftrightarrow q)$ symmetry between the target and incident photons whereas the generalized Hand convention does not. Using $\mathcal{F}_{\text{BS}}$, Brodsky and Schmidt [11] discovered that the box graph, $\mathcal{O}(\alpha^2)$, contribution to $(\sigma_A - \sigma_P)$ in polarized photon-photon fusion satisfies Eq. (2) with $Q^2 > 0$. The sum-rule (4) generalizes their result to all orders.

In the remainder of this paper we explore the symmetry properties of the box graph contribution to g_1^γ after we impose various kinematic cut-offs to separate the total phase space into "hard" and "soft" contributions. We discuss the application of these symmetry arguments to factorization in the QCD parton model. We then use the $(p \leftrightarrow q)$ symmetry of g_1^γ to show that Eq. (4) holds twist by twist in polarized deep inelastic scattering. Finally, we extend our results to the *gedanken* world of massless quarks in QCD where the Drell-Hearn-Gerasimov sum-rule is not guaranteed to hold.

2 $(p \leftrightarrow q)$ symmetry and photon-photon fusion

Consider the box graph contribution to photon-photon fusion. It is illuminating to evaluate the box graph with a cut-off on the transverse momentum squared of the struck quark relative to the photon-photon direction: $k_T^2 \geq \lambda^2$. The cut-off separates the total phase space into "hard" ($k_T^2 \geq \lambda^2$) and "soft" ($k_T^2 < \lambda^2$) contributions. One finds [20]:

$$\begin{aligned} g_1^\gamma(x, Q^2, P^2)|_{\rm hard} \quad = \quad & -\frac{\alpha}{\pi} \frac{\sqrt{1 - \frac{4(m^2+\lambda^2)}{s}}}{1 - \frac{4x^2P^2}{Q^2}} \Biggl[(2x-1)(1 - \frac{2xP^2}{Q^2}) \\ & \Biggl(1 - \frac{1}{\sqrt{1 - \frac{4(m^2+\lambda^2)}{s}}\sqrt{1 - \frac{4x^2P^2}{Q^2}}} \ln\Bigl(\frac{1 + \sqrt{1 - \frac{4x^2P^2}{Q^2}}\sqrt{1 - \frac{4(m^2+\lambda^2)}{s}}}{1 - \sqrt{1 - \frac{4x^2P^2}{Q^2}}\sqrt{1 - \frac{4(m^2+\lambda^2)}{s}}}\Bigr)\Biggr) \\ & + (x - 1 + \frac{xP^2}{Q^2}) \frac{\Bigl(2m^2(1 - \frac{4x^2P^2}{Q^2}) - P^2x(2x-1)(1 - \frac{2xP^2}{Q^2})\Bigr)}{(m^2+\lambda^2)(1 - \frac{4x^2P^2}{Q^2}) - P^2x(x - 1 + \frac{xP^2}{Q^2})}\Biggr] \end{aligned} \tag{9}$$

for each type of fermion liberated into the final state ‡ . Here m is the fermion mass, x is the Bjorken variable ($x = \frac{Q^2}{2\nu}$) and s is the center of mass energy squared

$$s = (p+q)^2 = Q^2\Bigl(\frac{1-x}{x}\Bigr) - P^2 \tag{10}$$

for the photon-photon collision. In perturbative QCD the box graph contribution to the spin structure function of a polarized gluon $g_1^{(g)}(x, Q^2, P^2)$ for $k_T^2 \geq \lambda^2$ is obtained from Eq. (9) by substituting $\frac{\alpha}{\pi}$ by $\frac{\alpha_s}{2\pi}$.

In general, the cut-off λ^2 may be chosen to be a function of x [16-20]:

$$\lambda^2 = \lambda_0^2 f_0(x) + P^2 f_1(x) + m^2 f_2(x). \tag{11}$$

‡Quark contributions to g_1^γ are obtained by multiplying the right hand side of Eq. (9) by the number of colors ($N_c = 3$).

If we set λ^2 to zero, thus including the entire phase space, then we obtain the full box graph contribution to g_1^γ. If we take λ^2 to be finite and independent of x, then the crossing symmetry of g_1^γ under the exchange of $(p \leftrightarrow q)$ is realized separately in each of the "hard" and "soft" parts of g_1^γ which correspond to phase space with $(k_T^2 > \lambda^2)$ and $(k_T^2 < \lambda^2)$ respectively. We could also choose an x-dependent cut-off on the struck quark's virtuality [20, 21]

$$m^2 - k^2 = P^2 x + \frac{k_T^2 + m^2}{(1-x)} > \lambda_0^2 = \text{constant}(x) \tag{12}$$

or a cut-off on the invariant mass squared of the quark-antiquark component of the light-cone wavefunction of the target photon [22, 24]

$$\mathcal{M}_{q\overline{q}}^2 = \frac{k_T^2 + m^2}{x(1-x)} + P^2 \geq \lambda_0^2 = \text{constant}(x). \tag{13}$$

Substituting Eqs. (11-13) into Eq. (9) we find that the "hard" and "soft" contributions to g_1^γ do not separately satisfy the $(p \leftrightarrow q)$ symmetry of $g_1(x, Q^2)$ if use an x-dependent cut-off to define the "hard" part of the total phase space. The reason for this is that the transverse momentum is defined perpendicular to the plane spanned by p_μ and q_μ in momentum space. The x-dependent cut-offs mix the transverse and longitudinal components of momentum. They induce a violation of crossing symmetry in $g_1|_{\text{hard}}(x, Q^2, P^2)$ under $(p \leftrightarrow q)$.

If we set P^2 and λ^2 to zero in Eq. (9) we obtain the box graph contribution to g_1^γ for a real photon target:

$$g_1^\gamma = -\frac{\alpha}{\pi}\sqrt{1 - \frac{4m^2}{s}}\Big[(2x-1)\Big(1 - \frac{1}{\sqrt{1-\frac{4m^2}{s}}}\ln\Big(\frac{1+\sqrt{1-\frac{4m^2}{s}}}{1-\sqrt{1-\frac{4m^2}{s}}}\Big) + 2(x-1)\Big)\Big]. \tag{14}$$

The discovery in Ref.[11] is that Eq. (4) vanishes for the box graph contribution — Eq. (14). The structure function g_1^γ in Eq. (14) can be written as the sum of two contributions $g_1^\gamma|_{\text{like}}$ and $g_1^\gamma|_{\text{unlike}}$ where the two fermions in the final state have the same spin $(g_1^\gamma|_{\text{like}})$ and opposite spins $(g_1^\gamma|_{\text{unlike}})$. Working in the limit $Q^2 \gg m^2$, Freund and Sehgal [6] have found that the first moments of $g_1^\gamma|_{\text{like}}$ and $g_1^\gamma|_{\text{unlike}}$ yield the explicit and anomalous chiral symmetry breaking contributions to the photon's axial charge. These two contributions cancel in the deep inelastic limit $(P^2 \ll m^2 \ll Q^2)$ [25, 21].

The $(p \leftrightarrow q)$ symmetry of $g_1|_{\rm hard}(x, Q^2, P^2)$ has application to the QCD parton model. The parton model description of polarized deep inelastic scattering involves writing the deep inelastic structure functions as the sum over the convolution of "soft" quark and gluon parton distributions with "hard" photon-parton scattering coefficients. The flavor-singlet part of g_1 may be written

$$g_1|_{\rm singlet} = \frac{1}{9}\Bigl(\sum_q \Delta q \otimes C^q + N_f \Delta g \otimes C^g\Bigr). \tag{15}$$

Here, Δq and Δg denote the quark and gluon parton distributions, C^q and C^g denote the corresponding hard scattering coefficients, and N_f is the number of quark flavors liberated into the final state. The parton distributions are target dependent and describe a flux of quark and gluon partons into the hard (target independent) photon-parton interaction which is described by the coefficients. The separation of g_1 into "hard" and "soft" is not unique and depends on the choice of factorization scheme [16-20].

We can use the kinematic cut-off on the partons' transverse momentum squared k_T^2 to define the factorization scheme and thus separate the hard and soft parts of the phase space for the photon-parton collision. Following Eq. (11), this cut-off may be x-dependent or x-independent. In the QCD parton model $g_1^{(g)}|_{\rm hard}(x, Q^2)$ is a suitable candidate for the hard coefficient C^g in photon-gluon fusion. Among the possible kinematic cut-offs, the x independent cut-off on the transverse momentum squared preserves the crossing symmetry of $g_1^{(g)}$ under $(p \leftrightarrow q)$ in both the hard gluonic coefficient $C^g = g_1^{(g)}|_{\rm hard}(x, Q^2)$ and the soft polarized quark distribution of the gluon $\Delta q^{(g)} = g_1^{(g)}|_{\rm soft}(x, Q^2)$.

The x-independent cut-off is especially suited to discussions about the axial anomaly in polarized deep inelastic scattering. Suppose that we evaluate the box graph contribution to the first moment of $g_1^{(g)}$ with an x-independent cut-off: $k_T^2 > \lambda^2$ where $(m^2, P^2 \ll \lambda^2 \ll Q^2)$. Then, we find the axial-anomaly [26, 27] as a contact photon-gluon interaction associated with $k_T^2 \sim Q^2$ [25]. On the other hand, the first moment of $g_1^{(g)}$, defined using the quark virtuality $(-k^2)$ cut-off yields "half of the anomaly" in the gluon coefficient through the mixing of transverse and longitudinal momentum components [20, 21]. The anomaly coefficient for the first moment is recovered with the invariant mass squared cut-off through a sensitive cancelation of large and small x contributions [21].

3 Twist expansion for g_1^γ when $Q^2 \to \infty$

The light-cone operator product expansion at large Q^2 relates the first moment of the structure function g_1^γ to the scale-invariant axial charges of the target photon [1, 2, 3] plus an expansion of higher-twist matrix elements:

$$\begin{aligned}
&\int_0^1 dx g_1^\gamma(x, Q^2, P^2) \qquad (16)\\
&= \left(\frac{1}{12}a^{(3)} + \frac{1}{36}a^{(8)}\right)\Big\{1 + \sum_{\ell\geq 1} c_{\mathrm{NS}\ell}\,\bar{g}^{2\ell}(Q)\Big\} + \frac{1}{9}a^{(0)}|_{\mathrm{inv}}\Big\{1 + \sum_{\ell\geq 1} c_{\mathrm{S}\ell}\,\bar{g}^{2\ell}(Q)\Big\}\\
&\quad + \sum_{j=1}^{\infty}\left(\frac{P^2}{Q^2}\right)^j \{\text{twist } (2+2j) \text{ operator matrix elements}\}\\
&\quad + \sum_{j=1}^{\infty}\left(\frac{m^2}{Q^2}\right)^j \sum_{k=0}^{\infty}\left(\frac{P^2}{Q^2}\right)^k \{\text{twist } (2+2k) \text{ operator matrix elements}\}.
\end{aligned}$$

where m is the quark mass. (We refer to [28] for a complete derivation of the twist-4 contributions to deep inelastic scattering from a nucleon target.)

For photon states $|\gamma(p,\lambda)\rangle$ with momentum p_μ and polarization λ

$$ia^{(k)}\epsilon_{\mu\nu\alpha\beta}p^\nu\epsilon^\alpha(\lambda)\epsilon^{*\beta}(\lambda) = \langle\gamma(p,\lambda)|J^{(k)}_{\mu 5}|\gamma(p,\lambda)\rangle_c \qquad (17)$$

where $k = (3, 8, 0)$ and the subscript c denotes the connected matrix element. The non-singlet isovector and SU(3) octet currents are

$$\begin{aligned}
J^{(3)}_{\mu 5} &= \left(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d\right)\\
J^{(8)}_{\mu 5} &= \left(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s\right) \qquad (18)
\end{aligned}$$

and

$$J^{(0)}_{\mu 5} = E(g)\ \left(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s\right)_{GI} \qquad (19)$$

is the scale invariant and gauge-invariantly renormalized singlet axial-vector operator. The renormalization group factor $E(g)$ [29] compensates for the non-zero anomalous dimension [30, 31, 32, 33] of the singlet axial-vector current $J^{(0)}_{\mu 5}/E(g)$. The flavor non-singlet $c_{\mathrm{NS}\ell}$ and singlet $c_{\mathrm{S}\ell}$ coefficients are calculable in ℓ-loop perturbation theory [34]. There are no twist-two, spin-one, gauge invariant photon or gluon operators which can contribute to the first moment of g_1^γ [35].

One can derive a rigorous sum-rule for the leading twist (=2) contribution to the first moment of g_1^γ in polarized deep inelastic scattering where one of the photons is deeply virtual ($Q^2 \to \infty$) and the other photon is either real [3] or carries small but finite virtuality [5]. Electromagnetic gauge-invariance implies [3] that the axial charges of a real photon vanish provided that there is no exactly massless Goldstone boson coupled to $J^{(k)}_{\mu 5}$, which is certainly true in nature with massive quarks. For real photons we find [3]:

$$\int_0^1 dx g_1^\gamma|_{\{\text{twist }2\}}(x,Q^2,P^2) = 0, \qquad (P^2=0,\ Q^2 \to \infty). \tag{20}$$

This deep inelastic sum-rule holds at every order in perturbation theory – starting with the box graph for photon - photon fusion. Comparing Eqs. (20) and (4) we find that the vanishing of the leading twist contribution to $\int_0^1 dx g_1^\gamma$ is a special case of the Drell-Hearn-Gerasimov sum-rule when the real photon is treated as the beam and the deeply virtual photon is treated as the target.

We now consider the higher-twist terms.

The higher twist terms receive contributions from both the "handbag" and "cat-ears" diagrams. To classify these terms we note that there are five scales in the physical problem: Q^2, P^2, ν, the quark mass m and a QCD scale Λ associated with non-perturbative bound-state dynamics. We integrate over the scale ν when we evaluate the first moment of g_1^γ.

To understand the higher-twist terms in Eq. (16) it is helpful to first consider the abelian QED contributions to g_1^γ. There are higher-twist terms proportional to non-zero powers of $\frac{P^2}{Q^2}$ and $\frac{m_e^2}{Q^2}$. The terms proportional to $\frac{P^2}{Q^2}$ vanish for a real photon target ($P^2 = 0$). The higher-twist terms proportional to $\frac{m_e^2}{Q^2}$ start with the leading twist (=2) operator matrix element. Fermion mass terms make a non-leading contribution to the Dirac trace over γ_μ matrices when we evaluate g_1^γ to any given order in α. They yield a unity matrix contribution to the trace so that the leading term in the Dirac trace is the twist-two operator matrix element. Since the photon's axial charges $a^{(k)}$ vanish when $P^2 = 0$ it follows that the higher-twist contributions to (16) vanish for a real photon target in QED.

In QCD we also have to consider the possible effects of Λ and whether we can have higher-twist terms proportional to $\frac{\Lambda^2}{Q^2}$ beyond the higher-twist terms listed in Eq. (16). This would also include vector meson dominated contributions to the cross section.

If we could calculate g_1^γ exactly in QCD, we would find an expression which is symmetric under $(p \leftrightarrow q)$. This symmetry imposes strong constraints on the possible Λ dependence of g_1^γ. As an example, recall that the box contribution $g_1^\gamma|_{\rm hard}$ in Eq. (9) is symmetric under $(p \leftrightarrow q)$ only with a special choice of infrared cut-off (independent of x). If we impose the physically sensible condition of not allowing Λ^2 to scale with the kinematic variables, then we find that any higher-twist contribution involving Λ^2 comes from rescaling the quark mass in one or more terms in the complete QCD expression for g_1^γ, viz. $m^2 \to (m^2 + \Lambda^2)$. That is, if there are higher-twist terms in g_1^γ proportional to $\frac{\Lambda^2}{Q^2}$, then they effectively induce a constituent-quark mass-term in the higher-twist expansion. These higher-twist terms thus also vanish for $P^2 = 0$ because the photon's axial charges vanish on-shell.

4 Massless QCD

It is interesting to extend our results to QCD with massless quarks. If we could turn the up, down and strange quark masses to zero in QCD, then the pion and the η would evidently become massless but, because of $U_A(1)$ dynamics [36], the η' would remain massive. Consider the gedanken world of massless QCD where we define real photons by first taking the light-quark masses to zero and then taking the photon virtuality to zero — that is, working in the limit $m^2 \ll P^2 \to 0$. In this *gedanken* world the real-photon's isotriplet $a^{(3)}$ and octet $a^{(8)}$ axial-charges would no longer vanish but instead would be equal to $-\frac{\alpha}{\pi} N_c$ where $N_c = 3$ is the number of colors [3, 5]. The singlet axial-charge $a_0|_{\rm inv}$ would remain zero since the photon matrix elements of $J^{(0)}_{\mu 5}$ would not contain a massless pole contribution (because of the massive η'). The non-vanishing of the non-singlet $a^{(k)}$ in massless QCD does not contradict our general result (4) because, even for the photon-photon fusion process (9), the low energy theorem [14, 15] which relates the Drell-Hearn-Gerasimov integral to the (vanishing) anomalous magnetic moment of the target photon is derived assuming that the fermions have a finite mass.

Gorskii, Ioffe, and Khodjamirian [37] have found a similar, anomalous, result in unpolarized photon-photon scattering. Consider the box graph cross-section for a hard transverse photon γ_T with virtuality Q^2 to scatter from a soft longitudinal photon γ_L with virtuality P^2: $\gamma_T \gamma_L \to l^+ l^-$ where l is the charged fermion liberated into the final state. This cross section vanishes when we take $P^2 \to 0$ in QED with a finite mass gap (the fermion mass $m \neq 0$) and also in the particular chiral limit $P^2 \ll m^2 \to 0$. However, the $\gamma_T\gamma_L$ cross-section is finite and non-vanishing in the alternative chiral limit defined by $m^2 \ll P^2 \to 0$.

5 Conclusions

We have shown that the first moment of the structure function $g_1^\gamma(x, Q^2, P^2)$ measured in polarized photon-photon collisions $\gamma(p)\gamma(q) \to X$ vanishes when either or both of the incident photons are on-shell. This sum rule follows from the Drell-Hearn-Gerasimov sum rule and simple $p \leftrightarrow q$ symmetry properties of the two-photon system. It holds in QED and QCD to all orders and at every twist provided that the fermions in the theory have non-vanishing mass.

Acknowledgments

We thank A. H. Mueller for helpful conversations. This work was supported in part by the United States Department of Energy under contract number DE–AC03–76SF00515, by Fondecyt (Chile) under grant 1960536, and by a Cátedra Presidencial (Chile).

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