SLAC-PUB-7767
hep-th/9803082
March 1998

Two-Form Fields and the Gauge Theory Description of Black Holes

Arvind Rajaraman *†

Stanford Linear Accelerator Center, Stanford, CA 94309

Abstract

We calculate the absorption cross section on a black threebrane of two-form perturbations polarized along the brane. The equations are coupled and we decouple them for s-wave perturbations. The Hawking rate is suppressed at low energies, and this is shown to be reflected in the gauge theory by a coupling to a higher dimension operator.

Submitted to Physical Review D

---

*e-mail address: arvindra@dormouse.stanford.edu

†Supported in part by the Department of Energy under contract no. DE-AC03-76SF00515.

I. INTRODUCTION

It has been proposed recently that the large N limit of maximally supersymmetric SU(N) Yang-Mills theory may be described by supergravity on an AdS background [1]. This proposal was motivated by the agreement between computations of Hawking radiation from black threebranes and the corresponding expectation from gauge theory [2–5,7,9].

Based on this conjecture, Gubser, Klebanov and Polyakov [7] and Witten [8] have given a concrete proposal for how to relate correlation functions in the gauge theory to supergravity computations. In their approach, one calculates the supergravity action in the AdS space subject to certain boundary conditions on the fields. The boundary conditions are treated as the sources for operators on the boundary. One can then read off the correlation function of these operators from the supergravity action.

For practical calculations, one would like to know the solutions at linearized level (at least) for all the bulk fields. This is the first step to being able to compute multipoint correlation functions in the gauge theory.

In this note, we shall consider perturbations of two-form potentials which are polarized parallel to the brane. These fields have coupled equations of motion (as was pointed out in [4].) We are able to decouple these equations in the case of s-wave perturbations, and extract the absorption cross section for quanta of these fields incident on the black hole.

We find that the absorption cross-section for these fields is suppressed at small frequencies relative to minimally coupled scalars. This is somewhat surprising because these scalars are not fixed in the sense of [10,11] and therefore are not expected to have suppressed absorption rates.

We find that the suppression of the absorption rate is reflected in the gauge theory in that these scalars are coupled to higher dimensional operators in the gauge theory, which naturally leads to lower absorption rates. One can formulate a conformally invariant coupling (along the lines of [8]) to describe this interaction. The results are in agreement with the semiclassical calculation.

Related issues have been discussed in [16–34].

II. THE SEMICLASSICAL ANALYSIS

A. The black hole

The black hole background is defined by the metric

$$\begin{aligned} ds^2 &= H^{-1/2}(-dt^2 + dx_a dx^a) + H^{1/2}(dx_i^2) \\ H &= 1 + \frac{R^4}{r^4} \end{aligned} \tag{1}$$

where $a = 1, 2, 3$ labels the coordinates parallel to the brane, $i = 4 \cdots 9$ labels the coordinates perpendicular to the brane.

The four-form field strength is

$$F_{0123r} = H^{-2}\left(\frac{R^4}{r^5}\right) \tag{2}$$

We are considering waves of two-form potentials. The relevant field equations at the linearized level are [14]

$$\nabla^\mu H_{\mu\nu\rho} = \left(\frac{2}{3}\right) F_{\nu\rho\kappa\tau\sigma} F^{\kappa\tau\sigma}$$
$$\nabla^\mu F_{\mu\nu\rho} = -\left(\frac{2}{3}\right) F_{\nu\rho\kappa\tau\sigma} H^{\kappa\tau\sigma} \tag{3}$$

where we have denoted the NSNS two-form field strength as $H_{\mu\nu\rho}$, and the RR two form field strength as $F_{\mu\nu\rho}$. We shall denote the corresponding potentials as $B_{\mu\nu}$ and $A_{\mu\nu}$ respectively.

The above equations show that the perturbations of the two two-form potentials are mixed. In particular, a perturbation of $A_{12} = \Phi$ mixes with perturbations of H_{03r}. In the case of s-waves, i.e. when there is no angular dependence, these equations can be decoupled, and an equation for Φ can be obtained. By symmetry, similar equations can be obtained for $A_{13}, A_{23}, B_{12}, B_{13}$ and B_{23}.

B. Decoupling the equations of motion

We start with the equation (we shall always assume the diagonal form of the metric)

$$\frac{1}{\sqrt{g} g^{rr} g^{33}} \partial_0 (\sqrt{g} g^{00} g^{33} g^{rr} H_{0r3}) = \left(\frac{2}{3}\right) F_{r3\mu\nu\rho} F^{\mu\nu\rho}$$
$$= 4 F_{r3012} F^{012} \tag{4}$$

which gives (assuming everything goes as $e^{-i\omega t}$, we set $\partial_0 = -i\omega$)

$$(i\omega) g^{00} H_{0r3} = \left(\frac{4}{3}\right) F_{r3012} g^{11} g^{22} g^{00} (i\omega) \Phi \tag{5}$$

that is,

$$H_{0r3} = \left(\frac{4}{3}\right) F_{r3012} g^{11} g^{22} \Phi \tag{6}$$

We now turn to the equation

$$\frac{1}{\sqrt{g} g^{11} g^{22}} \partial_r (\sqrt{g} g^{rr} g^{11} g^{22} \partial_r \Phi) - \omega^2 g^{00} \Phi = 12 F_{120r3} H^{0r3} \tag{7}$$

Using (6), we can simplify this to

$$\partial_r (H r^5 \partial_r \Phi) + \omega^2 H^{1/2} \Phi = 16 \frac{R^8}{r^5 H} \Phi \tag{8}$$

C. Solving the equations

We will solve this in various regions.

Far from the horizon ($r \gg R$), we can set $H = 1$, and $R = 0$. The equation is then

$$\frac{1}{r^5}\partial_r(r^5\partial_r\Phi) + \omega^2\Phi = 0$$

Using the standard substitution $\Phi = r^{-5/2}$, , we get

$$\left(\partial_r^2 - \frac{15}{4r^2} + \omega^2\right), \quad = 0$$

with the usual solution [3]

$$\Phi = c_1 r^{-2} J_2(\omega r) + c_2 r^{-2} N_2(\omega r) \tag{9}$$

In the intermediate region ($r \sim R \ll \omega^{-1}$), we set $\omega = 0$. The equation is then

$$\partial_r(Hr^5\partial_r\Phi) = 16\frac{R^8}{r^5 H}\Phi$$

with the solution

$$\Phi = c_3 H + c_4 H^{-1} \tag{10}$$

Finally, near the horizon ($r \ll R$), we approximate

$$H = \frac{R^4}{r^4}$$

The equation then becomes

$$r\partial_r(r\partial_r\Phi) + \left(\frac{\omega^2 R^4}{r^2} - 16\right)\Phi = 0$$

with the solution

$$\Phi = J_4\left(\frac{\omega R^2}{r}\right) + iN_4\left(\frac{\omega R^2}{r}\right) \tag{11}$$

where we have chosen the solution for an ingoing wave.

To match the intermediate solution, we need

$$c_3 = \frac{\omega^4 R^4}{16, (5)} \qquad c_4 = \frac{-96i}{\pi(\omega R)^4} \tag{12}$$

Matching the intermediate to the outer solution, we get

$$c_1 = \left(\frac{8}{\omega^2}\right)(c_3 + c_4) \qquad c_2 = -\left(\frac{\omega^2 R^4 \pi}{4}\right)(c_3 - c_4) \tag{13}$$

The solution for large r tends to $c_1cos(\omega r) + c_2sin(\omega r)$. The absorption cross section is then

$$A = 1 - \|\frac{c_2 + ic_1}{c_2 - ic_1}\|^2$$

which is easily evaluated to be

$$A = \left(\frac{\pi^2}{2^{14}3^2}\right)(\omega R)^{12}$$

The cross-section is then obtained by the formula

$$\sigma = \frac{32\pi^2}{\omega^5}A = \left(\frac{\pi^4}{2^9 3^2}\right)\omega^7 R^{12} \tag{14}$$

It may seem odd that we get a cross-section that goes to zero more quickly than ω^3 (the behaviour exhibited by minimal scalars), which is reminiscent of the behaviour of fixed scalars and intermediate scalars [11–13]. This may be unexpected since the scalar we are considering is not expected to be fixed (in the sense of [10]), since it can take on any value in the black hole background. It nevertheless has a Hawking rate that is suppressed, essentially because in the near horizon region it has an effective mass term similar to the effective mass term of fixed scalars. The presence of this effective mass term is confirmed by the analysis of [15], who have worked out the wave equations of all the supergravity fields in an AdS background.

In the next section, we shall see that the suppression of the Hawking rate is reflected in the gauge theory by the fact that the scalar we are considering couples to a higher dimension operator on the threebrane worldvolume.

III. THE GAUGE THEORY ANALYSIS

We now turn to the extraction of gauge theory correlators from the absorption amplitudes. This is a straightforward extension of the analysis of [7,8] .We shall attempt to clarify the relation between the two procedures.

We will focus on the near horizon equation

$$z\partial_z(z\partial_z\Phi) + \left(\omega^2 z^2 - 16\right)\Phi = 0 \tag{15}$$

where we have defined

$$z = \frac{R^2}{r}$$

We analytically continue to spacelike momenta, in which region the solutions are $K_4(\omega z)$ and $I_4(\omega z)$. We keep the solution $K_4(\omega z)$, which is the one which decays exponentially at the horizon $z \to \infty$.

The idea of [7,8] is that one solves the above equation for a given choice of boundary conditions for small z, which is taken to be the value of the field on the boundary. We then treat the boundary field as the source for an operator $\mathcal{O}$ which lives only on the boundary. The Green's functions of $\mathcal{O}$ are then generated by the functional obtained by substituting the full solution of (15) into the supergravity action.

In this case, the full solution is $K_4(\omega z)$, which for small z diverges as z^{-4}. Accordingly, we need to specify a cutoff. The problem is then that the boundary value is highly sensitive to the choice of cutoff.

It is natural, therefore, to associate the boundary field not to Φ directly, but rather to the boundary value of $\Phi_0 = z^4\Phi$. This is stable in the sense that if we move the cutoff from $z = z_0$ to (say) $z = 2z_0$, the value of Φ_0 does not change drastically. This is the same setup as in section 2.5 of [8].

We can then proceed as before, by coupling the field Φ_0 to a boundary operator $\mathcal{O}$, and extracting the correlation functions of $\mathcal{O}$ from the supergravity action.

Let us see explicitly how this works for the case of a massive scalar. We shall differ slightly from the method of [7], in that we shall set $\kappa = 1$, and explicitly follow all factors of R.

The equation of motion is

$$z^5\partial_z\left(\frac{1}{z^3}\partial_z\chi\right) - z^2\omega^2\chi - m^2\chi = 0$$

which has the solution

$$\chi = z^2K_\nu(wz) \qquad \nu^2 = m^2 + 4$$

In [7], the boundary condition chosen was

$$\chi \sim 1 \qquad at \quad z = R$$

We shall modify this condition in our case to

$$\chi \sim \frac{R^{\nu-2}}{z^{\nu-2}}$$

for small ωz, which fixes the solution to be

$$\chi = R^{\nu-2}\omega^\nu z^2 K_\nu(wz)$$

We now substitute this solution into the supergravity action. As shown in [7,8], this can be reduced to a surface term at the boundary $z = R$,

$$I[\chi] \sim R^8\left[\left(\frac{1}{z^3}\right)\chi\partial_z\chi\right]_R \tag{16}$$

To extract the absorption cross-section, we need the nonanalytic part of $I(\chi)$as in [5], which is provided by the logarithm in the expansion of $K_\nu(z)$,

$$K_\nu(z) \sim 2^{n-1},\,(n)z^{-\nu}(1+\cdots) + (-)^{n+1}\left(\frac{1}{2^n,\,(n+1)}\right)ln\left(\frac{1}{2}z\right)(z^\nu+\cdots)$$

where $\cdots$ represent higher orders in z. The leading nonanalytic term in (16) then scales as

$$R^8 R^{2\nu-4}\left(\frac{1}{z^3}\right)(\omega^{2\nu}z^4)(\omega^{-n}z^{-\nu-1})ln(wz)(\omega z)^\nu \sim R^{2\nu+4}\omega^{2\nu}ln(wz)$$

Upon Fourier transforming to position space, we find that the two point function of the boundary operator scales as

$$\langle\mathcal{O}(x)\mathcal{O}(0)\rangle \sim \partial^{2\nu}\frac{1}{x^4}$$

indicating that the dimension of the operator is

$$\Delta = 2+\nu = 2+\sqrt{4+m^2} \tag{17}$$

in exact agreement with [8].

The reason that the coupling is still conformal is that the boundary value χ_0 has now acquired a dimension. In this case, this can be seen in that if we shift the position of the boundary from $z=R$ to $z=\lambda R$, the relation between χ and χ_0 changes from

$$\chi_0 = r^{\nu-2}\chi$$

to

$$\chi_0 = \lambda^{\nu-2}r^{\nu-2}\chi$$

Since χ itself was a canonical scalar field, χ_0 is not (as otherwise the relation would not change under this rescaling.) In fact, it is a conformal density of dimension $2-\nu$ [8]. The coupling $\chi_0\mathcal{O}$ therefore has dimension 4, and is a conformally invariant term. This is in spite of the fact that we have introduced a higher dimension operator which will result in a suppressed Hawking rate.

It is simple to repeat this analysis for massive p-forms. The equation of motion is

$$z^{5-2p}\partial_z\left(\frac{1}{z^{3-2p}}\partial_z\chi^{(p)}\right) + z^2\omega^2\chi^{(p)} - m^2\chi^{(p)} = 0$$

with the solution

$$\chi^{(p)} = R^{\nu+p-2}\omega^\nu z^{2-p}K_\nu(wz) \qquad \nu^2 = m^2+(2-p)^2 \tag{18}$$

where we have normalized the solution to go as

$$\chi^{(p)} \sim \frac{R^{\nu+p-2}}{z^{\nu+p-2}}$$

for small ωz.

We can reduce the action to a surface term as before

$$I[\chi^{(p)}] \propto \left[\left(\frac{z^{2p-3}}{R^{2p-8}}\right)\chi^{(p)}\partial_z\chi^{(p)}\right]_R \tag{19}$$

Approximating the behaviour of $\chi^{(p)}$ at small z as before, we find the leading nonanalytic term

$$I \sim \frac{z^{2p-3}}{R^{2p-8}}(R^{2\nu+2p-4}\omega^{2\nu}z^{4-2p})(\omega^{-\nu}z^{-\nu-1})(\omega^{\nu}z^{\nu}ln(wz)) \sim R^{2\nu-4}\omega^{2\nu}ln(wR)$$

Hence the dimension of the operator on the boundary is

$$\Delta = 2 + \nu = 2 + \sqrt{m^2 + (2-p)^2} \tag{20}$$

which is the correct result [8,25], as the m^2 refers not to the eigenvalue of the Laplacian, but to the eigenvalue of the Maxwell operator ($\tilde{m}^2$ in [25].)

There is however a puzzle in the comparison of the gauge theory to the semiclassical calculation. The problem is that in (19), if we explicitly substitute the solution (18), the leading nonanalytic term cancels! This is because

$$I \propto \partial_z \chi^2$$

and in χ^2, the leading coefficient of $ln(\omega z)$ is z^0. Hence, upon differentiation, we find that the nonanalytic term $\ln(\omega)$ disappears. Another way of saying this is that if we treat p as a continuous variable, the coefficient of the action is proportional to $(p-2)$ and hence vanishes for two-forms.

It is clear that the true answer in the gauge theory cannot be zero, since the absorption cross section is nonzero. Also, the procedure of [8] does not seem to give zero for this case. This may be a problem of our normalization. If one treats p as continuous, it is possible that the normalization of $\chi^{(p)}$ should be taken to involve inverse powers of $(p-2)$ which will cancel the apparent zero in the above expression. Other possibilities may exist. We will treat this as an overall coefficient in the correlation function that we cannot determine, since we cannot normalize the operators unambiguously.

In particular, in the case we are considering, we have $p = 2$, and $m^2 = 16$, which yields $\Delta = 6$. Hence we have a coupling to a dimension 6 operator. The exact form of this operator has been discussed in more detail in [34].

We also find that

$$I \sim R^{12}\omega^8 ln(wR)$$

and since the cross section is related to the discontinuity of the above function near $\omega = 0$, we find (from [5])

$$\sigma \sim \frac{i}{\omega}R^{12}\omega^8(ln(-s+i\epsilon) - ln(s-i\epsilon)) \sim \omega^7 R^{12}$$

which agrees with (14).

We therefore get results in agreement with the semiclassical calculation. The exact coefficient is, however, undetermined. We emphasize that this is because the exact normalization of the operators has not been fixed. It may be necessary to calculate a three-point correlation function in order to resolve the ambiguity.

In conclusion, we have extracted the absorption rate for a two-form field incident on a black threebrane. We have shown that the Hawking rate is proportional to ω^7, a fact which follows from a coupling to a dimension 6 operator on the brane world volume. We have thus found that a non-fixed scalar can also have a suppressed cross-section.

IV. ACKNOWLEDGEMENTS

We would like to thank I. Klebanov and J. Rahmfeld for discussions.

This work was supported in part by the Department of Energy under contract no. DE-AC03-76SF00515.

REFERENCES

[1] J. M. Maldacena, "The Large N Limit of Superconformal Field Theories and Supergravity," hep-th/9711200.

[2] S. S. Gubser, I. R. Klebanov and A. W. Peet, "Entropy and Temperature of Black 3-branes," hep-th/9602135, Phys. Rev. **D54** (1996) 3915.

[3] I. R. Klebanov, "Worldvolume Approach to Absorption by Nondilatonic Branes," hep-th/9702076, Nucl. Phys. **B496** (1997) 231.

[4] S. S. Gubser, I. R. Klebanov and A. A. Tseytlin, "String Theory and Classical Absorption by Three Branes," hep-th/9703040, Nucl. Phys. **B499** (1997) 217.

[5] S. S. Gubser and I. R. Klebanov, "Absorption by Branes and Schwinger Terms in the World-volume Theory," hep-th/9708005, Phys. Lett. **B413** (1997) 41.

[6] A. M. Polyakov, "String Theory and Quark Confinement," hep-th/9711002.

[7] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, "Gauge Theory Correlators from Non-Critical String Theory," hep-th/9802109.

[8] E. Witten, "Anti-de Sitter Space and Holography," hep-th/9802150.

[9] S. S. Gubser, A. Hashimoto, I. R. Klebanov and M. Krasnitz, "Scalar Absorption and the Breaking of the World Volume Conformal Invariance," hep-th/9803023.

[10] S. Ferrara and R. Kallosh, "Supersymmetry and Attractors,", hep-th/9602136.

[11] B. Kol and A. Rajaraman, "Fixed Scalars and Suppression of Hawking Evaporation," hep-th/9608126.

[12] C. Callan, S. Gubser, I. Klebanov and A. Tseytlin, "Absorption of fixed scalars and the D-brane approach to black holes," hep-th//9610172.

[13] I. Klebanov, A. Rajaraman and A. Tseytlin, " Intermediate Scalars and the Effective String Model of Black Holes," hep-th/9704112.

[14] J.H. Schwarz, Nuc. Phys. **B226** (1983) 269.

[15] H.J. Kim, L.J. Romans and P. van Nieuwenhuizen, "The Mass Spectrum of Chiral N=2 D=10 Supergravity on S^5", Phys. Rev. **D32** (1985) 389.

[16] K. Sfetsos, K. Skenderis, "Microscopic Derivation of the Bekenstein-Hawking Entropy Formula for Non-extremal Black Holes," hep-th/9711138.

[17] S. Ferrara, C. Fronsdal, "Conformal Maxwell Theory as a Singleton Field Theory on AdS_5, IIB Three Branes and Duality," hep-th/9712239.

[18] N. Itzhaki, J. M. Maldacena, J. Sonnenschein, S. Yankielowicz, "Supergravity and The Large N Limit of Theories With Sixteen Supercharges," hep-th/9802042.

[19] M. Gunaydin and D. Minic, "Singletons, Doubletons and M Theory," hep-th/9802047.

[20] G. T. Horowitz, H. Ooguri, "Spectrum of Large N Gauge Theory from Supergravity," hep-th/9802116.

[21] S. Kachru, E. Silverstein, "4d Conformal Field Theories and Strings on Orbifolds," hep-th/9802183.

[22] M. Berkooz, "A Supergravity Dual of a (1,0) Field Theory in Six Dimensions," hep-th/9802195.

[23] V. Balasubramanian and F. Larsen, "Near Horizon Geometry and Black Holes in Four Dimensions," hep-th/9802198.

[24] S. Minwalla, "Particles on $AdS_{(4/7)}$and Primary Operators on $M2/5$ Brane Worldvolumes," hep-th/9803053.

[25] O. Aharony, Y.Oz, and Z. Yin, "M Theory on $AdS_p \times S^{11-p}$ and Superconformal Field Theories," hep-th/9803051.

[26] R. Kallosh, J. Kumar and A. Rajaraman, " Special Conformal Symmetry of Worldvolume Actions," hep-th/9712073.

[27] P. Claus, R. Kallosh, J. Kumar, P. Townsend, A. van Proeyen, "Conformal Theory of M2, D3, M5 and D1+D5 branes," hep-th/9801206.

[28] S. J. Rey, J. Yee, "Macroscopic Strings as Heavy Quarks of Large N Gauge Theory and Anti-de Sitter Supergravity," hep-th/9803001.

[29] J. M. Maldacena, "Wilson loops in large N field theories," hep-th/9803002.

[30] M. Flato and C. Fronsdal, "Interacting Singletons," hep-th/9803013.

[31] A. Lawrence, N. Nekrasov, C. Vafa, "On Conformal Field Theories in Four Dimensions," hep-th/9803015.

[32] I. Ya. Aref'eva and I. V. Volovich, "On Large N Conformal Theories, Field Theories in Anti-De Sitter Space and Singletons," hep-th/9803028.

[33] L. Castellani, A. Ceresole, R. D'Auria, S. Ferrara, P. Fre' and M. Trgiante, "G/H M-branes and AdS_{p+2} Geometries," hep-th/9803039.

[34] S. Ferrara, C. Fronsdal and A. Zaffaroni, "On $\mathcal{N} = 8$ Supergravity on AdS_5 and $\mathcal{N} = 4$ Superconformal Yang-Mills Theory," hep-th/9802203.