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Supersymmetry-Breaking Loops from Analytic Continuation into Superspace

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Abstract

We extend to all orders in perturbation theory a method to calculate supersymmetry-breaking effects by analytic continuation of the renormalization group into superspace. A central observation is that the renormalized gauge coupling can be extended to a real vector superfield, thereby including soft breaking effects in the gauge sector. We explain the relation between this vector superfield coupling and the "holomorphic" gauge coupling, which is a chiral superfield running only at 1 loop. We consider these issues for a number of regulators, including dimensional reduction. With this method, the renormalization group equations for soft supersymmetry breaking terms are directly related to supersymmetric beta functions and anomalous dimensions to all orders in perturbation theory. However, the real power of the formalism lies in computing finite soft breaking effects corresponding to high-loop component calculations. We prove that the gaugino mass in gauge-mediated supersymmetry breaking is "screened" from strong interactions in the messenger sector. We present the complete next-to-leading calculation of gaugino masses (2 loops) and sfermion masses (3 loops) in minimal gauge mediation, and several other calculations of phenomenological relevance.

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1 Introduction

Recently there has been a great deal of interest in building models in which supersymmetry breaking is communicated to the observable particles through renormalizable interactions [1]. A common feature of these models is that supersymmetry breaking occurs in the masses of "messenger" fields in the form

$$M = M_{\rm SUSY} + \delta M, \tag{1.1}$$

where $M_{\rm SUSY}$ is a supersymmetric mass term, and δM breaks supersymmetry. In most models of this kind constructed to date $\delta M \ll M_{\rm SUSY}$, and so the messenger threshold is approximately supersymmetric. Integrating out the messenger fields gives rise to supersymmetry breaking in the low-energy effective lagrangian below the scale M. A large amount of work has already been done on the calculation of the supersymmetry breaking effects from various types of interactions [2, 3]. In Ref. [3] it was shown how to compute the leading low-energy supersymmetry breaking effects in a large class of models using only one-loop renormalization group (RG) equations and tree-level matching, while direct calculations of the same quantities require the evaluation of 1- and 2-loop graphs.

The starting point of Ref. [3] is the observation that since the messenger threshold is approximately supersymmetric, one can use a formalism where all couplings and masses are treated as superfields, and the SUSY breaking terms correspond to nonzero θ-dependent spurion components of the couplings. In this framework, it is not hard to see that leading-log effects that are determined by the RG in the SUSY limit are related to *finite* SUSY-breaking effects. For example, the RG can be used to compute corrections of the form $(\ln M)/(16\pi^2)$, where M is a threshold mass. If M is a superfield, then this contribution has a SUSY-breaking component

$$\frac{1}{16\pi^2} \ln M|_{\theta^2\bar{\theta}^2} = \frac{1}{16\pi^2} \frac{M|_{\theta^2\bar{\theta}^2}}{M|_0}, \tag{1.2}$$

which contains a loop factor, but no logarithm. Effects of this type therefore correspond to finite loop effects that are not related to an RG calculation in components.

A simple power-counting argument can be used to show that in gauge-mediated models the leading SUSY-breaking terms in the low-energy effective lagrangian arise from this sort of threshold dependence in the dimensionless couplings. This allows one to compute 1- and 2- loop SUSY breaking effects using the 1-loop RG equations and tree-level matching, analytically continued into superspace. In Ref. [3] this technique was used to reproduce known results in a much simpler way, and also to derive new phenomenologically interesting results that would be much more difficult to compute directly.

In this paper, we extend the analysis of Ref. [3] to higher orders in perturbation theory. One motivation for this is to define an unambiguous procedure to perform the analytic continuation into superfield beyond one loop. We show that the gauge coupling is naturally extended to a real superfield that is not the sum of a chiral and an antichiral superfield. The $\theta^2\bar{\theta}^2$ component of the real gauge superfield plays a crucial role in reproducing the correct behavior of perturbation theory. Another motivation for this is to obtain new results of interest for testing models in the literature. In particular, we are able to compute gaugino, squark, and slepton masses in gauge-mediated models at the next-to-leading order in perturbation theory. Our result corresponds to an explicit calculation of 2- and 3-loop Feynman diagrams. One of our results is that the gaugino masses in gauge-mediated models are "screened" from corrections from the SUSY-breaking sector up to 4 loops. This implies that the gauge-mediation relations are preserved up to corrections of order $g_{\rm SM}^4/(16\pi^2)^2 \sim 10^{-4}$ even if the SUSY-breaking (or messenger) sector is strongly coupled. We also compute other interesting effects, like the gaugino masses in "mediator" models [4], the gauge-mediated effective potential induced along classically flat directions, both for D flat directions (2 loops) as well as for the scalar partner of the axion (3 loops).

This paper is organized as follows. In Section 2, we give a definition of renormalized coupling constants that can be viewed as superfield spurions to all orders in perturbation theory. We use as examples specific theories that allow simple supersymmetric regulators. In Section 3, we discuss this prescription in the case in which the theory is regulated using dimensional reduction. We also show that extending the couplings to superfields automatically selects the so-called $\overline{\rm DR}'$ scheme for the soft terms. In Section 4, we use our technique to prove the gaugino screening result mentioned above, and compute gluino, squark, and slepton masses in gauge mediation at the NLO. We also extend our results to D-term breaking of SUSY, and derive the gaugino mass in "mediator" models. In Section 5, we compute some other interesting SUSY-breaking effects in gauge-mediated theories. Section 6 summarizes our main results and contains our conclusions.

2 Renormalized Coupling Constants as Superfields

The main tool of our approach is the use of renormalization schemes in which the renormalized coupling constants can be treated as superfields. Much of our discussion can be viewed as a restatement of the insights of Shifman and Vainshtein [5] in the framework of renormalized perturbation theory. However, we will generalize the method to include supersymmetry breaking effects. For gaugino masses and A terms, this was first done in ref. [6]. Here we will simultaneously describe the running of the scalar masses. For related studies, see also ref. [7].

2.1 Invitation: the Wess–Zumino Model

In this subsection we consider a simple example that illustrates many of the main ideas we will use in more complicated theories. We consider a massless Wess–Zumino model with bare lagrangian

$$\mathcal{L}_0 = \int d^4\theta\, \mathcal{Z}_0 \Phi^\dagger \Phi + \left(\int d^2\theta\, \frac{\lambda}{3!} \Phi^3 + \text{h.c.} \right), \tag{2.1}$$

and higher-derivative regulator terms [8]

$$\mathcal{L}_{\text{reg}} = \int d^4\theta\, \mathcal{Z}_0 \Phi^\dagger \frac{\Box}{\Lambda^2} \Phi. \tag{2.2}$$

We can incorporate soft SUSY breaking by extending the bare couplings λ and $\mathcal{Z}_0$ to be θ-dependent (but x-independent) superfields.[1] (λ is a superpotential coupling, and is not renormalized.) We have regulated the theory in a supersymmetric manner, so we can treat the bare couplings as superfields even at the quantum level.

Because the theory is regulated in a way that preserves SUSY (including the spurious SUSY acting on the couplings), the divergences that appear order-by-order in perturbation theory can be absorbed by supersymmetric counterterms. That is, we can write

$$\mathcal{Z}_0 = \mathcal{Z}(\mu) + \delta\mathcal{Z}(\lambda, \mathcal{Z}(\mu), \Lambda/\mu), \tag{2.3}$$

where $\delta\mathcal{Z}$ is the matter wavefunction counterterm. Because the relation between the bare and renormalized couplings preserves SUSY, we see that the renormalized couplings can also be viewed as SUSY spurions.

More specifically, we can define the counterterms by computing supergraphs with renormalized couplings in the vertices and propagators and choosing the counterterms to cancel the divergences. In the SUSY limit where there is no θ dependence in $\mathcal{Z}_0$ and λ, the counterterms have the form [9]

$$\delta\mathcal{L} = \int d^4\theta\, \mathcal{Z}\, C(|\lambda|^2/\mathcal{Z}^3(\mu), |\Lambda|/\mu) \Phi^\dagger \Phi, \tag{2.4}$$

[1]Note that taking a superfield S to be x-independent does not violate SUSY, since it amounts to imposing the supersymmetric constraint $\partial_\mu S = 0$.

where the form of the function C follows from the fact that theory depends trivially on the overall normalization of the fields.

In the presence of soft SUSY breaking, the renormalized couplings $\mathcal{Z}$ and λ will also depend on θ, and there are new terms in the Feynman rules involving supercovariant derivatives acting on the couplings $\mathcal{Z}$ and λ. However, it is easy to see that such terms can be ignored for purposes of computing the counterterms [10]. Because our regulator preserves the spurion SUSY even in the presence of soft SUSY breaking, we know that the counterterms can still be chosen to be superfield functions of λ and $\mathcal{Z}$. But local superspace counterterms involving supercovariant derivatives of λ and $\mathcal{Z}$ are forbidden simply by dimensional analysis. We conclude that even in the presence of soft SUSY breaking, the counterterms are still given by Eq. (2.4). Note what has happened here: the renormalization of the theory with soft SUSY breaking is completely determined by a *supersymmetric* calculation. This is the advantage of treating the bare and renormalized couplings as superfields.

The fact that the theory depends in a trivial way on the scale of the fields can be expressed more formally by noting that the the bare lagrangian is invariant under

$$\Phi \mapsto e^{A}\Phi, \quad \mathcal{Z}_0 \mapsto e^{-(A+A^\dagger)}\mathcal{Z}_0, \quad \lambda \mapsto e^{-3A}\lambda, \tag{2.5}$$

where A is a θ-dependent (but x-independent) chiral superfield. The fact that the relation between the bare and renormalized parameters preserves this feature can be expressed by stating that the renormalized parameter $\mathcal{Z}$ transforms the same way as $\mathcal{Z}_0$:

$$\mathcal{Z} \mapsto e^{-(A+A^\dagger)}\mathcal{Z}. \tag{2.6}$$

If we view this as a $U(1)$ "gauge" transformation, then $\ln\mathcal{Z}$ (and $\ln\mathcal{Z}_0$) transform as a gauge field. This point of view will be extremely useful to us later.

The relation between the bare and renormalized quantities determined by Eq. (2.4)

$$\mathcal{Z}_0 = \mathcal{Z}(\mu)\left[1 + C(|\lambda|^2/\mathcal{Z}^3(\mu), |\Lambda|/\mu)\right], \tag{2.7}$$

determines the RG flow of the theory from $d\mathcal{Z}_0/d\mu = 0$. This gives

$$\mu\frac{d\ln\mathcal{Z}}{d\mu} = -\mu\frac{d}{d\mu}C(|\lambda|^2/\mathcal{Z}^3, |\Lambda|/\mu) \equiv \gamma(|\lambda|^3/\mathcal{Z}^3). \tag{2.8}$$

The $\theta = \bar\theta = 0$ component of γ is just the supersymmetric anomalous dimension. The renormalized soft scalar mass m^2 is defined by writing

$$\mathcal{Z} = Z\left[1 - \theta^2\bar\theta^2 m^2\right], \tag{2.9}$$

where Z is the renormalized wavefunction factor. The RG equation for the soft mass is determined by the $\theta^2\bar{\theta}^2$ component of Eq. (2.8):

$$\mu\frac{dm^2}{d\mu} = -\left.\gamma(|\lambda|^2/\mathcal{Z}^3)\right|_{\theta^2\bar{\theta}^2} = -\gamma'(|\lambda|^2/Z^3)\frac{3|\lambda|^2m^2}{Z^3}. \tag{2.10}$$

This formula is valid to all orders in perturbation theory. At the 1-loop level

$$\gamma = -\frac{1}{16\pi^2}\frac{|\lambda|^2}{\mathcal{Z}^3}, \tag{2.11}$$

and we recover the familiar result

$$\mu\frac{dm^2}{d\mu} = \frac{3}{16\pi^2}\,\bar{\lambda}^2 m^2, \tag{2.12}$$

where $\bar{\lambda} = |\lambda|Z^{-3/2}$ is the running coupling constant. Eq. (2.8) also gives the RG equation for A terms if we add a non-vanishing θ^2 component to $\mathcal{Z}(\mu)$.

In the following, we will generalize the procedure followed in this section to general renormalizable SUSY theories with soft SUSY breaking. The idea is to include soft SUSY breaking by extending the bare couplings K_0 to θ-dependent superfields. As long as the theory is regulated in a supersymmetric manner, the bare couplings can be viewed as spurion superfields even at the quantum level. We then define renormalized couplings $K(\mu)$ related to the bare couplings by a superfield relation

$$K_0 = G(K(\mu), \Lambda/\mu). \tag{2.13}$$

The renormalization of the couplings in the SUSY limit then determines the renormalization of the soft SUSY breaking terms as long as the relation does not involve supercovariant derivatives acting on $K(\mu)$. But in a vast class of theories, this is guaranteed by simple power counting and symmetry arguments. Eq. (2.13) therefore determines the complete RG flow of all soft SUSY breaking parameters. In the remainder of this Section, we explain how to carry out these steps for gauge theories, which present additional subtleties.

2.2 Holomorphic Coupling in Supersymmetric QED

We begin with SUSY QED, a $U(1)$ gauge theory with matter fields Φ and $\bar{\Phi}$ with charges $+1$ and -1, respectively. This theory can be regulated in a completely supersymmetric manner using a combination of Pauli–Villars fields to regulate matter loops and a higher-derivative regulator for the gauge fields. The bare lagrangian can be written as $\mathcal{L}_0 + \mathcal{L}_{\rm reg}$, where

$$\begin{aligned} \mathcal{L}_0 = \int d^4\theta\, \mathcal{Z}_0 \left(\Phi^\dagger e^V \Phi + \bar\Phi^\dagger e^{-V} \bar\Phi \right) \\ + \int d^2\theta\, \tfrac{1}{2} S_0 W^\alpha W_\alpha + \text{h.c.}, \end{aligned} \tag{2.14}$$

contains the "physical" couplings, and

$$\begin{aligned} \mathcal{L}_{\rm reg} = \int d^4\theta\, \mathcal{Z}_0 \left(\Omega^\dagger e^V \Omega + \bar\Omega^\dagger e^{-V} \bar\Omega \right) \\ + \int d^2\theta\, \Lambda_\Phi \Omega \bar\Omega + \text{h.c.} \\ + \int d^2\theta\, W^\alpha \frac{\Box}{4\Lambda_G^2} W_\alpha + \text{h.c.}, \end{aligned} \tag{2.15}$$

contains the regulator terms. Here, Ω and $\bar\Omega$ are Pauli–Villars fields (odd-statistics chiral superfields) and Λ_Φ and Λ_G are cutoffs for the matter and gauge fields, respectively. We will take the cutoffs to infinity with $\Lambda_\Phi \sim \Lambda_G$, so there is effectively a single cutoff. Note that the bare wavefunction factor $\mathcal{Z}_0$ appears both in front of the matter fields and the Pauli–Villars fields. This is necessary to regularize $\mathcal{Z}_0$-dependent subdivergences that occur at two loops and beyond. For reference, the components of S_0 are given by

$$S_0 = \frac{1}{2g_0^2} - \frac{i\Theta_0}{16\pi^2} - \theta^2 \frac{m_{\lambda,0}}{g_0^2}, \tag{2.16}$$

where Θ_0 is the (bare) vacuum angle and $m_{\lambda,0}$ is the bare gaugino mass.

We incorporate explicit soft SUSY breaking by allowing the bare coupling S_0 and $\mathcal{Z}_0$ to be superfields with nonzero θ components. Just as in the Wess–Zumino model, the fact that the regulator preserves SUSY means that the bare couplings can be viewed as superfields at the quantum level, and we can renormalize the theory by adding counterterms that are local (in superspace) and gauge invariant. We therefore define renormalized superfield couplings S and $\mathcal{Z}$ by

$$S_0 = S + \delta S, \quad \mathcal{Z}_0 = \mathcal{Z} + \delta\mathcal{Z}, \tag{2.17}$$

where the counterterms δS and $\delta\mathcal{Z}$ are superfield functions of S and $\mathcal{Z}$ determined order-by-order in perturbation theory to cancel the ultraviolet divergences.

For $\mathcal{Z}$ we can proceed exactly as in the Wess–Zumino model discussed above, but we immediately encounter difficulties when we try to renormalize the gauge coupling as a superfield. One way to see the problem is that the only manifestly gauge-invariant operator that can act as a gauge counterterm is

$$\mathcal{L} = \int d^2\theta \, \tfrac{1}{2} \delta S \, W^\alpha W_\alpha + \text{h.c.} \tag{2.18}$$

However, the result of a supergraph calculation is necessarily a $d^4\theta$ integral. At one loop, this is not a problem because the one-loop gauge diagrams are independent of all couplings (since the gauge coupling is in front of the kinetic term), and the counterterm can be proportional to

$$\int d^4\theta \, (D^\alpha V W_\alpha + \text{h.c.}) = \int d^2\theta \, W^\alpha W_\alpha + \text{h.c.} \tag{2.19}$$

However, beyond one loop, the coefficient of the counterterm depends on the superfield couplings, and the counterterm cannot be written as $d^4\theta$ integral.

This argument can be sharpened by using the fact that the counterterm δS is a *chiral* superfield. Because of this, δS must be a holomorphic function of S, Λ_Φ, Λ_G, and μ, independent of $S^\dagger$ as well as $\mathcal{Z}$. We therefore have

$$\delta S = f\left(S, \frac{\mu}{\Lambda_\Phi}, \frac{\Lambda_G}{\Lambda_\Phi} \right), \tag{2.20}$$

where f is a holomorphic function. Now, the divergence in the gauge coupling g is independent of the vacuum angle Θ to all orders in perturbation theory, since $F^{\mu\nu}\tilde{F}_{\mu\nu}$ is a total derivative, and therefore irrelevant in perturbation theory.[2] Therefore,

$$0 = \frac{\partial \, \text{Re}(f)}{\partial \, \text{Im}(S)} = - \, \text{Im} \, \frac{\partial f}{\partial S}. \tag{2.21}$$

Since f is a holomorphic function, the only possibility is that $\partial f / \partial S$ is independent of S, which implies

$$f(S) = a + bS, \tag{2.22}$$

where a and b are independent of S. We see that a is the 1-loop contribution, and b is identically zero (since the zero coupling limit corresponds to $S \to +\infty$). We conclude that there is no divergence in the vacuum polarization beyond one loop.[3] If this argument is to be believed, the coupling S satisfies the exact (to all orders in perturbation theory) RG equation

$$\mu \frac{dS}{d\mu} = -\frac{1}{8\pi^2}. \tag{2.23}$$

[2] We do not address the subtle question of renormalization beyond perturbation theory.

[3] Note that this argument does not assume that f is a power series in S. This is important for non-abelian gauge theories, where we will see that the perturbation series is non-analytic in S.

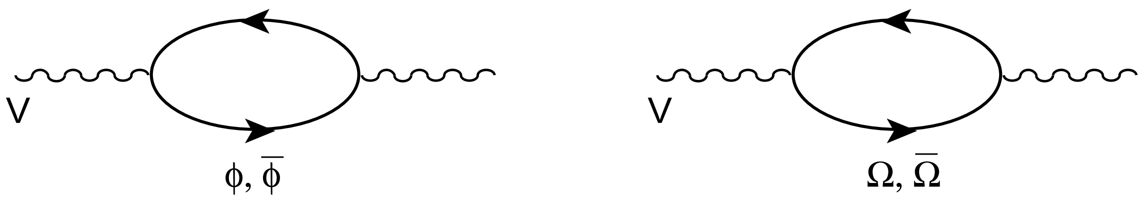

Figure 1: *One-loop diagrams contributing to the vector field propagator.*

This appears paradoxical, since it is known that the β-function has a (scheme-independent) contribution at two loops.

To understand what is going on, we compute the counterterm explicitly at one loop, keeping the couplings as superfields. The diagrams are shown in Fig. 1. We obtain the contribution to the 1PI effective action

$$\Gamma_{\rm 1PI} = -\frac{1}{2}\int d^4\theta \int d^4 p\, V \left[\gamma(p^2) + \delta S + \delta S^\dagger\right] p^2 P_T V + \text{finite}, \tag{2.24}$$

where P_T is a transverse superspace projector, and

$$\begin{aligned}\gamma(p^2) &= \frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\, \frac{-|\Lambda_\Phi|^2/\mathcal{Z}^2}{k^2(k^2 - |\Lambda_\Phi|^2/\mathcal{Z}^2)}\, \frac{-|\Lambda_\Phi|^2/\mathcal{Z}^2}{(k+p)^2((k+p)^2 - |\Lambda_\Phi|^2/\mathcal{Z}^2)} \\ &= \frac{1}{8\pi^2}\ln\frac{|\Lambda_\Phi|^2/\mathcal{Z}^2}{-p^2} + \text{finite}.\end{aligned} \tag{2.25}$$

The 1PI effective action can therefore be made finite by adding the counterterm

$$\delta S = -\frac{1}{8\pi^2}\ln\frac{\Lambda_\Phi}{\mu}, \tag{2.26}$$

where μ is a renormalization scale. Note that we cannot choose δS to depend on the "kinematic" cutoff $|\Lambda_\Phi|/\mathcal{Z}$, the scale at which the Pauli–Villars regulator cuts off the ultraviolet modes, simply because this quantity is not a chiral superfield. On the other hand, it is clear that physical quantities depend on Λ_Φ only through the combination $|\Lambda_\Phi|/\mathcal{Z}$, together with the bare parameters. This is the key to understanding the meaning of the renormalized coupling S.

More formally, we note that the bare lagrangian is invariant under

$$\begin{gathered}\Phi \mapsto e^A\Phi, \quad \bar\Phi \mapsto e^A\bar\Phi, \quad \Omega \mapsto e^A\Omega, \quad \bar\Omega \mapsto e^A\bar\Omega, \\ \mathcal{Z}_0 \mapsto e^{-(A+A^\dagger)}\mathcal{Z}_0, \quad \Lambda_\Phi \mapsto e^{-2A}\Lambda_\Phi, \quad S_0 \mapsto S_0,\end{gathered} \tag{2.27}$$

where A is a θ-dependent (but x-independent) chiral superfield. Because $\mathcal{Z}_0$ measures the scale of the fields in the regulated theory, we can choose the subtractions that defined the renormalized $\mathcal{Z}$ so that

$$\mathcal{Z} = \mathcal{Z}_0 f(S_0 + S_0^\dagger, |\Lambda_\Phi|/\mathcal{Z}_0, |\Lambda_G|, \mu), \tag{2.28}$$

which shows that we can assign the same transformation rule to $\mathcal{Z}$ as $\mathcal{Z}_0$.[4] From Eq. (2.26) we find that the renormalized $S(\mu)$ transforms as

$$S \mapsto S - \frac{A}{4\pi^2}. \tag{2.29}$$

Just as in the case of the Wess–Zumino model, we have found a symmetry under which $\mathcal{Z}$ can be interpreted as a background $U(1)$ gauge field. Eq. (2.29) is just a reflection of the Konishi anomaly [11], therefore we will refer to this symmetry as the (renormalized) "anomalous $U(1)$" symmetry.As a consequence of this symmetry, physical quantities can depend on S only in the combination

$$S + S^\dagger - \frac{1}{4\pi^2} \ln \mathcal{Z} = S_0 + S_0^\dagger + \frac{1}{8\pi^2} \ln \frac{|\Lambda_\Phi|^2/\mathcal{Z}^2}{\mu^2}. \tag{2.30}$$

(The right-hand side shows that this combination depends on the kinematic cutoff when expressed in terms of the bare parameters.) Notice that $S - S^\dagger$, which is proportional to the vacuum angle, cannot appear in any invariant, consistent with the fact that the vacuum angle is not physical in a theory with massless fermions. Because of the symmetry defined by Eq. (2.27) and Eq. (2.29), the relation between the bare and renormalized wavefunction factors has the form

$$\mathcal{Z}_0 = \mathcal{Z}(\mu)\ f\left(S + S^\dagger - \frac{1}{4\pi^2} \ln \mathcal{Z}, \frac{|\Lambda_\Phi|/\mathcal{Z}}{\mu}, \frac{|\Lambda_G|}{\mu}\right). \tag{2.31}$$

The RG flow of the theory is determined by $d\mathcal{Z}_0/d\mu = 0$. Due to the loop factor multiplying $\ln \mathcal{Z}$ in the above expression, an $(n+1)$-loop effects are often related to n-loop effects. There are many examples of this in the literature, and we also obtain new results of this type in subsequent sections.

Because the correlation functions depend on S only through Eq. (2.30), the relation between the coupling S and a gauge coupling defined directly in terms of 1PI Green's function is *non-analytic* in the couplings. As already observed in Refs. [5], this can resolve the apparent contradiction between a holomorphic coupling that runs at one loop and the conventional definition of the gauge coupling that runs at all loops.

We now note that the quantity

$$\tilde{R} \equiv S + S^\dagger - \frac{1}{4\pi^2} \ln \mathcal{Z} \tag{2.32}$$

[4]This may become clearer when we give a 1PI definition of $\mathcal{Z}$ in the next Subsection.

that appears in Eq. (2.30) is a good candidate for a *real* renormalized superfield coupling. $\tilde{R}$ is a finite quantity that parameterizes the couplings of the theory, and it does not have any unphysical dependence on the scale of the fields. Also, the $\theta = 0$ and θ^2 components of R give the correct RG equations for the gauge coupling and gaugino mass to 2-loops. (In fact, Eq. (2.32) is identical in form to the famous equation of Refs. [5], but note our equation involves only renormalized quantities.) In the next Subsection, we will explain the relation between $\tilde{R}$ and a renormalized gauge coupling defined from the 1PI action, and address the meaning of the $\theta^2\bar{\theta}^2$ component of $\tilde{R}$.

We close our discussion of SUSY QED by remarking that there is a completely analogous $U(1)$ symmetry with a well defined action on the *bare* couplings. The "gauge transformation" $\Phi \mapsto e^A \Phi$ has an anomaly, and so the bare gauge coupling must also transform to compensate for the transformation. In our regulator, this can be seen from the fact that the Pauli–Villars fields transform under the symmetry, so the anomaly can be obtained as the matrix element of the Pauli–Villars mass term in a background gauge field. More generally, it is clear that any holomorphic regulator yields the anomaly, and the result is that the theory is invariant under the transformation

$$\Phi \mapsto e^A \Phi, \quad \mathcal{Z}_0 \mapsto e^{-(A+A^\dagger)} \mathcal{Z}_0, \quad S_0 \mapsto S_0 - \frac{A}{4\pi^2} \tag{2.33}$$

with the regulator Lagrangian *invariant*. This "bare" or "Wilsonian" anomalous $U(1)$ is also a very useful symmetry [12].

2.3 Real Superfield Coupling in Supersymmetric QED

We now give another definition of the renormalized gauge coupling, obtained directly from the 1PI effective action by subtraction at a Euclidean momentum point. This corresponds more closely to the "physical" coupling that describes the momentum dependence of the effective charge. More to the point, this definition of the gauge coupling can be directly understood in terms of component calculations, allowing us to make contact between our formalism and conventional calculations.

In a component calculation, it is natural to define the renormalized gauge coupling and gaugino mass in terms of an appropriate 1PI correlation function at a Euclidean kinematic point. We now show that a definition of this type gives rise to a real superfield R whose lowest components are the gauge coupling and gaugino mass.

Consider the supersymmetric limit first. To define the renormalized gauge coupling we must consider the gauge invariant bilinears in W_α in the 1PI action. Since we include quantum effects we must focus on $d^4\theta$ integrals. By a simple operator analysis one finds there exists just one independent term

$$\Gamma_{\rm 1PI} = \int d^4p \int d^4\theta \gamma(p^2) W^\alpha \frac{D^2}{-8p^2} W_\alpha + \text{h.c.} + \cdots \tag{2.34}$$

$$= \int d^4p \int d^2\theta \, \tfrac{1}{2}\gamma(p^2) W^\alpha W_\alpha + \text{h.c.} + \cdots, \tag{2.35}$$

where the last identity follows simply by integrating over half of superspace. Therefore, γ can contain the contribution from the tree-level and loop contributions to the ordinary gauge kinetic term. We can therefore define the renormalized gauge coupling simply by subtracting at a Euclidean momentum point:

$$\frac{1}{g^2(\mu)} \equiv \gamma(p^2)\Big|_{p^2=-\mu^2}. \tag{2.36}$$

The role of the operator of Eq. (2.34) in generating the all order β-function was already emphasized in Ref. [5].

We can similarly define a renormalized wavefunction superfield by considering the terms in the 1PI action that contribute to the matter kinetic term

$$\Gamma_{\rm 1PI} = \int d^4p \int d^4\theta \, \zeta(p^2) \left[\Phi^\dagger e^V \Phi + \bar{\Phi}^\dagger e^{-V} \bar{\Phi}\right] + \cdots, \tag{2.37}$$

and defining

$$\mathcal{Z} = \zeta(p^2)\Big|_{p^2=-\mu^2}. \tag{2.38}$$

In the presence of soft SUSY-breaking sources in S and $\mathcal{Z}$, the gauge kinetic terms in the 1PI effective action are

$$\Gamma_{\rm 1PI} = \int d^4p \int d^4\theta \, \gamma(p^2) W^\alpha \frac{D^2}{-8p^2} W_\alpha + \text{h.c.} + \mathcal{O}(D_\alpha S, D_\alpha \mathcal{Z}, \ldots) \tag{2.39}$$

where $\gamma(p^2)$ is now a *vector* superfield function of the couplings $S + S^\dagger$ and $\mathcal{Z}$, and $\mathcal{O}(D_\alpha S, \ldots)$ represents terms involving at least one supercovariant derivative acting on the sources. By studying all possible WW and $W\bar{W}$ terms involving supercovariant derivatives, it can be shown that they always lead to terms of second order in the soft masses, *i.e.* they are $\mathcal{O}(m^2/p^2)$. These terms therefore do not contribute to the gauge kinetic term and gaugino mass term in the 1PI action. It therefore makes sense to define a renormalized superfield coupling by

$$R(\mu) \equiv \gamma(p^2)\Big|_{p^2=-\mu^2}. \tag{2.40}$$

Everything in this definition is manifestly supersymmetric, so the relation between this renormalized coupling and the bare couplings is SUSY covariant. The interpretation of the components of R is given by

$$\int d^4\theta\, \gamma W^\alpha \frac{D^2}{-8p^2} W_\alpha = \left[\int d^2\theta\, \tfrac{1}{2}\left(\gamma|_0 + \theta^2\, \gamma|_{\theta^2}\right) W^\alpha W_\alpha + \text{h.c.}\right] + \gamma|_{\theta^2\bar\theta^2}\, \frac{\lambda^\alpha \sigma^\mu_{\alpha\dot\beta} p_\mu \bar\lambda^{\dot\beta}}{-p^2}. \tag{2.41}$$

The lowest components of R are therefore the coefficients of the gauge kinetic term and gaugino mass term, and we identify

$$\frac{1}{g^2(\mu)} \equiv R(\mu)|_0\,, \quad -\frac{m_\lambda(\mu)}{g^2(\mu)} \equiv R(\mu)|_{\theta^2}\,. \tag{2.42}$$

Note that this renormalization scheme is mass-independent.

The $\theta^2\bar\theta^2$ component of R multiplies a non-local SUSY-breaking contribution to the 1PI action. It is instructive to ask what distinguishes this $\mathcal{O}(m^2)$ effect from the other $\mathcal{O}(m^2)$ WW and $W\bar W$ operators induced by the terms involving covariant derivatives acting on the couplings. To do so it is useful to work in components. Since there are three component fields A_μ, λ, and D, there are in general three independent $\mathcal{O}(m^2)$ corrections to the corresponding self-energies:

$$\begin{aligned} \Pi_A^{\mu\nu}(p^2) &= (p^2 g^{\mu\nu} - p^\mu p^\nu)\left(1 + \frac{\kappa_A^2}{p^2}\right), \\ \Pi_\lambda(p^2) &= \not{p}\left(1 + \frac{\kappa_\lambda^2}{p^2}\right) \\ \Pi_D(p^2) &= \left(1 + \frac{\kappa_D^2}{p^2}\right), \end{aligned} \tag{2.43}$$

where $\kappa_{A,\lambda,D} = \mathcal{O}(m^2)$. A simple operator analysis shows that the terms involving supercovariant derivatives acting on couplings generate $\mathcal{O}(m^2)$ corrections that always satisfy the supertrace sum rule $3\kappa_A^2 - 4\kappa_\lambda^2 + \kappa_D^2 = 0$. On the other hand the $\theta^2\bar\theta^2$ component of R is associated to a non zero supertrace $2\ R|_{\theta^2\bar\theta^2} = 3\kappa_A^2 - 4\kappa_\lambda^2 + \kappa_D^2$. If one computes the effect of the dressed self-energies in Eq. (2.43) on the matter self-energy, one finds that the only divergent contribution is proportional to the supertrace. This simple exercise clarifies why the $\theta^2\bar\theta^2$ component of R, although associated with a non-local operator, nonetheless enters into the RG flow equations of the softly broken theory.

We now discuss the relation between the real superfield gauge coupling discussed here and the holomorphic gauge coupling described in the previous Subsection. Since both are perfectly valid parameterizations of the renormalized gauge coupling, we can express R in terms of the holomorphic coupling S and $\mathcal{Z}$. The coupling R is clearly invariant under the field rescaling Eq. (2.27), so

$$R(\mu) = f\left(S(\mu) + S^\dagger(\mu) - \frac{1}{4\pi^2}\ln\mathcal{Z}(\mu)\right). \tag{2.44}$$

Demanding that the holomorphic and real couplings coincide at tree level gives

$$R(\mu) = S(\mu) + S^\dagger(\mu) - \frac{1}{4\pi^2}\ln\mathcal{Z}(\mu) + \frac{c}{8\pi^2} + \mathcal{O}(S + S^\dagger)^{-1}, \tag{2.45}$$

where c is a 1-loop scheme-dependent constant. Notice that this expression automatically gives the correct 2-loop β function. Eq. (2.45) is identical to the famous formula of Ref. [5] that relates the 1PI and "Wilsonian" gauge couplings. However, it is important to remember that the coupling S in our Eq. (2.45) is a renormalized coupling constant.

2.4 Holomorphic Coupling in Supersymmetric Yang–Mills Theory

We now consider some additional features that arise in non-abelian gauge theories, using the example of a pure SUSY Yang–Mills theory with gauge group $SU(N)$. We regulate this theory in a supersymmetric way by embedding it into a finite theory with softly broken $\mathcal{N} = 2$ SUSY. The additional fields in the regulated theory consist of a chiral field Φ in the adjoint representation (the $\mathcal{N} = 2$ superpartner of the $\mathcal{N} = 1$ gauge multiplet) and $2N$ hypermultiplets consisting of chiral fields Ω^J and $\bar{\Omega}_J$ ($J = 1, \ldots, 2N$) in the fundamental and antifundamental representations, respectively.

The bare lagrangian of the theory (written in $\mathcal{N} = 1$ superspace) is

$$\begin{aligned}\mathcal{L}_0 = &\int d^2\theta\, S_0 \,\mathrm{tr}\left[W^\alpha W_\alpha - \tfrac{1}{4}\bar{D}^2(e^{-V}\Phi^\dagger e^V)\Phi\right] + \text{h.c.} \\ &+ \int d^4\theta \left[\Omega_J^\dagger e^V \Omega^J + \bar{\Omega}^{J\dagger} e^{-V^T}\bar{\Omega}_J\right] + \left(\int d^2\theta\, \sqrt{2}\,\Omega^J \Phi \bar{\Omega}_J + \text{h.c.}\right) \\ &+ \int d^2\theta\, \left[\Lambda_\Omega \Omega^J \bar{\Omega}_J + \Lambda_G \,\mathrm{tr}(\Phi^2)\right] + \text{h.c.}\end{aligned} \tag{2.46}$$

The coefficient of the $\Omega^J \Phi \bar{\Omega}_J$ interaction is fixed by $\mathcal{N} = 2$ SUSY. The $\mathcal{N} = 2$ SUSY is broken explicitly down to $\mathcal{N} = 1$ by the Φ mass term (the mass term for Ω^J and $\bar{\Omega}_J$ is $\mathcal{N} = 2$ invariant). $\mathcal{N} = 2$ theories are finite beyond one loop [13]. With our choice of matter, the 1-loop beta function vanishes and therefore, in the background gauge, there are no divergences. The parameters Λ_Ω and Λ_G therefore act as cutoffs for SUSY Yang–Mills theory, with the fields Φ, Ω^J, and $\bar{\Omega}_J$ playing the role of regulator fields. We will eventually take the limit $\Lambda_\Omega, \Lambda_G \to \infty$ with $\Lambda_\Omega \sim \Lambda_G$, so that there is effectively a single cutoff.

We now show that the finiteness of this theory persists when S_0 is a chiral superfield with nonzero θ components. Any divergences in the 1PI effective action must be local (in $\mathcal{N} = 1$ superspace) expressions involving the superfield couplings of the theory. Because this theory is renormalizable, the divergences must have the same form as terms in the lagrangian. There are no divergences when S_0 is a number, so any divergences must be proportional to SUSY-covariant derivatives acting on S_0. But such terms have positive mass dimension, so there can be no divergences proportional to dimension-4 operators. The only remaining possibility is that there are divergences proportional to

$$\int d^2\theta\, \bar{D}^2 S_0^\dagger\, \Omega^J \bar{\Omega}_J + \text{h.c.} \quad \text{or} \quad \int d^2\theta\, \bar{D}^2 S_0^\dagger\, \text{tr}(\Phi^2) + \text{h.c.} \tag{2.47}$$

Such divergences can be excluded by considering the (anomaly-free) transformation

$$\begin{gathered} \Omega^J \mapsto e^{i\alpha}\Omega^J, \qquad \bar{\Omega}_J \mapsto e^{i\alpha}\bar{\Omega}_J, \qquad \Phi \mapsto e^{-2i\alpha}\Phi, \\ \Lambda_\Omega \mapsto e^{-2i\alpha}\Lambda_\Omega, \qquad \Lambda_G \mapsto e^{4i\alpha}\Lambda_G, \end{gathered} \tag{2.48}$$

under which $\bar{D}^2 S_0^\dagger$ is invariant.

This establishes that the theory above is finite, and therefore provides a regulator for the SUSY Yang–Mills theory we want to study. We still need to renormalize the theory in order to take the limit $\Lambda_\Omega, \Lambda_G \to \infty$. The renormalized lagrangian is[5]

$$\mathcal{L} = \int d^2\theta\, S\, \text{tr}(W^\alpha W_\alpha) + \text{h.c.}, \tag{2.49}$$

where S is defined by

$$S_0 = S + \delta S. \tag{2.50}$$

The counterterm δS is fixed order-by-order in perturbation theory to cancel the divergences as $\Lambda_\Omega, \Lambda_G \to \infty$.

[5]The renormalized lagrangian can be thought of as the "effective lagrangian" below the scales $\Lambda_\Omega, \Lambda_G$. However, we must choose the couplings in the "fundamental lagrangian" $\mathcal{L}_0$ as a function of Λ_Ω and Λ_G so that the couplings in the "effective lagrangian" are held fixed as the cutoff is removed. This can be thought of as "fixing the parameters from low-energy experiment".

At one loop, the vacuum polarization in the background gauge is proportional to

$$-\frac{N}{16\pi^2}\ln\frac{|\Lambda_G|^2/(S+S^\dagger)^2}{-p^2}-\frac{2N}{16\pi^2}\ln\frac{|\Lambda_\Omega|^2}{-p^2}+\text{finite}+\delta S+\delta S^\dagger, \tag{2.51}$$

where the "physical" cutoff for the Φ contribution is $|\Lambda_G|/(S+S^\dagger)$ due to the non-canonical kinetic term for the gauge multiplet. At this order, the theory can be renormalized in a holomorphic way by choosing

$$\delta S=\frac{N}{16\pi^2}\ln\frac{\Lambda_G}{\mu}+\frac{2N}{16\pi^2}\ln\frac{\Lambda_\Omega}{\mu}, \tag{2.52}$$

where μ is a renormalization scale.

Because the theory is regulated in a supersymmetric manner, the same argument used in Sect. 2.2 shows that there are no counterterms beyond one loop to all orders in perturbation theory.[6] We can therefore choose the counterterm to be given by Eq. (2.52) to all orders in perturbation theory. The renormalized gauge coupling defined in this way satisfies the *exact* RG equation

$$\mu\frac{dS}{d\mu}=\frac{3N}{16\pi^2}. \tag{2.53}$$

As in SUSY QED, the fact that the holomorphic gauge coupling has a 1-loop beta function is closely connected to the fact that the subtraction depends on Λ_Ω and $S+S^\dagger$ separately. Logarithmic divergent loops always involve the "kinematic" cutoff $|\Lambda_G|/(S+S^\dagger)$, and therefore the renormalized expansion coefficient is

$$\begin{aligned}S+S^\dagger+\frac{N}{8\pi^2}\ln(S+S^\dagger)=S_0+S_0^\dagger&-\frac{N}{8\pi^2}\ln\frac{|\Lambda_G|/(S+S^\dagger)}{\mu}\\&-\frac{2N}{8\pi^2}\ln\frac{|\Lambda_\Omega|}{\mu}\end{aligned} \tag{2.54}$$

We can also define a real superfield coupling from the 1PI effective action similarly to what was done for SUSY QED. In this scheme, there is a real gauge coupling superfield R defined to be the coefficient of the V propagator term in the 1PI effective action. R must depend on the combination Eq. (2.54), and we find

$$R=S+S^\dagger+\frac{N}{8\pi^2}\ln(S+S^\dagger)+\mathcal{O}(S+S^\dagger)^{-1}. \tag{2.55}$$

[6]Note that the perturbation series is nonanalytic in S, as can be seen from Eq. (2.51). However, the arguments of Sect. 2.2 do not require the perturbation series to be a power series in S, and are therefore valid in this case as well.

2.5 General Gauge Theories

We have so far treated only simple theories where we know how to construct a manifestly supersymmetric regulator. However, we now argue that our results apply to any SUSY gauge theory as long as a supersymmetric regulator exists. The general arguments above tell us that the only divergence in the gauge coupling occurs at one loop, and has the form

$$\delta S = \frac{3T_G}{16\pi^2} \ln \frac{\Lambda_G}{\mu} - \sum_r \frac{T_r}{16\pi^2} \ln \frac{\Lambda_r}{\mu}, \tag{2.56}$$

where T_r is the Dynkin index of the r representation. Here Λ_G is a cutoff parameter for gauge loops and Λ_r is a cutoff parameter for matter fields in the representation r. Note that in order for this formula to make sense, Λ_G and Λ_r must be chiral superfield spurions, as they are in the examples considered previously. On the other hand, the "kinematic" cutoff (the momentum scale at which loop momenta are damped) cannot be a chiral superfield, for the simple reason that it must be real. As we have seen, Eq. (2.56) is consistent with the 2-loop RG equations provided that the kinematic cutoff for matter loops is $\Lambda_{r,\text{kin}} = |\Lambda_r|/Z_r$. The relation between the kinematic gauge cutoff and Λ_G is more complicated, as seen in the example of SUSY Yang–Mills. In any case, in order to reproduce the correct 2-loop beta function, physical quantities must depend on the combination

$$R = S + S^\dagger + \frac{T_G}{8\pi^2} \ln(S + S^\dagger) - \sum_r \frac{T_r}{8\pi^2} \ln Z_r + \text{2-loop corrections}, \tag{2.57}$$

which is the real gauge coupling superfield. In the following we will give further evidence for the generality of our conclusions by showing how they arise in dimensional reduction, a regulator that can in principle be used for any SUSY theory.

3 Dimensional Reduction

So far we have been dealing with regulators that apply only to special theories. However, in order to be able to calculate higher order effects in any theory, including the supersymmetric extension of the Standard Model, the only practical regulator is dimensional reduction (DRED) [14, 15]. In this section we show how the holomorphic and real gauge couplings arise in DRED. We also show that the procedure of analytically continuing the renormalized couplings into superspace picks out the so-called $\overline{\text{DR}}'$ scheme [16] in which the ϵ-scalar mass does not appear in physical quantities.

3.1 Real and Holomorphic Gauge Coupling in Dimensional Reduction

The renormalization of SUSY gauge theories in the framework of DRED was clarified more than a decade ago by Grisaru, Milewski and Zanon (GMZ) [17]. They pointed out that in $d = 4 - 2\epsilon$ dimensions, there is an additional supersymmetric and gauge-invariant local operator

$$\mathcal{O}_{\rm GMZ} = g_\epsilon^{\mu\nu}\, {\rm tr}(\Gamma_\mu \Gamma_\nu), \tag{3.1}$$

where $g_\epsilon^{\mu\nu}$ is the metric in the 2ϵ "compactified" dimensions, and Γ_μ is the superfield gauge connection defined by

$$\Gamma^\mu = \frac{1}{2}\sigma^\mu_{\alpha\dot\beta}\bar D^{\dot\beta}\left(e^{-V} D^\alpha e^V\right). \tag{3.2}$$

This operator is an $\mathcal{O}(\epsilon)$ (or "evanescent") operator, with the property that

$$\int d^4\theta\, \mathcal{O}_{\rm GMZ} = \epsilon \int d^2\theta\ {\rm tr}(W^\alpha W_\alpha) + {\rm h.c.} \tag{3.3}$$

(Note that $g_\epsilon^{\mu\nu}\Gamma_\mu\Gamma_\nu$ is real.) Therefore, the quantity $\int d^4\theta\, \mathcal{O}_{\rm GMZ}$ is a dimension-4 term that can appear as a counterterm for the gauge kinetic term.

Taking this into account, the bare lagrangian is

$$\mathcal{L}_0 = \left(\int d^2\theta\, S_0\, {\rm tr}(W^\alpha W_\alpha) + {\rm h.c.}\right) + \int d^4\theta\, T_0 g_\epsilon^{\mu\nu}\, {\rm tr}(\Gamma_\mu\Gamma_\nu) + \text{matter terms}. \tag{3.4}$$

We can incorporate soft SUSY breaking by extending S_0 and T_0 to θ-dependent superfield spurions. Because DRED preserves SUSY, we can treat S_0 and T_0 as superfields even at the quantum level. The meaning of the higher components of T_0 is given by

$$\begin{aligned}\int d^4\theta\, T_0 g_\epsilon^{\mu\nu}\, {\rm tr}(\Gamma_\mu\Gamma_\nu) = \epsilon \left[\int d^2\theta\, \left(T_0| + \theta^2\, T_0|_{\theta^2}\right) {\rm tr}(W^\alpha W_\alpha) + {\rm h.c.}\right] \\ + T_0|_{\theta^2\bar\theta^2}\, g_\epsilon^{\mu\nu} A_\mu A_\nu.\end{aligned} \tag{3.5}$$

That is, the lowest components of T_0 are contributions to the gauge coupling and gaugino mass, and the $\theta^2\bar\theta^2$ component is the ϵ-scalar mass.

We now renormalize the theory by writing

$$S_0 = \mu^{-2\epsilon}\left(S + \delta S\right), \qquad T_0 = \mu^{-2\epsilon}(T + \delta T), \tag{3.6}$$

where δS and δT are counterterms that are determined order by order in perturbation theory to absorb the $1/\epsilon$ divergences. Note that we include a finite renormalized value for T. This corresponds to including evanescent effects: the scalar and θ^2 components of T are $\mathcal{O}(\epsilon)$ contributions to the gauge coupling and gaugino mass, and the $\theta^2\bar{\theta}^2$ component of T is a renormalized ϵ-scalar mass parameter. We will return to the significance of these parameters below. If we compute using supergraphs, all divergences appear in the 1PI effective action in the form $\int d^4\theta\, \mathcal{O}/\epsilon^n$, where $\mathcal{O}$ is a local (in superspace) gauge-invariant supersymmetric operator, so the counterterms can be defined to preserve the SUSY acting on the coupling constants.

Ref. [17] show that at one loop, the divergences can be absorbed in δS, but at two loops and higher, all divergences must be absorbed in δT. This sheds considerable light on the origin of the 2-loop running of the gauge coupling, as follows. At 2 loops (and higher), a $1/\epsilon^2$ pole in δT will appear as a result of subdivergences. By Eq. (3.3), this corresponds to a $1/\epsilon$ pole in the counterterm for the gauge coupling, which affects the beta function. The fact that a $1/\epsilon^2$ pole arises only from subdivergences explains why the higher-loop contributions to the gauge coupling beta function are determined by the anomalous dimensions of the matter fields.

New features arise if we include soft SUSY breaking by extending the couplings to superfields. At one loop, we find an ultraviolet divergent contribution to the ϵ-scalar mass:

$$\delta \tilde{m}_A^2 = \frac{g^2}{4\pi^2}\frac{1}{\epsilon}\left[-T_G|m_{\tilde{g}}|^2 + \sum_r T_r m_r^2\right]. \tag{3.7}$$

Although this is a finite effect, it is known that renormalization of the ϵ-scalar interactions is required to preserve unitarity [18, 19]. (Indeed an explicit calculation of Poppitz and Trivedi [20] shows that *infrared* divergences arise at 2-loops if the ϵ-scalar mass is not renormalized.)

To subtract the divergence in the ϵ-scalar mass in a way that preserves SUSY acting on the couplings, we must add the 1-loop counterterm

$$\delta T = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left[T_G \ln(S + S^\dagger) - \sum_r T_r \ln \mathcal{Z}_r\right]. \tag{3.8}$$

The logs ensure that the counterterm for the ϵ-scalar mass has the correct dependence on the gauge coupling and is independent of the wavefunction of the matter fields. Note that the scalar and θ^2 components of δT give rise to *finite* contributions to the gauge coupling and gaugino mass. This restores the dependence of the renormalized gauge coupling on $\ln \mathcal{Z}$ and $\ln(S + S^\dagger)$.

We now have all the ingredients we need to define the renormalized holomorphic and real gauge coupling superfields in DRED. The holomorphic gauge coupling is defined simply by S. Because δS contains only 1-loop divergences (and S_0 is μ

independent), S runs only at one loop. On the other hand, because of the subtraction in Eq. (3.8), the components of S do not give the renormalized gauge coupling and gaugino mass. Rather, these are given by the lowest components of a superfield R, defined by

$$R \equiv S + S^\dagger + \epsilon T + \delta T^{(1)}, \tag{3.9}$$

where $\delta T^{(1)}$ is the coefficient of $1/\epsilon$ in δT. From Eq. (3.8), we see that the quantities R and S satisfy precisely the relation derived in the previous Section for other regulators and renormalization schemes:

$$R = S + S^\dagger + \frac{T_G}{8\pi^2} \ln(S + S^\dagger) - \sum_r \frac{T_r}{8\pi^2} \ln \mathcal{Z}_r + \mathcal{O}((S + S^\dagger)^{-1}). \tag{3.10}$$

The definition Eq. (3.9) also shows that physical quantities must depend on S through R, since it is the components of R that multiply the kinetic terms and gaugino mass terms in the lagrangian.

We need to understand what scheme in component calculations is picked out by the procedure above. It is useful to define a bare gauge coupling superfield

$$R_0 \equiv S_0 + S_0^\dagger + \epsilon T_0 \tag{3.11}$$

in terms of which the bare gauge coupling and gaugino mass are

$$\frac{1}{g_0^2} = R_0|_0\,, \qquad -\frac{m_{\lambda,0}}{g_0^2} = (S_0 + \epsilon T_0)|_{\theta^2} = R_0|_{\theta^2}\,. \tag{3.12}$$

while the renormalized couplings are (see Eq. (3.9))

$$\frac{1}{g^2} = R|_0\,, \qquad -\frac{m_\lambda}{g^2} = R|_{\theta^2}\,. \tag{3.13}$$

The relation between the bare and renormalized couplings is therefore determined by the components of

$$R_0 = \mu^{-2\epsilon} \left(R + \frac{\delta S^{(1)}}{\epsilon} + \sum_{n=2}^{\infty} \frac{\delta T^{(n)}}{\epsilon^{n-1}} \right), \tag{3.14}$$

where $\delta T^{(n)}$ is the coefficient of $1/\epsilon^n$ in δT. We assume that δS and δT consist of pure $1/\epsilon$ poles. This corresponds to modified minimal subtraction ($\overline{\text{MS}}$) if we rescale μ appropriately, writing $\mu = \bar{\mu}\sqrt{e^\gamma/4\pi}$ and writing all expressions in terms of $\bar{\mu}$. Eqs. (3.12) and (3.13) then show that g and m_λ are precisely the renormalized couplings in $\overline{\text{DR}}$.

When we consider the inclusion of matter with soft scalar masses, the scheme picked out by the procedure above is identical to $\overline{\mathrm{DR}}'$ [16]. To understand the issues involved, note that there appears to be an extra renormalized parameter in DRED, corresponding to an ϵ-scalar mass. This parameter has a non-trivial RG evolution, and so cannot be set to zero at all scales. However, the ϵ-scalar mass is an evanescent effect, and does not give rise to an additional parameter at the quantum level. The way this works is that if we renormalize the theory with an arbitrary ϵ-scalar mass parameter, it only appears in physical quantities in the combination [16]

$$m^2_{r,\overline{\mathrm{DR}}}(\mu) - \frac{g^2_{\overline{\mathrm{DR}}}(\mu) C_r}{8\pi^2} \tilde{m}^2_{A,\overline{\mathrm{DR}}}(\mu) + \mathcal{O}(g^4). \tag{3.15}$$

One can then define the scheme $\overline{\mathrm{DR}}'$ by declaring the combination above to be the renormalized soft scalar mass. $\overline{\mathrm{DR}}'$ is therefore the scheme in which the ϵ-scalar mass does not appear in any renormalized expression *for arbitrary values of* μ.

In terms of the superfield couplings, the renormalized ϵ-scalar mass corresponds to the term $\epsilon\theta^2\bar{\theta}^2$ in R. But because we subtract all the $1/\epsilon$ poles in R, the 1PI action is a finite function of R. Therefore, there is no *explicit* dependence on $\tilde{m}^2_A$ in physical quantities, for any value of μ. This is sufficient to prove that the scheme we have defined is identical with $\overline{\mathrm{DR}}'$. Our procedure extends the definition of $\overline{\mathrm{DR}}'$, given in ref. [16] at the 2-loop level, to all orders in perturbation theory.

Note that the inclusion of the evanescent ϵT term in (3.9) is essential for R to satisfy the d-dimensional RG equation

$$\mu \frac{dR}{d\mu} = 2\epsilon R + \beta(R). \tag{3.16}$$

This is easy to check at lowest order by considering the RG equation for T. Therefore, in our scheme $\tilde{m}^2_A$ plays a role similar to that of the $\mathcal{O}(\epsilon)$ term in the d-dimensional RG equation for g^2: it insures μ independence of the bare coupling g^2_0, but is irrelevant in calculations.

To see more explicitly the connection to naïve $\overline{\mathrm{DR}}$, consider the relation between the bare and renormalized wavefunctions for the matter fields

$$\mathcal{Z}_{r,0} = \mathcal{Z}_r \left[1 + \sum_{n=1}^{\infty} \frac{\delta \mathcal{Z}^{(n)}_r(R)}{\epsilon^n} \right]. \tag{3.17}$$

Taking the $\theta^2\bar{\theta}^2$ components of both sides gives

$$m^2_{r,0} = m^2_r - \frac{d}{dR}\left[\delta \mathcal{Z}^{(1)}_r(R)\right] \tilde{m}^2_A + 1/\epsilon \text{ poles}. \tag{3.18}$$

In our scheme, the renormalized scalar mass is $m_r^2 = -\ln \mathcal{Z}|_{\theta^2\bar\theta^2}$, while the finite term on the right-hand side is the scalar mass in $\overline{\rm DR}$ (*not* $\overline{\rm DR}'$), since it corresponds to minimal subtraction. Comparing Eqs. (3.18) and (3.15), we see that m_r^2 is identical to $m^2_{r,\overline{\rm DR}'}$ to 2 loops. (But note that our scheme is defined to all orders in perturbation theory.)

Let us summarize the main results. In the supersymmetric limit where the explicit soft breaking is turned off, we can renormalize the theory by (modified) minimal subtraction, defining renormalized couplings in the $\overline{\rm DR}$ scheme. Our result is that if we include renormalized soft terms by analytically continuing both the renormalized couplings and the counterterms (defined as functions of the renormalized couplings) into superspace via

$$\frac{1}{g^2} \to R, \qquad Z_r \to \mathcal{Z}_r, \tag{3.19}$$

this defines a valid subtraction scheme for the softly-broken theory. This picks out a unique scheme for the soft terms to all orders in perturbation theory, which we call $\overline{\rm SDR}$ for supersymmetric dimensional reduction. (At two loops, $\overline{\rm SDR}$ coincides with $\overline{\rm DR}'$, so we can think of it as an all-orders definition of $\overline{\rm DR}'$.) In $\overline{\rm SDR}$, the RG equations for all soft parameters is determined by the RG equations in the SUSY limit, to all orders in perturbation theory. For instance, in gauge mediated models (see the next section), the analytic continuation of Eq. (3.19) is simply performed by substituting $M \to M + \theta^2 F$ in the effective couplings of the low-energy supersymmetric Standard Model.

We close with two comments on the superfield coupling R defined above. Note that the *finite* $\theta^2\bar\theta^2$ component of R defined in DRED corresponds to an *infinite* contribution to the ϵ-scalar mass. In our definition of R from the 1PI effective action, the $\theta^2\bar\theta^2$ component of R was related to a nonlocal effect. It is interesting to see the connection between these effects explicitly by considering softly broken SQED as in Section 2.2, but dimensionally reduced to $4 - 2\epsilon$ dimensions. After subtracting the $\mathcal{Z}$ independent $1/\epsilon$ divergence the gauge self-energy has the form

$$\begin{aligned}\Gamma_{\rm 1PI} = \frac{1}{4\pi^2}\int d^4\theta \, \ln \mathcal{Z} \left[\frac{1}{\epsilon} g_\epsilon^{\mu\nu} \, {\rm tr}(\Gamma_\mu \Gamma_\nu + {\rm h.c.}) + {\rm tr}\left(W^\alpha \frac{D^2}{-p^2} W_\alpha + {\rm h.c.}\right)\right] \\ + (\mathcal{Z}\text{-independent}) + \mathcal{O}(D_\alpha \mathcal{Z}, \ldots).\end{aligned} \tag{3.20}$$

If we write this out in terms of components of $\mathcal{Z}$, we see that the the terms involving $\mathcal{Z}|$ and $\mathcal{Z}|_{\theta^2}$ are local and exactly cancel between the two terms in brackets. What is left, from $\ln \mathcal{Z}|_{\theta^2\bar\theta^2}$, is just a divergent ϵ-scalar mass, see Eq. (3.7), and a nonlocal correction to the gaugino self-energy, see (2.43). Anyway, we must subtract the divergent ϵ-scalar with a superfield counterterm as (3.8), so that in the subtracted 1PI, the dependence on $\ln \mathcal{Z}$ is all coming from the non-local operator. This shows that the "chiral" components of R defined in DRED and by 1PI subtraction differ only by finite analytic ($\mathcal{Z}$-independent) terms, that is, by a change in scheme. In this sense, the two definitions are equivalent.

A closely-related issue involves the relation between the origin of the $\ln \mathcal{Z}$ term in R in DRED and in the general discussion of SQED given earlier, where it was inferred from the anomalous $U(1)$ symmetry. It is conventionally said that there is no rescaling or chiral anomaly in DRED, and it may appear that there is no direct connection between these arguments. However, an intriguing clue can be seen by considering the bare Lagrangian with couplings S_0, T_0, and $\mathcal{Z}_0$. This Lagrangian has the symmetry

$$T_0 \mapsto T_0 + A + A^\dagger, \quad S_0 \mapsto S_0 + \epsilon A, \quad \mathcal{Z}_0 \mapsto \mathcal{Z}_0, \tag{3.21}$$

which ensures that physical quantities depend on the combination $S_0 + S_0^\dagger + \epsilon T_0$. However, arbitrary values of T_0 lead to inconsistencies (loss of unitarity and IR divergences). Up to two loops the choice

$$T_0 = -\frac{1}{4\pi^2}\frac{1}{\epsilon}\ln \mathcal{Z}_0 \tag{3.22}$$

eliminates the problems. But with this choice, physical quantities depend on the combination $S_0 + S_0^\dagger - \ln \mathcal{Z}_0/4\pi^2$, which is just what is required to obtain the anomalous $U(1)$. We believe that these are very suggestive connections that come close to exposing the anomaly in DRED, and we plan on exploring this point more completely elsewhere.

3.2 Two-loop Renormalization Group equations in $\overline{\mathrm{DR}}'$

We can check explicitly that the scheme defined above is equivalent to $\overline{\mathrm{DR}}'$ at NLO by computing the 2-loop RG equations for the gluino and sfermion masses. Consider the real gauge coupling, given by

$$R(\mu) = S(\mu) + S^\dagger(\mu) + \frac{T_G}{8\pi^2}\ln\left[S(\mu) + S^\dagger(\mu)\right] - \frac{T_r}{8\pi^2}\ln \mathcal{Z}_r(\mu), \tag{3.23}$$

where S is the holomorphic gauge coupling. The gaugino mass is given by $m_\lambda = -\ln R|_{\theta^2}$, so its NLO β function is easily derived from Eq. (3.23):

$$\mu\frac{dm_\lambda}{d\mu} = -\frac{g^2}{(8\pi^2)^2}\left(T_G b - 2\sum_r T_r C_r\right) m_\lambda. \tag{3.24}$$

where $b = 3T_G - \sum_r T_r$. This equation agrees with the explicit component calculations in $\overline{\rm DR}$. A similar derivation, based on the Konishi anomaly, was given by Hisano and Shifman [6]. A new feature of the present treatment is that R also governs the evolution of the dimension-2 soft terms. To see this, consider

$$R|_{\theta^2\bar\theta^2} = \frac{1}{8\pi^2}\left[-T_G m_\lambda^2 + \sum_r T_r m_r^2\right]. \tag{3.25}$$

According to our discussion above, $R|_{\theta^2\bar\theta^2}$ corresponds to a $1/\epsilon$ counterterm for the ϵ-scalar mass. Eq. (3.25) agrees with what is found in explicit component calculations [20]. (Notice that the quantity on the right-hand side is proportional to the supertrace weighted by the Dynkin indices.) Now, consider the 2-loop RG equation for matter fields in $\overline{\rm DR}$ [21, 22]

$$\mu\frac{d\ln Z_r}{d\mu} = \frac{1}{8\pi^2}\left\{2C_r g^2 + \frac{g^4}{8\pi^2}C_r\left[3T_G - T - 2C_r\right]\right\}, \tag{3.26}$$

where $T = \sum_r T_r$. Its continuation into superspace simply amounts to the substitution $g^2 \to 1/R$, $Z \to \mathcal{Z}$. The RG equation for the scalar masses is then obtained by taking the $\theta^2\bar\theta^2$ component of Eq. (3.26). This gives

$$\mu\frac{dm_r^2}{d\mu} = -\frac{C_r}{8\pi^2}\left\{4g^2 m_\lambda^2 + \frac{g^4}{8\pi^2}\left[2T_G m_\lambda^2 - 2\sum_s T_s m_s^2 + 6(3T_G - T - 2C_r)m_\lambda^2\right]\right\}, \tag{3.27}$$

which agrees with the result in $\overline{\rm DR}'$ [22, 10, 19, 16]. The same check can be done for the evolution of A- and B-terms and in the presence of Yukawa interactions.

4 Gauge-mediated Supersymmetry Breaking

We now show how to apply the formalism of the previous Section to perform calculations in gauge-mediated SUSY breaking (GMSB) models. We begin by briefly reviewing the calculation of the leading gaugino and scalar masses in GMSB, as performed in Ref. [3]. We then turn to new calculations at higher loop orders. The main new result in this Section is that the gaugino masses are insensitive to the couplings in the messenger sector up to four loops. This "screening theorem" means that it is possible to make precise predictions for gaugino masses even when the SUSY breaking dynamics is strongly coupled. The scalar masses are not screened in this way, and are therefore sensitive to strong SUSY-breaking dynamics. We also compute the NLO corrections to SUSY-breaking masses in GMSB, which correspond to 2-loop corrections for gaugino masses and 3-loop corrections to the scalar masses.

4.1 Leading Results

In this Subsection, we briefly review the main results of Ref. [3] for completeness. Consider the fundamental theory

$$\begin{aligned}\mathcal{L}' = \int d^4\theta \left[\mathcal{Z}'_Q \left(Q^\dagger e^{V(Q)} Q + \bar{Q}^\dagger e^{V(\bar{Q})} \bar{Q} \right) + \sum_r \mathcal{Z}'_r q_r^\dagger e^{V(r)} q_r \right] \\ + \int d^2\theta \, S' \operatorname{tr}(W^\alpha W_\alpha) + \text{h.c.} \\ + \int d^2\theta \, \lambda X Q \bar{Q} + \text{h.c.},\end{aligned} \tag{4.1}$$

where $Q, \bar{Q}$ are the messengers, q_r are observable sector fields, and X is a singlet. X is a background chiral superfield that parameterizes the effect of SUSY breaking via

$$\lambda X = M + \theta^2 F, \tag{4.2}$$

with the assumption $F \ll M^2$. Our notation is appropriate to the case where there is a single gauge group, but our formulas are trivial to generalize to the case of product gauge groups. Below the scale M, the effective lagrangian is

$$\begin{aligned}\mathcal{L} = \int d^4\theta \sum_r \mathcal{Z}_r q_r^\dagger e^{V(r)} q_r \\ + \int d^2\theta \, S \operatorname{tr}(W^\alpha W_\alpha) + \text{h.c.} + \cdots,\end{aligned} \tag{4.3}$$

where the omitted terms consist of higher-dimension operators. The low-energy gauge coupling is given by tree-level matching and one-loop running to be

$$S(\mu) = S'(\mu_0) + \frac{b'}{16\pi^2} \ln \frac{M}{\mu_0} + \frac{b}{16\pi^2} \ln \frac{\mu}{M}, \tag{4.4}$$

where

$$b' = b - N, \qquad b = 3T_G - \sum_r T_r, \tag{4.5}$$

are the beta function coefficients in the full and effective theories, respectively. $N \equiv \sum_Q T_Q$ is the "messenger index". Here μ_0 is an ultraviolet scale where the theory is defined; this means that we must evaluate derivatives holding the running couplings at the scale μ_0 fixed.

The dependence of the low-energy effective Lagrangian on the SUSY-breaking effects is given simply by making the replacement

$$M \to X \tag{4.6}$$

in the dependence of the effective couplings S and $\mathcal{Z}_r$. (Notice that to simplify the notation we have absorbed λ in the definition of X). It is this "analytic continuation" that is at the heart of the method of Ref. [3]. We can now read off the gaugino mass from

$$m_\lambda(\mu) = -g^2(\mu)\,\frac{\partial S(\mu)}{\partial X}\bigg|_0 F = \frac{Ng^2(\mu)}{16\pi^2}\,\frac{F}{M}, \tag{4.7}$$

where the notation "$|_0$" denotes setting $\theta = \bar{\theta} = 0$ and $X = M$. Note that this automatically gives the correct RG improvement of the gaugino mass. Eq. (4.7) involves the holomorphic gauge coupling, which is equivalent to the real superfield coupling at one loop. The use of the real gauge coupling is crucial for the higher-order calculations we do later.

We now consider the contribution to the gaugino mass coming from higher-dimension operators in the effective lagrangian [3]. Operators in the effective lagrangian consist of analytic terms in the light fields and the background X and their derivatives divided by powers of X. The lowest-dimension operator respecting the $U(1)_R$ symmetry that can contribute to the gaugino mass is

$$\delta\mathcal{L} = \frac{cg^2}{16\pi^2}\int d^4\theta\left[\frac{X^\dagger D^2 X}{|X|^4}\,\mathrm{tr}(W^\alpha W_\alpha) + \text{h.c.}\right]. \tag{4.8}$$

Eq. (4.8) gives a contribution to the gaugino mass of order

$$\delta m_\lambda \sim m_\lambda\,\frac{|F|^2}{|M|^4}. \tag{4.9}$$

This is negligible if $F \ll M^2$. It is easy to see that all other higher-dimension operators also give contributions to the gaugino and scalar masses that are suppressed by powers of F_X^2/M^4.

We now turn to the calculation of the scalar mass, where the correct continuation into superspace is $M \to \sqrt{XX^\dagger}$ [3]. We compute the matter field wavefunction coefficients $\mathcal{Z}_r$, whose θ-dependence contains SUSY breaking from the dependence on the threshold at M:

$$m_r^2(\mu) = -\,\frac{\partial^2 \ln \mathcal{Z}_r(\mu)}{\partial X^\dagger \partial X}\bigg|_0 |F|^2 = -\frac{1}{4}\,\frac{\partial^2 \ln \mathcal{Z}_r(\mu)}{(\partial \ln |X|)^2}\,\left|\frac{F}{M}\right|^2, \tag{4.10}$$

where we have used the fact that $\ln \mathcal{Z}_r$ is a vector superfield, and therefore depends on X through $|X|$. The 1-loop RG equation for $\mathcal{Z}_r$ is

$$\gamma_r = \mu\frac{d\ln \mathcal{Z}_r(\mu)}{d\mu} = \frac{C_r}{4\pi^2}\,\frac{1}{S+S^\dagger}, \quad \gamma'_r = \mu\frac{d\ln \mathcal{Z}'_r(\mu)}{d\mu} = \frac{C_r}{4\pi^2}\,\frac{1}{S'+S'^\dagger}. \tag{4.11}$$

Computing $\mathcal{Z}_r$ using 1-loop running and tree-level matching, we have

$$\ln \mathcal{Z}_r(\mu) = \int_{\mu_0}^{M} \frac{d\mu'}{\mu'}\, \gamma'_r(\mu') + \int_{M}^{\mu} \frac{d\mu'}{\mu'}\, \gamma_r(\mu'). \tag{4.12}$$

This gives

$$\begin{aligned} \frac{\partial \ln \mathcal{Z}_r(\mu)}{\partial \ln |X|} &= \int_{|X|}^{\mu} \frac{d\mu'}{\mu'} \frac{\partial \gamma_r(\mu')}{\partial \ln |X|} \\ &= \frac{C_r}{4\pi^2} \int_{|X|}^{\mu} \frac{d\mu'}{\mu'} \frac{\partial}{\partial \ln |X|} \left(\frac{1}{S(\mu') + S^\dagger(\mu')} \right). \end{aligned} \tag{4.13}$$

Note that the explicit $|X|$ dependence from the limits of integration cancels in the derivative because of the tree-level matching conditions. From Eq. (4.4), we see that

$$S(\mu) + S^\dagger(\mu) = S'(\mu_0) + S'^\dagger(\mu_0) + \frac{b'}{16\pi^2} \ln \frac{X^\dagger X}{\mu_0^2} + \frac{b}{16\pi^2} \ln \frac{\mu^2}{X^\dagger X}, \tag{4.14}$$

which depends on X only through $|X|$, as required. We then obtain

$$\frac{\partial \ln \mathcal{Z}_r(\mu)}{\partial \ln |X|} = -\frac{C_r}{4\pi^2} \int_{X}^{\mu} \frac{d\mu'}{\mu'} \frac{b' - b}{8\pi^2} \left(\frac{1}{S(\mu') + S^\dagger(\mu')} \right)^2. \tag{4.15}$$

Computing one more derivative yields

$$\left. \frac{\partial^2 \ln \mathcal{Z}_r(\mu)}{\partial (\ln |X|)^2} \right|_{\mu = M} = -\frac{2 C_r N}{(8\pi^2)^2}\, g^4(M), \tag{4.16}$$

where we used the definition of the messenger index $N \equiv b - b'$. This gives a scalar mass

$$m_r^2(M) = \frac{C_r N \alpha^2(M)}{8\pi^2} \left| \frac{F}{M} \right|^2. \tag{4.17}$$

It is remarkable that the finite part of a 2-loop graph can be evaluated from a 1-loop RG computation. In the present approach, this arises because $\mathcal{Z}_r$ depends on $|X|$ only through the values of running couplings, and derivatives with respect to $|X|$ therefore bring in extra loop factors.

4.2 Gaugino Screening

We now consider corrections to the gaugino mass. Very generally, we will find that contributions from messenger interactions to the gaugino mass are suppressed by additional loop factors beyond the naïve expectation, a result we refer to as "gaugino screening". We will see in Sect. 4.4 that the scalar masses are not similarly screened.

The main point is that the holomorphic gauge coupling is given *exactly* by

$$S(\mu) = \frac{b' - b}{16\pi^2} \ln X + (X\text{-independent}), \tag{4.18}$$

where b (b') is the beta function coefficient in the effective theory below (above) the messenger scale. (If the SM gauge group has a standard embedding into a larger messenger group above the messenger scale, then b' is the beta function of the larger group.) The physical gaugino mass must be read off from the θ-dependent components of the real superfield gauge coupling. (As explained above, the holomorphic gauge coupling has unphysical field rescaling invariance that is not present in physical quantities.) The real gauge coupling is related to the holomorphic gauge coupling by

$$\begin{aligned} R(\mu) = S(\mu) + S^\dagger(\mu) + \frac{T_G}{8\pi^2} \ln \left[S(\mu) + S^\dagger(\mu) \right] - \sum_r \frac{T_r}{8\pi^2} \ln \mathcal{Z}_r \\ + \mathcal{O}(S + S^\dagger)^{-1}. \end{aligned} \tag{4.19}$$

The dependence on the wavefunction factors $\mathcal{Z}_r$ contains the information about the 2-loop RG behavior of the physical couplings. Since S is just given exactly by Eq. (4.18), and since the sum over r runs only over the *light* fields, R is not affected at the NLO by the messenger interactions. That's all there is to the proof!

Because the leading dependence on the messenger interactions comes from $\mathcal{Z}_r$ in Eq. (4.19), it is easy to see that

$$\frac{\delta m_\lambda}{m_\lambda} \sim \left(\frac{g}{4\pi} \right)^4 \left[\frac{g_{\text{mess}}^2}{16\pi^2} + \ln \frac{M'}{M} \right] . \tag{4.20}$$

The $(g/4\pi)^4$ factor arises because the messenger fields interact with matter only at 2 loops. The first term in square brackets represents a threshold correction due to a messenger coupling g_{mess},while the term $\ln(M'/M)$ represents the sensitivity to mass splittings among the messengers. Such mass splittings will arise if the various messengers have different Yukawa couplings λ to the same source X (see Eq. (4.1)). In the next Subsection, we perform explicit calculations of the gaugino masses and NLO, and we will see how the screening theorem manifests itself in detail. In the remainder of this Subsection, we confine ourselves to some qualitative remarks.

Consider for example the dependence on the messenger Yukawa coupling λ. At leading order, the low-energy gaugino masses are independent of λ, but one may naively expect important quantum corrections if λ is large. This is not an artificial possibility: if the Yukawa coupling arises from composite dynamics, the value of λ will be close to the perturbative limit $\lambda \sim 4\pi$ at the compositeness scale [23]. In this case, $g^2_{\rm mess}/(16\pi^2) \sim 1$ in Eq. (4.20), but $\delta m_\lambda / m_\lambda$ is still suppressed by two weak loops. Therefore, the gauge-mediation gaugino mass relation are rather insensitive to strong dynamics of the messenger fields even if λ is close to the perturbative limit.

Another interesting example is the case in which different messenger fields have different Yukawa couplings to the same supersymmetry breaking source X. In other words, the various messengers have different masses M but the same ratio F/M. For example, in a GUT model with a messenger scale much lower than the GUT scale, the running of the messenger Yukawa couplings between the GUT scale and the messenger scale can induce splittings of the messenger masses of order $(g^2/(16\pi^2)\ln(M_{GUT}/M)$, which can be $\mathcal{O}(1)$ even if the messenger Yukawa interactions are unified at the GUT scale. Now, Eq. (4.20) shows that, even for $\mathcal{O}(1)$ messenger mass splittings, the minimal GMSB relation between the different gaugino masses are only violated by $\mathcal{O}((g/4\pi)^4)$. Therefore, the gaugino masses do not depend on the assumption of universality of the messenger Yukawa couplings at the messenger scale even at NLO, as long as the Yukawa couplings are of the same order and there is a single source X of SUSY breaking.

Similar considerations apply to models with vector messengers. In such models, the vacuum expectation value that breaks SUSY also breaks a larger gauge group down to the standard-model subgroup. There are therefore massive gauge bosons charged under the standard-model gauge group that act as SUSY-breaking messengers. Ref. [3] computed the leading contribution of vector messengers to the scalar and gaugino masses, and showed that the contribution to the scalar mass-squared is negative. The leading contribution to the gaugino mass from the vector messengers also arises at 4 loops, and again has the order of magnitude given in Eq. (4.20), where $g_{\rm mess}$ is now the messenger gauge coupling. This is important because the messenger gauge coupling can be strong at the messenger scale. (For example, this occurs in the models of Refs. [24]).

4.3 Gaugino Masses at the Next-to-Leading Order

We now compute the NLO corrections to the gaugino masses in $\overline{\rm DR}$. In components these corrections correspond to threshold effects at the messenger mass scale described by 2-loop Feynman graphs, together with the two-loop RG evolution from the messenger scale to the physical scale. In our approach, these corrections can be extracted from the expression of the real superfield R. In $\overline{\rm DR}$, the NLO matching at the messenger scale is simply obtained by requiring continuity of $R(\mu)$ at the threshold of the physical messenger mass [25]

$$\mu_X^2 = \frac{XX^\dagger}{\mathcal{Z}_M^2(\mu_X)}. \tag{4.21}$$

Here $\mathcal{Z}_M$ is the wavefunction factor for the messenger fields. Following the notation introduced in sect. 4.1, primed (unprimed) quantities refer to the theory above (below) the messenger mass scale. In terms of the value of R' at an arbitrary high-energy scale μ_0, much larger than the messenger scale μ_X, at the low-energy scale μ we find

$$R(\mu) = R'(\mu_X) + \frac{b}{16\pi^2}\ln\frac{\mu^2}{\mu_X^2} + \frac{T_G}{8\pi^2}\ln\frac{\mathrm{Re}S(\mu)}{\mathrm{Re}S(\mu_X)} - \sum_r \frac{T_r}{8\pi^2}\ln\frac{\mathcal{Z}_r(\mu)}{\mathcal{Z}_r(\mu_X)}, \tag{4.22}$$

$$\begin{aligned} R'(\mu_X) = R'(\mu_0) &+ \frac{b'}{16\pi^2}\ln\frac{\mu_X^2}{\mu_0^2} + \frac{T_G}{8\pi^2}\ln\frac{\mathrm{Re}S'(\mu_X)}{\mathrm{Re}S'(\mu_0)} \\ &- \sum_r \frac{T_r}{8\pi^2}\ln\frac{\mathcal{Z}'_r(\mu_X)}{\mathcal{Z}'_r(\mu_0)} - \frac{N}{8\pi^2}\ln\frac{\mathcal{Z}_M(\mu_X)}{\mathcal{Z}_M(\mu_0)} \ . \end{aligned} \tag{4.23}$$

Here $S(\mu)$ is the gauge coupling at one loop (see Eq. (4.14)), and $R'(\mu_0) = \mathrm{Re}\, S'(\mu_0)$ gives a SUSY-preserving boundary condition on the gauge coupling. The sums in the previous equations extend over the different matter superfields. Substituting Eq. (4.22) into Eq. (4.23), we obtain

$$\begin{aligned} R(\mu) = R'(\mu_0) &+ \frac{b}{16\pi^2}\ln\frac{\mu^2}{\mu_0^2} + \frac{b'-b}{16\pi^2}\ln\frac{XX^\dagger}{\mu_0^2\mathcal{Z}_M^2(\mu_0)} \\ &+ \frac{T_G}{8\pi^2}\ln\frac{\mathrm{Re}S(\mu)}{\mathrm{Re}S'(\mu_0)} - \sum_r \frac{T_r}{8\pi^2}\ln\frac{\mathcal{Z}_r(\mu)}{\mathcal{Z}'_r(\mu_0)} \ . \end{aligned} \tag{4.24}$$

Notice that in this expression the explicit dependence on $\mathcal{Z}_M(\mu_X)$ has dropped out. An implicit dependence appears from higher-order contributions in the matter wave-function renormalization $\mathcal{Z}_r(\mu)$. However, the NLO expression for the gaugino mass, which requires only the leading contribution to $\mathcal{Z}_r(\mu)$, is independent of $\mathcal{Z}_M(\mu_X)$. This is a manifestation of the "gaugino screening" theorem discussed in Sect. 4.2. See see that at this order in perturbation theory, the gaugino masses are not affected by new messenger interactions. Similarly, the W-ino and B-ino masses have no α_3 corrections from messenger thresholds, but only from their RG evolution below the messenger mass.

To obtain the expression of the gaugino mass, we take the F component of Eq. (4.24):

$$m_\lambda(\mu) = -g^2(\mu) R(\mu)|_{\theta^2}$$

$$= \frac{1}{1 - g^2(\mu) T_G/(8\pi^2)} \left\{ \frac{g^2(\mu)}{16\pi^2} N \frac{F}{M} + \sum_r \frac{g^2(\mu)}{8\pi^2} T_r \ln \mathcal{Z}_r(\mu)|_{\theta^2} \right\} . \quad (4.25)$$

This equation gives the NLO expression of the gaugino mass in terms of the SUSY-breaking part of the light matter wave functions $\mathcal{Z}_r(\mu)$ at the leading order. To complete the calculation, we now compute $\ln \mathcal{Z}_r(\mu)|_{\theta^2}$ for matter fields including both gauge and Yukawa interactions. For simplicity, we give the result for a simple gauge group, but the generalization to a product group is completely straightforward. The relevant 1-loop RG equations are:

$$\mu \frac{d}{d\mu} \ln \mathcal{Z}_r = \frac{C_r}{4\pi^2} g^2 - \frac{d_r}{8\pi^2} y^2 , \quad (4.26)$$

$$\mu \frac{d}{d\mu} y^2 = \frac{y^2}{4\pi^2} \left(\frac{D}{2} y^2 - C g^2 \right) , \quad (4.27)$$

$$\mu \frac{d}{d\mu} g^{-2} = \frac{b}{8\pi^2} , \quad (4.28)$$

where y is the running Yukawa coupling (physically normalized by appropriate wave-function factors). Here, d_r is the number of fields circulating in the Yukawa loop, and

$$C \equiv \sum_r C_r, \quad D \equiv \sum_r d_r, \quad (4.29)$$

with the sum extended to the field participating in the Yukawa interaction. If g is the QCD coupling, and y is the top-quark Yukawa coupling, we have $C = 8/3$ and $D = 6$. Taking the F component of the solution of Eq. (4.26) for $\mathcal{Z}_r(\mu)$, we obtain the final expression for the gaugino mass including QCD (α_3) and top-quark Yukawa ($\alpha_t = y_t^2/(4\pi)$) corrections

$$m_{\lambda_J}(\mu) = \frac{\alpha_J(\mu)}{4\pi} N \frac{F}{M} \left[1 + T_J \frac{\alpha_J(\mu)}{2\pi} + \frac{4\alpha_3(\mu)}{9\pi} (\xi - 1) \sum_r T_r \right.$$
$$\left. + \frac{\alpha_t(\mu)}{6\pi} I(\xi) \sum_r T_r d_r \right] , \quad (4.30)$$

where

$$\xi = \frac{\alpha_3(X)}{\alpha_3(\mu)}, \quad I(\xi) = 1 - \frac{16}{7}\xi + \frac{9}{7}\xi^{16/9} , \quad (4.31)$$

where the sum is taken over the colored light fields. This result has been recently confirmed by an explicit component calculation [26]. Notice that in the above equations the dependence on the physical messenger mass appears via ξ, and it is of the form given in Eq. (4.20).

In order to obtain the pole gaugino mass we have to include also the finite one-loop corrections at the infrared threshold. For the gluino, in the $\overline{\rm DR}$ scheme they are given by [27]

$$m_{\lambda_3}^{\rm pole} = m_{\lambda_3}(\mu)\left\{1+\frac{3\alpha_3(\mu)}{4\pi}\left[\ln\left(\frac{\mu^2}{m_{\lambda_3}^2}\right)+\mathcal{F}\left(\frac{\tilde{m}_q^2}{m_{\lambda_3}^2}\right)\right]\right\}~, \tag{4.32}$$

$$\mathcal{F}(x) = 1+2x+2x(2-x)\ln x+2(1-x)^2\ln|1-x|~. \tag{4.33}$$

The function $\mathcal{F}$ includes the effect of the gluon-gluino and quark-squark loops in the approximation in which all squarks have equal mass $\tilde{m}_q$. Since we have neglected weak corrections, the $SU(2)\times U(1)$ gaugino masses receive no contributions from infrared thresholds. The final expressions for the three gaugino masses improved by α_3 and α_t corrections are then given by

$$m_{\lambda_3}^{\rm pole} = \frac{\alpha_3(\mu)}{4\pi}N\frac{F}{M}\left\{1+\frac{3\alpha_3(\mu)}{4\pi}\left[\ln\left(\frac{\mu^2}{m_{\lambda_3}^2}\right)+\mathcal{F}\left(\frac{\tilde{m}_q^2}{m_{\lambda_3}^2}\right)+2+\frac{32}{9}(\xi-1)\right]\right.$$
$$\left. + \frac{\alpha_t(\mu)}{3\pi}I(\xi)\right\}~, \tag{4.34}$$

$$m_{\lambda_2}^{\rm pole} = \frac{\alpha_2(\mu)}{4\pi}N\frac{F}{M}\left[1+\frac{2\alpha_3(\mu)}{\pi}(\xi-1)+\frac{\alpha_t(\mu)}{2\pi}I(\xi)\right]~, \tag{4.35}$$

$$m_{\lambda_1}^{\rm pole} = \frac{5\alpha_1(\mu)}{12\pi}N\frac{F}{M}\left[1+\frac{22\alpha_3(\mu)}{15\pi}(\xi-1)+\frac{13\alpha_t(\mu)}{30\pi}I(\xi)\right]~, \tag{4.36}$$

where $\alpha_2=\frac{5}{3}\alpha_1$ at the unification scale.

The NLO correction to the gluino mass $(m_{\lambda_3}^{\rm pole})^{\rm NLO}/(m_{\lambda_3})^{\rm LO}-1$ is shown in Fig. 2. We have assumed $(m_{\lambda_3})^{\rm LO}=600$ GeV and $\tan\beta=2$, but the result is very insensitive to this choice. In particular, the value of $\tan\beta$ is unimportant because the top-quark Yukawa contribution in Eq. (4.34) is negligible. The NLO contribution from messenger loops, which is obtained by setting $\xi=1$ in Eq. (4.34), is about +4–5%. However, the NLO gauge RG evolution contributes a negative contribution (see Eq. (4.34) and Fig. 2) that almost completely cancels the messenger contribution for very large running ($M\simeq 10^{15}$ GeV). The finite gluon-gluino loop gives also a large positive contribution of about +10–12% to the gluino mass. This effect is partially compensated by the quark-squark loops, if the ratio $m_{\tilde{q}}^2/M_3^2$ is not large, as in the case of several messenger flavors ($N>1$). This explains why the NLO correction to the gluino mass is very important for small M and N, but significantly decreases for larger values of M and N (see Fig. 2).

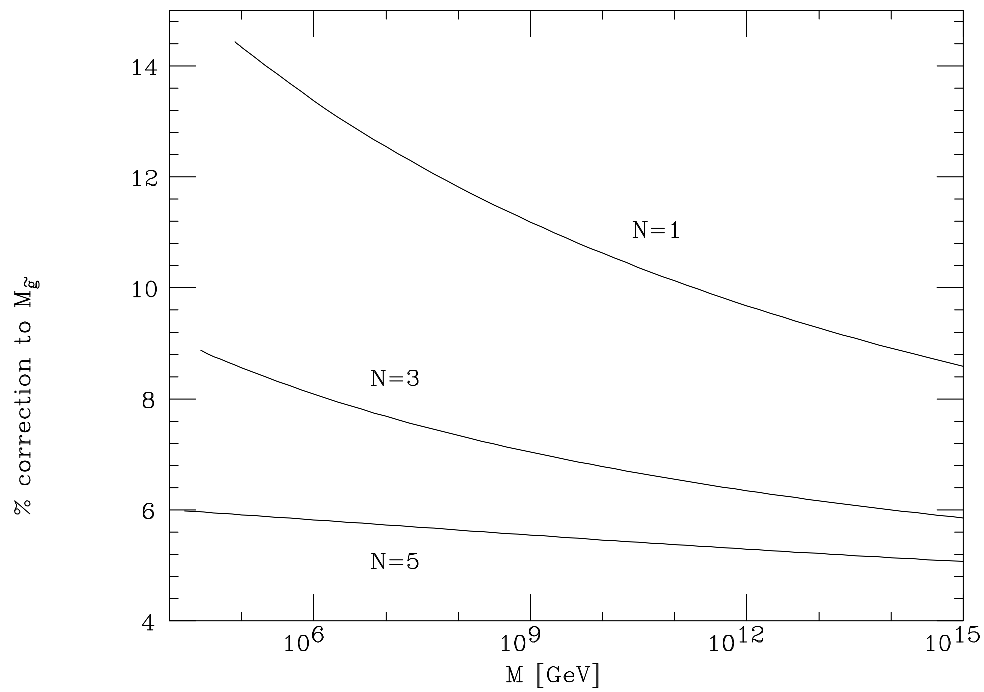

Figure 2: *NLO correction to the gluino pole mass, as a function of the messenger mass scale M, for messenger index $N = 1, 3, 5$. We have taken a leading-order value of the gluino mass of 600 GeV and $\tan\beta = 2$, but the results are rather insensitive to these choices. The curves are interrupted at values of M that require $F = M^2$ to obtain the required gluino mass.*

The QCD corrections to the $SU(2)\times U(1)$ gaugino masses vanish at the messenger scale, as expected from the "screening theorem" previously discussed. The effects from the RG running, shown in Eqs. (4.35)–(4.36), tend to cancel between the gauge and Yukawa term, and give a contribution to the weak gaugino masses that is at most of few percent.

4.4 Scalar Masses at the Next-to-Leading Order

We can now also compute the NLO corrections to the squark and slepton masses in $\overline{\mathrm{DR}}'$, which correspond to 3-loop diagrams. The RG equation for the wave-function renormalization of a matter field r is

$$\mu\frac{d}{d\mu}\ln\mathcal{Z}_r=\gamma_r. \tag{4.37}$$

The gauge contribution to the anomalous dimension γ_r at the NLO is given by [21, 22]

$$\gamma_r=C_r\frac{g^2}{4\pi^2}+C_r\left[3T_G-2C_r-T\right]\frac{g^4}{4(2\pi)^4}\ . \tag{4.38}$$

The SUSY-breaking scalar mass is obtained from Eq. (4.10)

$$\begin{aligned}\tilde m_r^2(\mu)&=-\frac{1}{4}\frac{\partial^2\ln\mathcal{Z}_r(\mu)}{(\partial\ln|X|)^2}\left|\frac{F}{M}\right|^2\\&=-\frac{1}{4}\left|\frac{F}{M}\right|^2\frac{\partial^2}{(\partial\ln|X|)^2}\left[\int_{\mu_0}^{\mu_X}\frac{d\mu'}{\mu'}\gamma_r'(\mu')\right.\\&\left.+\int_{\mu_X}^{\mu}\frac{d\mu'}{\mu'}\gamma_r(\mu')\right]\ ,\end{aligned} \tag{4.39}$$

where γ_r (γ_r') is the anomalous dimensions below (above) the physical messenger scale μ_X (see Eq. (4.21)). Note that γ_r in the low-energy theory depends implicitly on μ_X from the matching conditions at the messenger threshold. Notice also that the lowest matching correction for the wave function at the messenger scale μ_X is at 2-loops. This corresponds to the addition of an $\mathcal{O}(\alpha(X)^2/16\pi^2)$ term inside square brackets in Eq. (4.39). The resulting correction to the squark mass is $\mathcal{O}(\alpha^4)$.

For simplicity, we will give the expression of the scalar masses evaluated at the messenger scale, as the 2-loop running from μ_X to the low-energy scale μ is well known [22, 10, 19]. In this case, the action of $\partial^2/(\partial\ln|X|)^2$ on Eq. (4.39) gives, at the NLO in gauge interactions,

$$m_r^2(\mu_X)=-\frac{1}{4}\left|\frac{F}{M}\right|^2\frac{\partial\ln\mu_X}{\partial\ln|X|}\,\frac{\partial}{\partial\ln\mu_X}\left[\gamma_r'(R'(\mu_X))-\gamma_r(R(\mu_X))-\gamma_r(R(\mu))\right]\Big|_{\mu=\mu_X}$$

$$= \frac{g^4}{4}\left|\frac{F}{M}\right|^2 (1-\gamma_M(\mu_X))\left[\frac{\partial(\gamma_r'(\mu)-\gamma_r(\mu))}{\partial g^2}\frac{\partial R'(\mu)}{\partial \ln\mu} - \frac{\partial \gamma_r}{\partial g^2}\frac{\partial R(\mu)}{\partial \ln \mu_X}\right]_{\mu=\mu_X} \quad (4.40)$$

Here $\gamma_M = d\ln \mathcal{Z}_M / d\ln\mu$ is the anomalous dimension of the messenger superfield at the leading order, which depends not only on gauge interactions, but also on any new additional interactions of the messengers. In particular, including the Yukawa interaction in Eq. (4.1), we find

$$\gamma_M = \frac{C_M g^2}{4\pi^2} - \frac{\lambda^2}{8\pi^2}\ . \quad (4.41)$$

This explicitly shows that the "screening theorem", valid for gaugino masses, does not apply to scalar masses.

We can now evaluate the derivatives of R, using the expressions obtained in the previous section:

$$\left.\frac{\partial R'(\mu_X)}{\partial \ln \mu_X}\right|_0 = \frac{b'}{8\pi^2}\ , \quad (4.42)$$

$$\left.\frac{\partial R(\mu,\mu_X)}{\partial \ln \mu_X}\right|_0 = -\frac{N}{8\pi^2}\left(1+\frac{T_G}{8\pi^2}g^2\right)\ . \quad (4.43)$$

Notice that in Eq. (4.42) we have kept only the leading term in the perturbative expansion, since in Eq. (4.40) it multiplies the factor $\partial(\gamma_r'-\gamma_r)/\partial g^2$, which is an NLO quantity. Putting it all together, we obtain the final expression for the scalar masses at the NLO

$$\tilde{m}_r^2(\mu_X) = \frac{C_r\alpha^2(\mu_X)N}{8\pi^2}\left|\frac{F}{M}\right|^2(1-\gamma_M) \times\left[1+\frac{\alpha(\mu_X)}{2\pi}(T_G-2C_r+N)\right]\ . \quad (4.44)$$

Assuming that the messengers belong to fundamentals of $SU(5)$, the NLO expression for the QCD contribution to squark masses is

$$\tilde{m}_q^2(\mu_X) = \frac{\alpha_3^2(\mu_X)N}{6\pi^2}\left|\frac{F}{M}\right|^2\left[1+\frac{\alpha_3(\mu_X)}{2\pi}\left(N-\frac{7}{3}\right)+\frac{\lambda_3^2(\mu_X)}{8\pi^2}\right]\ . \quad (4.45)$$

Here λ_3 is the messenger Yukawa coupling for the color triplet. Notice that $\mathcal{O}(\alpha_3^3)$ contribution to squark masses from the messengers tends to cancel the contribution from gauge and matter fields, as long as N is not too large. NLO corrections to slepton masses from QCD and new messenger interactions come only from the factor

$(1-\gamma_M)$ in Eq. (4.40). Since, in our case, weak-doublet messengers are color neutral, the $SU(2)$ contribution to left-handed slepton masses is corrected only by the factor $(1+\lambda_2^2(\mu_X)/8\pi^2)$. However, for a generic choice of messengers, the QCD corrections are non-vanishing. Notice also that in general $\lambda_2 \neq \lambda_3$, although they may be related in a GUT. Finally, the improved expression for the right-handed slepton mass is

$$\tilde{m}^2_{e_R}(\mu_X) = \frac{5\alpha_1^2(\mu_X)N}{24\pi^2}\left|\frac{F}{M}\right|^2\left[1-\frac{8\alpha_3(\mu_X)}{15\pi}+\frac{3\lambda_2^2(\mu_X)}{40\pi^2}+\frac{\lambda_3^2(\mu_X)}{20\pi^2}\right]. \quad (4.46)$$

In Eq. (4.46), μ_X can correspond to the mass scale of either the triplet or the doublet messenger mass. The difference between the two definitions is $\mathcal{O}(\alpha_1^3)$, which is negligible in our approximation.[7] On the other hand, μ_X in Eq. (4.45) has to be interpreted as the triplet messenger mass, since we include terms $\mathcal{O}(\alpha_3^3)$.

In conclusion, because of the absence of a "screening theorem", the NLO corrections to scalar masses are quite dependent on the model assumptions. They are sensitive to new messenger interactions, like the messenger Yukawa couplings, and they depend on the messenger representations in a way that cannot be described only by the messenger index N.

4.5 D-type Supersymmetry Breaking

We now consider leading SUSY breaking effects in theories where the dominant source of SUSY breaking is a D-type soft mass for the messengers rather than a F-type mass, as considered previously. Some of these results have already been derived in the language of renormalized couplings in Sect. 3.2. We discuss them here in a manifestly "Wilsonian" picture, that is, by simply computing in the theory with given bare parameters. We do this in part for variety, and in part to show how these results follow from the "Wilsonian" anomalous $U(1)$ symmetry.[8] Consider a gauge theory with bare lagrangian

$$\mathcal{L}_0 = \int d^2\theta\, S_0\,\mathrm{tr}(W^\alpha W_\alpha) + \mathrm{h.c.} + \int d^4\theta\, \mathcal{Z}_{r,0}\Phi_r^\dagger e^{V^{(r)}}\Phi_r, \quad (4.47)$$

regulated in a supersymmetric manner. Assume that the theory contains bare soft masses, parameterized by

$$\mathcal{Z}_{r,0} = Z_{r,0}\left[1-\theta^2\bar{\theta}^2 m_{r,0}^2\right]. \quad (4.48)$$

[7] Higher orders in the electroweak couplings can be computed following the same procedure used to obtain Eq. (4.44), with the introduction of separate messenger thresholds. For an application of the method of Ref. [3] to the case of multiple messenger thresholds, see Ref. [28].

[8] This symmetry is extremely useful in obtaining physically interesting results for non-holomorphic soft terms in strongly coupled SUSY gauge theories with small soft breakings [29].

As discussed above, this theory is invariant under the "Wilsonian" anomalous $U(1)$ transformation

$$\Phi_r \mapsto e^{A_r}\Phi_r, \quad \mathcal{Z}_{r,0} \mapsto e^{-(A_r + A_r^\dagger)}\mathcal{Z}_{r,0}, \quad S_0 \mapsto S_0 + \sum_r \frac{T_r}{8\pi^2}A_r, \tag{4.49}$$

with the regulator held fixed.

At one loop, the matter terms in the 1PI effective action are

$$\Gamma_{\rm 1PI} = \int d^4p \int d^4\theta\, \zeta(p^2)\Phi_r^\dagger e^{V^{(r)}}\Phi_r + \text{finite}, \tag{4.50}$$

where

$$\zeta(p^2) = \mathcal{Z}_{r,0}\left[1 - \frac{1}{4\pi^2}\frac{C_r}{S_0 + S_0^\dagger}\ln\frac{\Lambda}{\mu}\right]. \tag{4.51}$$

Here, Λ is the ultraviolet cutoff. Invariance under the transformation Eq. (4.49) allows us to conclude that $\Gamma_{\rm 1PI}$ depends on $S_0 + S_0^\dagger$ only in the invariant combination

$$S_0 + S_0^\dagger - \sum_r \frac{T_r}{8\pi^2}\ln\mathcal{Z}_{r,0}. \tag{4.52}$$

This allows us to infer the 2-loop dependence of the matter kinetic term in $\Gamma_{\rm 1PI}$ from Eq. (4.51). We can then obtain

$$\begin{aligned} m_r^2(\mu) &= -\ln\zeta(p^2 = -\mu^2)\Big|_{\theta^2\bar\theta^2} \\ &= m_{r,0}^2 - \frac{g_0^4 C_r}{32\pi^4}\left(\sum_r T_r m_{r,0}^2\right)\ln\frac{\Lambda}{\mu}. \end{aligned} \tag{4.53}$$

From this, we can read off the 2-loop RG equation for the soft masses arising from gauge interaction with other soft masses:

$$\mu\frac{dm_r^2}{d\mu} = \frac{2g^4 C_r}{(8\pi^2)^2}\sum_r T_r m_r^2. \tag{4.54}$$

(Note we have not specified a definition for the renormalized gauge coupling, but the result is invariant under changes of scheme for the gauge coupling.)

If the gauge group contains a $U(1)$ factor, there is an additional contribution to the RG equation for the scalars from an induced Fayet–Illiopoulos term. In superspace, a Fayet–Illiopoulos term can be viewed as a "kinetic mixing" between the $U(1)$ gauge field and that of the anomalous $U(1)$ symmetries for the various matter fields. Note that in the presence of bare soft masses, there is no symmetry forbidding such a term, so we have an addition contribution to the bare lagrangian

$$\delta\mathcal{L}_0 = \int d^2\theta \, \tfrac{1}{2}\kappa_{r,0} W_1 \mathcal{W}_{r,0} + \text{h.c.}, \tag{4.55}$$

where W_1 is the $U(1)$ gauge field strength and

$$(\mathcal{W}_{r,0})_\alpha \equiv -\frac{1}{4}\bar{D}^2 D_\alpha \ln \mathcal{Z}_{r,0} = \theta_\alpha m^2_{r,0} \tag{4.56}$$

is the field strength of the anomalous $U(1)$. Eq. (4.55) contains a linear term in the $U(1)$ auxiliary gauge field D_1, forcing $\langle D_1 \rangle \neq 0$ and giving an additional contribution to the scalar mass. It is the running of this contribution that we now compute.

The Fayet–Illiopoulos term is renormalized at one loop, and we obtain

$$\Gamma_{\text{1PI}} = \int d^2\theta \, \tfrac{1}{2} \left(\kappa_{r,0} + \frac{q_r}{16\pi^2} \ln \frac{\Lambda/\mathcal{Z}_0}{\mu} \right) W_1 \mathcal{W}_{r,0} + \text{h.c.} + \text{finite}. \tag{4.57}$$

Combining this result with the 1-loop renormalization of the matter wavefunction given in Eq. (4.51), we obtain an induced vacuum expectation value

$$\langle D \rangle = -\frac{g_1^2}{16\pi^2} \left(\sum_r q_r m^2_{r,0} + \sum_{J,r} \frac{g_J^2}{4\pi^2} C_r^J q_r m^2_{r,0} \right) \ln \frac{\Lambda}{\mu}, \tag{4.58}$$

where the sum on J runs over the factors of the gauge group, and q_r is the $U(1)$ charge of the field r. From this we can read off an additional contribution to the RG equation for the soft mass:

$$\mu \left. \frac{dm_r^2}{d\mu} \right|_D = \frac{g_1^2}{16\pi^2} \left(\sum_r q_r m_r^2 + \sum_{J,r} \frac{g_J^2}{4\pi^2} C_r^J q_r m_r^2 \right). \tag{4.59}$$

Recall that in Sect. 3 we showed that the RG equations for the soft masses above correspond to $\overline{\text{DR}}'$. The present derivation shows that these RG equations follow as long as the theory is regulated and subtracted in a supersymmetric fashion. To further amplify this point, we give an illustrative application of these methods where we compute a soft mass as a finite calculable effect.

Consider a toy model with bare lagrangian

$$\begin{aligned} \mathcal{L}_0 = & \int d^2\theta \, S_0 \, \text{tr}(W^\alpha W_\alpha) + \text{h.c.} \\ & + \int d^4\theta \, \Big[\mathcal{Z}_{r,0} \left(Q_r^\dagger e^{V(r)} Q_r + \bar{Q}_r^\dagger e^{V(\bar{r})} \bar{Q}_r \right) \\ & \qquad + \mathcal{Z}_{q,0} q^\dagger e^{V(q)} q \Big] \\ & + \int d^2\theta \, M_r Q_r \bar{Q}_r + \text{h.c.}, \end{aligned} \tag{4.60}$$

where $r = 1, 2$ are two copies of the same gauge representation. Suppose that the messengers $Q_{1,2}$ have bare soft masses given by

$$\mathcal{Z}_{1,0} = Z_{Q,0}\left[1 - \theta^2\bar{\theta}^2 m_0^2\right], \quad \mathcal{Z}_{2,0} = Z_{\bar{q},0}\left[1 + \theta^2\bar{\theta}^2 m_0^2\right]. \tag{4.61}$$

With this choice, the full theory has $\operatorname{Str}\mathcal{M}^2 = 0$, where $\mathcal{M}$ is the full mass matrix of the fields in the theory. However, if $M_1 \neq M_2$, the effective theory below the scale M_1 has nonvanishing mass supertrace. The value of this supertrace is therefore a calculable effect in this theory.

We could use the RG equations derived above to compute the soft masses in the low energy theory. We present here an alternative derivation of the supertrace that clarifies the methods used above. We assume $M_2 \ll M_1$, and compute the q soft mass in the low-energy theory below the scale M_2. With the choice of parameters made above, we can write

$$\mathcal{Z}_{1,0} = e^{U_0}, \quad \mathcal{Z}_{2,0} = e^{-U_0}, \quad U_0 = -\theta^2\bar{\theta}^2 m_0^2. \tag{4.62}$$

We can view U_0 as a "gauge" field for a single $U(1)$ under which Q_1 and $\bar{Q}_1$ have charge $+1$, Q_2 and $\bar{Q}_2$ have charge -1, M_1 has charge -2, and M_2 has charge $+2$. Moreover, this $U(1)$ symmetry is *anomaly free*, so we do not have to appeal directly to a Wilsonian picture of the anomaly.

We now integrate out Q and construct the effective lagrangian below the scale M_2. This has the form

$$\mathcal{L}'' = \int d^4\theta\, \mathcal{Z}_q'' q^\dagger e^{V(q)} q + \text{gauge terms}, \tag{4.63}$$

where the $U(1)$ symmetry enforces

$$\mathcal{Z}_q'' = f(|M_1|e^{-U_0}, |M_2|e^{U_0}). \tag{4.64}$$

We can determine the function f by 1-loop running and tree-level matching to a scale $\mu \ll M_2$:

$$\ln \mathcal{Z}_q(\mu) = \ln \mathcal{Z}_{q,0} + \frac{2C_q}{b} \ln \frac{g_0^2}{g^2(|M_1|)} + \frac{2C_q}{b'} \ln \frac{g^2(|M_1|)}{g^2(|M_2|)} + \frac{2C_q}{b''} \ln \frac{g^2(|M_2|)}{\mu}. \tag{4.65}$$

where b (b') [b''] are the beta function coefficients in the full theory (effective theory below M_1) [effective theory below M_2]. Using the 1-loop expressions for the gauge coupling g, and making the substitution $|M_1| \to |M_1|e^{-U_0}$, $|M_2| \to |M_2|e^{U_0}$, we obtain

$$\begin{aligned} m_q^2(\mu) = -\frac{C_q m_0^2}{4\pi^2} \bigg\{ & \frac{b - b'}{b''}\left[g^2(|M_2|) - g^2(|M_1|)\right] \\ & + \frac{b - 2b' + b''}{b''}\left[g^2(\mu) - g^2(|M_2|)\right] \bigg\}. \end{aligned} \tag{4.66}$$

The first term corresponds precisely to the running of the soft mass between the scales M_1 and M_2, and the second term to the running between M_2 and μ. There is no contribution from above the scale M_1 because the contributions from the two messengers cancel.

4.6 "Mediator" Models

We now consider GMSB models where SUSY breaking is communicated less directly to the observable sector. We find that very generally in such models, the gaugino screening mechanism described in Sect. 4.2 implies the gaugino mass is suppressed compared to the scalar masses by more loop factors than suggested by a naïve analysis.

We consider the "mediator" models introduced in Ref. [4]. We suppose that a SUSY-breaking sector communicates SUSY breaking to vectorlike fields Q and $\bar{Q}$. The fields Q and $\bar{Q}$ are not charged under the standard-model gauge group. Rather, they are in a vector-like representation of a "mediator" gauge group $G_{\rm med}$. The connection to the observable sector is made through a vectorlike pair of fields T and $\bar{T}$ that are charged under both the standard-model gauge group and $G_{\rm med}$. These fields have a supersymmetric mass term M_T in the lagrangian, which may be the result of a dynamical mechanism [4]. The lagrangian of this theory is

$$\begin{aligned}\mathcal{L}'' = \int d^4\theta \Big[& \mathcal{Z}''_Q \left(Q^\dagger e^{V^{(Q)}_{\rm med}} Q + \bar{Q}^\dagger e^{V^{(\bar{Q})}_{\rm med}} \bar{Q} \right) + \sum_r \mathcal{Z}''_r q_r^\dagger e^{V^{(r)}_{\rm SM}} q_r \\ & + \mathcal{Z}''_T \left(T^\dagger e^{V^{(T)}_{\rm med}} e^{V^{(T)}_{\rm SM}} T + \bar{T}^\dagger e^{V^{(\bar{T})}_{\rm med}} e^{V^{(\bar{T})}_{\rm SM}} \bar{T} \right) \Big] \\ & + \int d^2\theta \left[M_T T\bar{T} + S''_{\rm med} \operatorname{tr}(W^2_{\rm med}) + S''_{\rm SM} \operatorname{tr}(W^2_{\rm SM}) \right] + \text{h.c.} \\ & + \delta\mathcal{L}(Q, \bar{Q}, \ldots),\end{aligned} \tag{4.67}$$

where $\delta\mathcal{L}$ contains the interactions that break SUSY.

The holomorphic standard-model gauge coupling below the messenger scales M and the scale M_T is given exactly by

$$S_{\rm SM}(\mu) = S''_{\rm SM}(\mu_0) + \frac{b''_{\rm SM}}{16\pi^2} \ln \frac{M_T}{\mu_0} + \frac{b_{\rm SM}}{16\pi^2} \ln \frac{\mu}{M_T}, \tag{4.68}$$

where $b''_{\rm SM}$ and $b_{\rm SM}$ are the standard-model beta function coefficients in the effective theory with and without the field T, respectively. This is independent of M, so the leading contribution to the gaugino mass comes from the $\ln \mathcal{Z}_r$ term in the real effective gauge coupling $R_{\rm SM}$, see Eq. (4.19). The leading M-dependent contribution to $\mathcal{Z}_r$ arises at 4 loops, so the gaugino mass arises at 5 loops in this model, as opposed to the estimate of ref. [4]. Since scalar mass-squared terms arise at 4 loops, the gaugino mass is suppressed compared to the scalar masses in this model, posing a fine-tuning problem.

To make this argument concrete, and to illustrate the power of our techniques, we explicitly compute the gaugino mass in the case where SUSY breaking is communicated to the fields q and $\bar{q}$ by the vacuum expectation value of a singlet field X:

$$\delta\mathcal{L} = \int d^2\theta\, \lambda X Q \bar{Q} + \text{h.c.}, \tag{4.69}$$

with $\langle X \rangle, \langle F_X \rangle \neq 0$. The reader uninterested in details can skip the remainder of this Subsection.

We will do the calculation for the case where

$$M = \lambda \langle X \rangle \gg M_T. \tag{4.70}$$

We further assume that G_{med} is weakly coupled and unbroken down to the scale M_T. Below the scale M, the light fields are T, X, Q_r, V_{med}, and V_{SM}, and the effective lagrangian $\mathcal{L}'$ consists of the terms in Eq. (4.67) that depend on these fields. Below the scale M_T, the only light fields are X, Q_r, V_{med}, and V_{SM}, and we denote the effective lagrangian by $\mathcal{L}$.

Both the scalar and gaugino masses can be read off from $\mathcal{Z}_r$, the wavefunction renormalization factor in the low-energy theory. We therefore compute

$$\ln \mathcal{Z}_r(\mu) = \int_{\mu_0}^{M} \frac{d\mu'}{\mu'}\, \gamma_r''(\mu') + \int_{M}^{M_T} \frac{d\mu'}{\mu'}\, \gamma_r'(\mu') + \int_{M_T}^{\mu} \frac{d\mu'}{\mu'}\, \gamma_r(\mu'), \tag{4.71}$$

where γ_r (γ_r') [γ_r''] denotes the anomalous dimension in the theory $\mathcal{L}$ ($\mathcal{L}'$) [$\mathcal{L}''$]; and M (M_T) is the matching scale at the mass of q (T), defined similarly to Eq. (4.21). For example,

$$\gamma_r' = \mu \frac{d \ln \mathcal{Z}_r'}{d\mu} = \frac{C_r}{4\pi^2} \left[S_{\text{SM}}' + S_{\text{SM}}'^{\dagger} - \frac{2T_T}{8\pi^2} \ln \mathcal{Z}_T' + \cdots \right]^{-1}, \tag{4.72}$$

where we have displayed the dependence on $\mathcal{Z}_T'$ required by the "anomalous $U(1)$" invariance. This is important, because $\mathcal{Z}_T'$ depends on M at 2 loops, giving the leading M dependence of the anomalous dimensions. We have

$$\frac{\partial \ln \mathcal{Z}_r(\mu)}{\partial \ln |X|} = \int_{M}^{M_T} \frac{d\mu'}{\mu'} \frac{\partial \gamma_r'(\mu')}{\partial \ln |X|} + \int_{M_T}^{\mu} \frac{d\mu'}{\mu'} \frac{\partial \gamma_r(\mu')}{\partial \ln |X|}, \tag{4.73}$$

where

$$\frac{\partial \gamma_r'}{\partial \ln |X|} = \frac{4C_r T_T}{(8\pi^2)^2} \frac{1}{(S'_{\rm SM} + S'^{\dagger}_{\rm SM})^2} \frac{\partial \ln \mathcal{Z}'_T}{\partial \ln |X|}. \tag{4.74}$$

(T is not a light field in $\mathcal{L}$, so there is no contribution from scales below M_T.) We therefore have

$$\frac{\partial \ln \mathcal{Z}_r(\mu)}{\partial \ln |X|} = \frac{4C_r T_T}{(8\pi^2)^2} \int_M^{M_T} \frac{d\mu'}{\mu'} \frac{1}{(S'_{\rm SM}(\mu') + S'^{\dagger}_{\rm SM}(\mu'))^2} \frac{\partial \ln \mathcal{Z}'_T(\mu')}{\partial \ln |X|}. \tag{4.75}$$

(We see that the M-dependent part of $\mathcal{Z}_r$ is independent of the renormalization scale μ.) The dependence of $\mathcal{Z}'_T$ on the messenger threshold is identical to the calculation in GMSB, and we obtain

$$\frac{\partial \ln \mathcal{Z}_r(\mu)}{\partial \ln |X|} = \frac{8C_r C_T T_T^2}{(8\pi^2)^4} \int_{M_T}^{M} \frac{d\mu}{\mu}\, g'^4_{\rm SM}(\mu) \int_{\mu}^{M} \frac{d\mu'}{\mu'}\, g'^4_{\rm mess}(\mu'). \tag{4.76}$$

From this, we can obtain the gaugino mass

$$\begin{aligned} m_\lambda(\mu) &= \frac{g^2_{\rm SM}(\mu)}{2} \frac{\langle F_X \rangle}{\langle X \rangle} \sum_r \frac{T_r}{8\pi^2} \frac{\partial \ln \mathcal{Z}_r(\mu)}{\partial \ln |X|} \\ &= \frac{4C_r C_T T_T^2 \left[\sum_r T_r\right]}{(8\pi^2)^5}\, g^2_{\rm SM}(\mu) \\ &\qquad \times \int_{M_T}^{M} \frac{d\mu'}{\mu'}\, g'^4_{\rm SM}(\mu') \int_{\mu}^{M} \frac{d\mu''}{\mu''}\, g'^4_{\rm mess}(\mu''). \end{aligned} \tag{4.77}$$

Notice that the result scales like $m_\lambda = \alpha^3_{\rm SM} \alpha^2_{\rm mess} \ln^2 M/M_T$, indicating that two loops are accounted for by 1-loop evolution.

5 Effects from Other Thresholds

Up to now, we have been focusing on effects that can be computed from the dependence on the messenger threshold. However, there are interesting models with other thresholds than can give rise to important SUSY-breaking effects in the low-energy theory. In this Section we analyze some illustrative examples.

5.1 Flat Direction Effective Potential

In the limit where SUSY is unbroken, the minimal supersymmetric standard model has a large space of flat directions, directions in field space where the classical potential vanishes identically. (For an exhaustive list, see Ref. [30].) All of these flat directions will be lifted by SUSY breaking, and we are interested in computing the effective potential far out along one of these flat directions. For GMSB, the effective potential can be evaluated from 2-loop component diagrams such as those evaluated in Ref. [31], with the motivation of studying the cosmology of these flat directions. We will show how to compute the effective potential without evaluating loop diagrams.

We will explain our technique using a toy theory with an "observable sector" consisting of a $U(1)$ gauge theory with N_q pairs of chiral fields q and $\bar{q}$ with charges $+1$ and -1, respectively. These are coupled to a "messenger sector" consisting of N_Q pairs of chiral fields Q and $\bar{Q}$ and a singlet field X that parameterizes SUSY breaking. The lagrangian is

$$\begin{aligned}\mathcal{L}'' = \int d^4\theta \Big[& \mathcal{Z}_q'' \left(q^\dagger e^V q + \bar{q}^\dagger e^{-V} \bar{q} \right) \\ & + \mathcal{Z}_Q'' \left(Q^\dagger e^V Q + \bar{Q}^\dagger e^{-V} \bar{Q} \right) + \mathcal{Z}_X'' X^\dagger X \Big] \\ & + \int d^2\theta \, \tfrac{1}{2} S'' W^\alpha W_\alpha + \text{h.c.} \\ & + \int d^2\theta \, \lambda X q \bar{q} + \text{h.c.}\end{aligned} \tag{5.1}$$

Even though X is a background field, we must include a "kinetic" term for X to account for the anomalous dimension of operators that depend on X. (This operator is just the contribution to the cosmological constant.)

This theory has a single classical flat direction with $\langle q \rangle = \langle \bar{q} \rangle$. We want to compute the effective potential for $\langle q \rangle = \langle \bar{q} \rangle \gg \langle X \rangle$. In this case, the largest threshold in the theory is at the scale

$$M_1 = g(M_1) |\langle q \rangle|, \tag{5.2}$$

where g is the $U(1)$ gauge coupling. At this scale, the $U(1)$ gauge group is completely broken. The fields that are light below this scale are Q, $\bar{Q}$, and the flat direction $q = \bar{q}$, parameterized by a field Y defined as

$$q = \langle q \rangle + Y, \qquad \bar{q} = \langle \bar{q} \rangle + Y. \tag{5.3}$$

The background field X is also present in the low-energy theory. The effective lagrangian below the scale M_1 is therefore

$$\begin{aligned}\mathcal{L}' = \int d^4\theta \, & \left[\mathcal{Z}_Q' \left(Q^\dagger e^V Q + \bar{Q}^\dagger e^{-V} \bar{Q} \right) + \mathcal{Z}_X' X^\dagger X + \mathcal{Z}_Y' Y^\dagger Y \right] \\ & + \int d^2\theta \, X Q \bar{Q} + \text{h.c.} + \cdots,\end{aligned} \tag{5.4}$$

where the elipses denote higher-dimension operators.

The next threshold of interest is the messenger threshold at $M = \lambda\langle X\rangle$. Below this scale, the effective lagrangian contains only the fields X and Y, and it is given by

$$\mathcal{L} = \int d^4\theta \left[\mathcal{Z}_X X^\dagger X + \mathcal{Z}_Y Y^\dagger Y \right] + \cdots. \tag{5.5}$$

We are interested in the effective potential for Y in this effective lagrangian. When we continue the couplings into superspace, there will be contributions to the effective potential for Y from the Y dependence of $\mathcal{Z}_X$ as well as the X dependence of $\mathcal{Z}_Y$. The field Y does not have renormalizable interactions below the scale M_1, so $\mathcal{Z}_Y$ does not depend on X at the renormalizable level. The contribution to the effective potential we are interested in is therefore

$$V_{\text{eff}}(|Y|) = -|\langle F_X\rangle|^2 \mathcal{Z}_X(|Y|). \tag{5.6}$$

We compute $\mathcal{Z}_X$ using tree-level matching and 1-loop running. Using the RG equations

$$\mu \frac{d \ln \mathcal{Z}''_X}{d\mu} = -\frac{N_Q}{4\pi^2} \frac{\lambda^2}{\mathcal{Z}''_X \mathcal{Z}''^2_Q}, \qquad \mu \frac{d \ln \mathcal{Z}'_X}{d\mu} = -\frac{N_Q}{4\pi^2} \frac{\lambda^2}{\mathcal{Z}'_X \mathcal{Z}'^2_Q}, \tag{5.7}$$

we obtain

$$\mathcal{Z}_X = \mathcal{Z}''_X(\mu_0) - \frac{N_Q \lambda^2}{4\pi^2} \int_{\mu_0}^{M_1} \frac{d\mu}{\mu} \frac{1}{\mathcal{Z}''^2_Q(\mu)} - \frac{N_Q \lambda^2}{4\pi^2} \int_{M_1}^{M} \frac{d\mu}{\mu} \frac{1}{\mathcal{Z}'^2_Q(\mu)}, \tag{5.8}$$

where μ_0 is a fixed renormalization scale used to define the theory. Note that $\mathcal{Z}_X$ is independent of renormalization scale. Since we are interested in the Y dependence, we compute

$$\frac{\partial \mathcal{Z}_X}{\partial \ln |Y|} = \frac{N_Q \lambda^2}{4\pi^2} \int_{M_1}^{M} \frac{d\mu}{\mu} \frac{1}{\mathcal{Z}'^2_Q(\mu)} \frac{\partial \ln \mathcal{Z}'_Q(\mu)}{\partial \ln |Y|}. \tag{5.9}$$

$\mathcal{Z}'_Q$ does not run in the effective theory $\mathcal{L}'$, so we have $\mathcal{Z}'_Q(\mu) = \mathcal{Z}''_Q(M_1)$, which gives

$$\frac{\partial \ln \mathcal{Z}'_Q(\mu)}{\partial \ln |Y|} = \frac{g^2(M_1)}{8\pi^2}. \tag{5.10}$$

In this way, we obtain

$$|Y| \frac{\partial V_{\text{eff}}}{\partial |Y|} = \frac{N_Q \lambda^2 |\langle F_X\rangle|^2}{(4\pi^2)^2} \frac{g^2(M_1)}{\mathcal{Z}^2_Q(M_1)} \ln \frac{M_1}{M}. \tag{5.11}$$

Note that M_1 depends on $|Y|$, so this result automatically gives the RG-improved form of the effective potential.

5.2 (S)axion Potential

There are a number of models for physics beyond the standard model that involve the spontaneous breaking of a global symmetry at large energy scales. For example, "invisible" axion models invoke the breaking of a global $U(1)_{\rm PQ}$ symmetry at scales 10^{10}–10^{12} GeV in order to solve the strong CP problem. Other global symmetries that may be spontaneously broken include lepton number and flavor symmetries.

The breaking of a global symmetry will give rise to a massless Nambu–Goldstone boson (NGB) for every broken generator. If the global symmetry is broken at a scale where SUSY is (approximately) unbroken in the visible sector, then the light bosons must form complete chiral supermultiplets. There are therefore extra scalars whose mass is protected by SUSY.[9] We call these fields SNGB's. The SNGB fields parameterize non-compact directions in the vacuum manifold in the limit where SUSY is exact, and different points along the flat direction correpond to different values for the scale at which the global symmetry is broken. The SNGB fields will acquire a potential after SUSY breaking, which determines the vacuum expectation values along the flat direction.

As an example, we consider an axion model with colored fields R and $\bar{R}$ whose mass is determined by the vacuum expectation value of a field Φ. If we write

$$\Phi = \langle \Phi \rangle + A, \tag{5.12}$$

the imaginary part of A is the axion, while the real part is the SNGB. The lagrangian is

$$\begin{aligned} \mathcal{L}'' = & \int d^4\theta\, \mathcal{Z}_R \left(R^\dagger e^{V^{(R)}} R + \bar{R}^\dagger e^{V^{(\bar{R})}} \bar{R} \right) \\ & + \int d^2\theta\, \kappa \Phi R \bar{R} + \text{h.c.} + \cdots , \end{aligned} \tag{5.13}$$

where we have omitted the messenger sector and standard-model fields, see Eq. (4.1). The fields R, $\bar{R}$ therefore have a mass

$$M_R = \frac{\kappa \langle \Phi \rangle}{\mathcal{Z}_R(M_R)}. \tag{5.14}$$

Below this scale, the effective lagrangian $\mathcal{L}'$ is simply that of ordinary GMSB together with a kinetic term for the field Φ (see Eq. (4.1)). Below the messenger threshold M the effective lagrangian $\mathcal{L}$ is that of the standard model together with kinetic terms

[9] If a non-abelian symmetry is broken, some of the Nambu–Goldstone bosons can belong to the same chiral supermultiplet, but it can be shown that there are always some "extra" scalars.

for the singlets Φ and X. The wavefunction parameter $\mathcal{Z}_X$ in this effective lagrangian depends on the R mass, and this contains the leading contribution to the effective potential for the saxion field.

We can compute $\mathcal{Z}_X$ using 1-loop running and tree-level matching:

$$\mathcal{Z}_X = \mathcal{Z}''_X(\mu_0) + \int_{\mu_0}^{M_R} \frac{d\mu}{\mu}\, \mathcal{Z}''_X(\mu)\gamma''_X(\mu) + \int_{M_R}^{M} \frac{d\mu}{\mu}\, \mathcal{Z}'_X(\mu)\gamma'_X(\mu), \tag{5.15}$$

where γ_X is the anomalous dimension of X as defined in Eq. (5.7). The parameter $\mathcal{Z}_X$ does not run in this effective theory, so we need not specify a renormalization scale for it. We compute

$$\begin{aligned}\frac{\partial \mathcal{Z}_X}{\partial \ln|\Phi|} &= \int_{M_R}^{M} \frac{d\mu}{\mu} \frac{\partial}{\partial \ln|\Phi|} \left[\mathcal{Z}'_X(\mu)\gamma'_X(\mu)\right] \\ &= \frac{N_Q|\lambda|^2}{4\pi^2} \int_{M_R}^{M} \frac{d\mu}{\mu} \frac{1}{\mathcal{Z}'^2_Q(\mu)} \frac{\partial \ln \mathcal{Z}'_Q(\mu)}{\partial \ln|\Phi|}.\end{aligned} \tag{5.16}$$

The right-hand side is evaluated using

$$\begin{aligned}\frac{\partial \ln \mathcal{Z}'_Q}{\partial \ln|\Phi|} &= \int_{M_R}^{\mu} \frac{d\mu'}{\mu'} \frac{\partial \gamma'_Q(\mu')}{\partial \ln|\Phi|} \\ &= -\frac{C_Q}{4\pi^2} \int_{M_R}^{\mu} \frac{d\mu'}{\mu'}\, g'^4(\mu') \frac{\partial}{\partial \ln|\Phi|} \left(\frac{1}{g'^2(\mu')}\right) \\ &= \frac{C_Q T_r}{(4\pi^2)^2} \int_{M_R}^{\mu} \frac{d\mu'}{\mu'}\, g'^4(\mu'),\end{aligned} \tag{5.17}$$

which gives

$$|\Phi| \frac{\partial V_{\text{eff}}}{\partial|\Phi|} = -\frac{T_r C_Q N_Q}{(4\pi^2)^3} |\langle F_X\rangle|^2 \int_{M}^{M_R} \frac{d\mu}{\mu} \frac{1}{\mathcal{Z}'^2_Q(\mu)} \int_{\mu}^{M_R} \frac{d\mu'}{\mu'}\, g'^4(\mu'). \tag{5.18}$$

As before, this gives the RG-improved form for the effective potential. Note that the slope of the potential is negative, indicating that the saxion vacuum expectation value is driven away from the origin.

In the opposite limit $M_R \ll M$, it is easy to see that the potential also decreases as a function of M_R. In the effective theory at the scale M, R and $\bar{R}$ get a positive soft mass-squared from GMSB while Φ has zero soft mass. However, the Yukawa coupling $\kappa\Phi R\bar{R}$ drives the Φ soft mass^2 negative in running between M and the scale M_R, where R and $\bar{R}$ are integrated out. (This contribution is analogous to the negative contribution to the Higgs mass-squared from the top Yukawa coupling.)

Thus in all regions, the potential prefers to push the saxion vacuum expectation value, and hence the axion decay constant, to larger values. Therefore new interactions are needed between the axion and GMSB sectors in order to stabilize the axion decay constant in the cosmological and astrophysically desirable window between 10^{10}–10^{12} GeV.

6 Conclusions

In this paper, we have shown that the renormalization of soft SUSY-breaking terms is completely determined by the renormalization of SUSY-preserving terms if the regulator is supersymmetric. This allows us to calculate certain SUSY-breaking effects in gauge-mediated theories by performing a supersymmetric calculation and "analytically continuing" the result into superspace. The method is very powerful, and allows the calculation of interesting effects at 3-loop order and higher by purely algebraic manipulations.

The formal results that justify these calculations are easy to state in superspace if the soft SUSY breaking terms are parameterized by θ-dependent terms in the supersymmetric couplings. If the theory is regulated in a supersymmetric manner, then SUSY is formally preserved if we regard the bare couplings as superfield spurions. Our result is that there is a definition of the *renormalized* couplings that can be similarly grouped into supermultiplets. Specifically, the renormalized couplings K_R are related to the bare couplings K_0 via a superfield relation of the form

$$K_R(\mu) = f(K_0, \Lambda, \mu). \tag{6.1}$$

The function f determines the renormalization of the supersymmetric couplings as well as the soft SUSY breaking terms, and is the basis for the analytic continuation into superspace. An analogous relation holds between the (renormalized) couplings of an effective theory and the couplings in a more fundamental theory.

This leads naturally to a definition of the renormalized gauge coupling chiral superfield

$$S(\mu) = \frac{1}{2g_h^2(\mu)} - \frac{i\Theta}{16\pi^2} - \theta^2 \frac{m_{\lambda,h}(\mu)}{g_h^2(\mu)} \tag{6.2}$$

as a holomorphic object that is renormalized only at one loop (to all orders in perturbation theory). However, the subtraction that defines $S(\mu)$ is not invariant under constant rescaling of the fields, so the components of $S(\mu)$ do not correspond directly the usual renormalized couplings. The real superfield

$$R = S + S^\dagger - \frac{T_G}{16\pi^2} \ln(S + S^\dagger) - \sum_r \frac{T_r}{16\pi^2} \ln \mathcal{Z}_r + \mathcal{O}((S + S^\dagger)^{-1}). \tag{6.3}$$

is invariant under the field rescaling. (Here, r runs over the matter representations of the gauge group G and T_r is the index of r; $\mathcal{Z}_r$ is the wavefunction factor for the fields in the representation r.) We show that the lowest components of R

$$\frac{1}{g^2(\mu)} = R(\mu)| , \qquad \frac{m_\lambda(\mu)}{g^2(\mu)} = R(\mu)|_{\theta^2} , \tag{6.4}$$

are precisely the 1PI gauge coupling and gaugino mass defined by Euclidean subtraction or by minimal subtraction in dimensional reduction. The $\mathcal{O}((S + S^\dagger)^{-1})$ corrections account for possible scheme dependence in the definition of R. Eq. (6.3) and much of the story leading up to it is very similar to the results of Refs. [5], but we emphasize that all quantities are finite renormalized quantities, and no reference is made to the Wilsonian renormalization group.

The $\theta^2\bar{\theta}^2$ component R is given at lowest order by

$$R|_{\theta^2\bar{\theta}^2} = \frac{1}{8\pi^2}\left[-T_G m_\lambda^2 + \sum_r T_r m_r^2\right]. \tag{6.5}$$

and governs the RG evolution of dimension-2 soft terms. In dimensional reduction $R|_{\theta^2\bar{\theta}^2}$ corresponds to a $1/\epsilon$ counterterm for the ϵ-scalar mass. $R|_{\theta^2\bar{\theta}^2}$ can also be given a 1PI interpretation: it corresponds to a non-local $1/p^2$ correction to the propagator of the gauge supermultiplet. In the context of dimensional reduction and (modified) minimal subtraction, our results imply that the simple extension $1/g^2(\mu) \to R(\mu)$ automatically picks out the so-called $\overline{\mathrm{DR}}'$ scheme.

In practice, this result allows one to simply compute the SUSY breaking components of R (for example) by computing the lowest component as a function of the supersymmetric bare couplings (or couplings in an underlying renormalized theory). This is a supersymmetric calculation, but taking θ-dependent components of the result determines the low-energy SUSY breaking parameters. For instance, we have shown that the 2-loop RG equations for soft terms in $\overline{\mathrm{DR}}'$ are directly derived from the supersymmetric β-functions and anomalous dimensions.

More remarkably, this approach can be used to relate leading-log effects computed using the renormalization group to finite effects, since the result of taking higher θ components of a logarithm gives effects that are not logarithmically enhanced:

$$\frac{1}{16\pi^2}\ln M\bigg|_{\theta^2\bar{\theta}^2} = \frac{1}{16\pi^2}\frac{M|_{\theta^2\bar{\theta}^2}}{M|}. \tag{6.6}$$

In this way, we can obtain finite SUSY breaking effects at high loop order from simple algebraic calculations. Models with low-energy supersymmetry breaking mediated by perturbative interactions are the natural arena to apply our method. Indeed, it is precisely in these theories that it makes more sense to worry also about subleading RG evolution: this is because the boundary conditions for soft terms are in principle calculable with comparable accuracy.

Our technique was used to compute a variety of effects at 2-loop order and beyond. We computed for the first time the complete subleading corrections to the gaugino masses (2-loop) and scalar masses (3-loop) in gauge-mediated models; we showed how to compute the effective potential for SUSY flat directions lifted by gauge mediation (2- and 3-loop). We also proved that gaugino masses are screened from higher-loop corrections involving couplings in the messenger sector. Therefore, in the standard gauge mediated scenario, gaugino masses are rather insensitive on details of the model. Moreover, this result also shows that if the gaugino masses are not generated at one loop (as in the standard case) they will be generated only from the light matter fields, and will generally be too light. This shows that gauge mediation is the unique way to generate scalar and gaugino masses of the same order through loop effects.

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