Some Thoughts on COMMUTATION RELATIONS and MEASUREMENT ACCURACY *

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Abstract

We show that measuring the trajectories of charged particles to finite accuracy leads to the commutation relations needed for the derivation of the free space Maxwell equations using the *discrete ordered calculus* (DOC). We note that the finite step length derivation of the discrete difference version of the single particle Dirac equation implies the discrete version of the p, q commutation relations for a free particle. We speculate that a careful operational analysis of the change in momenta occurring in a step-wise continuous solution of the discrete Dirac equation could supply the missing source-sink terms in the DOC derivation of the Maxwell equations, and lead to a finite and discrete ("renormalized") quantum electrodynamics (QED).

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1 INTRODUCTION

Our derivation of the free space Maxwell equations using the *discrete ordered calculus* (DOC)[2] mentioned that the postulated commutation relations between position and velocity could be interpreted as a consequence of a fixed discrepancy between first measuring position and then velocity or visa versa. However, these commutation relations were not given a careful physical justification in terms of our finite measurement accuracy philosophy [5]. A second deficiency, which in fact caused us to warn the reader that we had only derived one part of the *formalism* of classical electrodynamics rather than the theory itself, was that no attempt was made to identify the sources and sinks of the "fields" and derive the inhomogeneous Maxwell equations from them. We took a step in that direction by our derivation[3] of a finite and discrete version of the 1+1 free space Dirac equation from a fixed step-length *Zitterbewegung* postulate using finite difference equations. Although it was noted that an attempt had been made by me[4] to attribute the *Zitterbewegung* to the conservation of spin or particle number in the presence of random electromagnetic fluctuations, no attempt was made to relate these interactions to the source terms needed to complete the argument in the Maxwell equations paper. Neither Kauffman nor I have attempted to relate the non-commutativity known to arise from the Dirac equation to the commutation relations needed to derive the Maxwell equations in our finite and discrete context. In this paper I take a few steps to remedy both defects, but more work is needed.

2 ELECTROMAGNETIC MEASUREMENT OF A CHARGED PARTICLE TRAJECTORY

In earlier work I have made use of what I called "the counter paradigm" to cut the Gordian knot of specifying what a physicist means when he says that a particle was or was not present in a finite spacial volume for a finite time duration. As a first approximation, I assume that this volume is the "sensitive volume" of a counter, and the time duration is the time during which the recording device attached to

the counter could have recorded an event, often called a "firing". This I call a NO-YES event, depending on whether the counter did not or did "fire". A more careful treatment specifies the probability of "spurious events", i.e. cases when the counter "should have fired" but did not (counter inefficiency), and the probability of cases when the counter "should not have fired", but according to the record did in fact fire (background events). Ted Bastin has often objected that this abrupt transition from the laboratory to Boolean logic sweeps too much under the rug, and I have often replied that to justify this way of talking about laboratory practice would require a book. Fortunately, Peter Galison has taken ten years to write the book I needed. He separates the history of the material culture of particle physics into a "logic" tradition contrasted with an "image" tradition. My "counter paradigm" finds its appropriate niche as part of the logic tradition. Galison shows that by now the two alternatives have fused in the mammoth "detectors" which are integrated into the accelerators in all high energy particle physics laboratories [1]. It took over a century for this language and practice to mature, and a decade to make a convincing argument as to why it should be accepted by philosophers. I now have a simple tactic open. I can ask any critic of my conceptual leap from counter firings to NO-YES events to first convince me that Galison's defense of the mainstream tradition is inadequate. Only then will I feel any need to take his or her criticism seriously.

This ploy allows me to use conventional language in my descriptions of laboratory measurement. In particular I can now construct a simple paradigm for what I mean by the measurement of the electromagnetic trajectory of a particle. First recall that by a "particle" I mean[5] "a conceptual carrier of conserved quantum numbers between events". I can take the simplest interpretation of two sequential counter firings a fixed distance L apart with a time interval T between them to be that a particle conserving mass, momentum and energy passed between them with velocity L/T. I assume available a "source" of particles which allows a large number of repetitions of these paired sequential events to occur. This data set is assumed to provide both statistical and systematic accuracy adequate for calibrating the *changes* in the magnitude and/or direction of this velocity caused by inserting electromagnetic devices into the path defined by sequential counter firings

The electromagnetic device we consider first, inserted between two counters previously used to measure velocity, is simply two parallel conducting plates with a hole through them across which a constant voltage can be applied. This voltage is measured by standard techniques. When the voltage is negligible, our original source and sink counters still give a velocity $v = L/T$ for each particle "passing through the two holes", showing that we can maintain the same particulate interpretation of the two sequential events with the plates in place, even though we do *not* "measure" the presence of the particles between the plates. We now apply a voltage V across the plates, which are large enough compared to the holes so that, according to standard electrostatic theory, the electric field between the plates and along the direction of motion of the particle is $\mathcal{E} = V/\Delta d$ where Δd is the separation between the plates. We now study the *change* in the velocity of a particle of the type being studied (i.e. produced in the same way or available from the same source) during a time when the voltage across the plates is held at V. Counter firings before the presumed arrival and after the presumed departure of the particle at the device allow us to say that the particle arrived at the position of the plates with velocity v_1 and left with velocity v_2. We then say that the particles have a charge e, a (rest) mass m, an energy E_1 before they enter the first hole, and an energy E_2 after leaving the second hole when, for various experiments, the velocity change produced by the device is equivalent to an energy change

$$\Delta E = E_2 - E_1 = \pm e\mathcal{E}\Delta q; \quad \mathcal{E} = V/\Delta d \tag{1}$$

with

$$E_1 = \frac{m}{\sqrt{1 - (v_1^2/c^2)}}; \quad E_2 = \frac{m}{\sqrt{1 - (v_2^2/c^2)}} \tag{2}$$

We then take this as our paradigm for the *measurement* of an electric field in a region of length Δd of strength $\mathcal{E}$.

We emphasize that this measurement requires a *change* in the velocity of the particle. The minimum change to which we can reliably assign a number *quantizes* our *measurement accuracy* at the level of technology we are using. Note that our paradigm assumes *constant* velocity between measurements in field-free regions. [Recall that we *derived* a discrete version of the constant velocity law from bit-string physics in

our foundational paper[6], Sec. 6.5, pp 94-95.] Alternatively, if we know the field (or voltage) and the (constant velocity) trajectories before and after the device, we can use the *same* device as a paradigm for *position* measurement to an accuracy Δd. By fleshing out this paradigm, we can recursively use electromagnetic language to justify the construction of laboratory counters which have a conceptual connection to those used in our counter paradigm.

Our paradigm for magnetic field (or momentum) measurement assumes that we have two double plates across each of which independently adjustable voltages can be applied. We call the entrance hole of the first pair 1 and the exit hole 2, and for the second plate the entrance hole 3 and the exit hole 4; thus the gaps are d_{12} and d_{34}, and the trajectory is 1,2,3,4. The plates are located geometrically in the laboratory in such a way that a path connecting the exit hole 2 from the first pair to the entrance hole 3 into the other can be an arc of a circle of radius R whose center lies in a plane with the two gaps; the gaps between the plates are two (short) arcs of that circle. The arc between the two devices is of length $R\Delta\theta$. The magnetic field we wish to measure is perpendicular to the plane of the circle and is of constant strength $\mathcal{B}$, along this arc. This is "guaranteed" by the geometry and the standard theory of magnetostatic fields. According to electromagnetic theory, this field does not change the energy of the particle, or the magnitude of its velocity, but does cause the *direction* of the velocity to change. This change is simply described in terms of the momentum $\mathbf{P}$ of vector magnitude

$$\mathbf{P} = \frac{m\mathbf{v}}{\sqrt{1 - (v^2/c^2)}}; \quad |\mathbf{v}| = \frac{R\Delta\theta}{t_3 - t_2} \tag{3}$$

where the time t_2 when the particle exits hole 2 and the time t_3 when it enters hole 3 are usually inferred rather than directly measured; $\mathbf{v}$ is the vector velocity of constant magnitude with a (varying) direction assumed tangent to the arc. The radius of the circle is related to the magnitude of the momentum by

$$R = \frac{eP}{c\mathcal{B}} \tag{4}$$

and the change in momentum (due to change in direction since the magnitude is

constant) by

$$\Delta P = 2P \ sin^2 \Delta\theta/2 = P(1 - cos \ \Delta\theta) \tag{5}$$

As as in the case of electric field measurement, we can consider this arrangement as either a measurement of the field $\mathcal{B}$ at (perpendicular to) the arc $R\Delta\theta$ geometrically defined or as a measurement of the velocity of the particle along that arc. But as a velocity measurement, it is important to realize that there is an ambiguity as to whether this is the measurement of velocity *after* the particle has traversed the first double plate 12, which could be a counter measuring position, or a measurement of velocity *before* it traverses the second double plate 34.

If all we have available are not individual particle detectors, but only devices that measure the charged current flowing along the trajectory, the arrangement discussed above can only be used to measure e/m and not charge and mass separately. Such experiments were, historically, sufficient to convince the proponents of various models of the charge distribution "within the electron" (Abraham, Lorentz, Poincaré) that their models were wrong, and that the Einstein equation connecting mass to velocity used above was correct even though it violated their way of thinking about space and time ([1], Sec. 9.6, pp 810-816). Galison shows by this historically examined case that experimental tradition and the material culture of physics allow theoretical physicists on opposite sides of what Kuhn would call a "paradigm shift" to agree on the significance of experimental results..

The fact that electric and magnetic fields acting on a moving charge effect changes in velocity along or at right angles to the direction of motion respectively allows one to build a "velocity selector" by setting up a region of electrostatic and magnetostatic fields in which the fields are at right angles to each other and both are at right angles to the direction of motion of the charge. The force on the charge due to the electric field is $e\mathcal{E}$ while the force due to the magnetic field is $ev\mathcal{B}/c$ and the geometry we have specified requires these forces to be in the same direction. Consequently there is a unique direction for which they cancel, provided the velocity has magnitude $v = c\mathcal{E}/\mathcal{B}$. A particle of that charge with any other velocity will be deflected away from this direction.

At first glance, such a device would seem to allow us to measure position and velocity "simultaneously". But this is not correct. So long as the charged particle has this velocity and the magnitude and direction of the fields does not change along this straight line trajectory, no force acts and the particle maintains constant velocity. However, we have no way of knowing *where* it is within this region, and hence when it enters and leaves it, without a measurement. But this measurement will change the velocity. So we must measure when the particle enters the region *and* when it leaves the region in order to know how long and when it is in the region with that velocity. As before, we can first measure position and then velocity or first measure velocity, and then position but not both simultaneously. An extended discussion of this case should allow us to see that three points on the trajectory are needed to establish the field at the intermediate point, and four if we are to measure both **E** and **B**. On another occasion we hope to be able to go on to derive the free field commutation relations by such considerations (or directly from our DOC equations), and not just the uncertainty principle restrictions obtained by Bohr and Rosenfeld.

In closing we note that, even though we started out to devise a paradigm for electromagnetic field measurement, we have ended up deriving from this paradigm the DOC postulate that we can first measure position and then velocity or first measure velocity and then position, but not both simultaneously. We hope that this discussion makes it less of a mystery why the DOC postulate leads so directly to the formalism of free-field electromagnetism.

3 FROM FREE DIRAC PARTICLES TO FIELD SOURCES AND SINKS

The derivation of the finite difference version of the free particle Dirac equation[3] for fixed step length $\hbar/mc$ with step velocity $\pm c$ tells us immediately that we can cut the trajectory of a free particle into segments of constant velocity between "points" at which the velocity changes discontinuously. On the other hand our DOC equations for the free space electromagnetic field [2] support solutions corresponding to the

propagation of crossed electromagnetic fields with velocity c and constant frequency which, for finite segments, can be interpreted as "photons" if they have the right amplitude. All we seem to need to produce a quantum electrodynamics which is finite and discrete, and hence "born renormalized", would seem to be to assign a charge to the massive particle which satisfies the Dirac equation in such a way that its discrete changes in velocity correspond to the emission or absorption of such photons. I hope to do this on another occasion. The details will obviously take some time to work out, but will provide a lot of fun along the way.

Since this amounts to solving a finite and discrete "three particle problem", an approach to the same theory which starts more directly from bit-string physics would be to treat the photon as a bound state of a particle-antiparticle pair in the relativistic three body theory now under active development [7].

References

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