

4d Conformal Theories and Strings on Orbifolds

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We propose correspondences between 4d quantum field theories with $\mathcal{N} = 2, 1, 0$ (super)conformal invariance and Type IIB string theory on various orbifolds. We argue using the spacetime string theory, and check using the beta functions (exactly for $\mathcal{N} = 2, 1$ and so far at 1-loop for the gauge couplings in the $\mathcal{N} = 0$ case), that these theories have conformal fixed lines. The latter case potentially gives well-defined non-supersymmetric vacua of string theory, with a mechanism for making the curvature and cosmological constant small at nontrivial string coupling.

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1. Introduction

Part of the recent renaissance in string theory has involved the study of string or M-theory configurations decoupled from gravity. The scaling arguments which decouple gravity do not directly depend on supersymmetry. It is therefore of interest to consider nontrivial geometrical and/or brane configurations in backgrounds with varying amounts of spacetime supersymmetry, to learn lessons about field theories with different numbers of supersymmetries and the backgrounds in which they reside. In particular, such techniques are applicable to the study of nonsupersymmetric field theories and string backgrounds.

Several recent developments have culminated in a proposal of “duality” between 4d $\mathcal{N} = 4$ supersymmetric $U(N)$ Yang-Mills theory (realized on N D3 branes) and Type IIB string theory on $AdS_5 \times S^5$ [1,2,3,4,5]. The conjecture [2] has been further developed in for example [6,7]. Similar conjectures relate other conformal field theories to other string or supergravity backgrounds. These exciting conjectures offer both a possibility of providing a nonperturbative definition of various string backgrounds in terms of field theories (in a way similar to M(atrrix) theory [8]), and a way of using supergravity/string theory to learn about conformal field theory.

In this note, we provide evidence that various 4d field theories with 4d $\mathcal{N} = 0, 1, 2$ (super)conformal invariance can be used to provide a nonperturbative definition of IIB string theory in orbifold backgrounds with 0, 8, and 16 supercharges. Our basic strategy is to study orbifolds of type IIB on $AdS_5 \times S^5$ which preserve the AdS structure but break some of the supersymmetries. Then as in [2], the $SO(4, 2)$ symmetry of the AdS space should translate into a superconformal group on the (orbifolded) D3 brane theory. Although from a supergravity point of view non-freely-acting orbifolds are singular, in string theory they are not and we can study them reliably.

The type IIB theory in each case has a coupling constant g . Gauge theories on the other hand in general do not have a meaningful dimensionless coupling parameter since the coupling runs. Therefore in order for the correspondence to work we must find that the beta function vanishes in the field theory. In the cases with supersymmetry, the spacetime coupling can take on any value because of the impossibility of generating a potential for the dilaton. So for this reason in those cases, there should not only be a zero of the beta function, but a fixed line (and in general a fixed hypersurface in coupling space whose dimension is given by the number of deformations of the spacetime string theory which preserve the AdS structure). Furthermore, even in the cases without supersymmetry, we

have an AdS factor in the spacetime theory. The AdS symmetry should therefore remain order by order in type IIB string perturbation theory. Since the AdS symmetry translates into the $SO(4,2)$ conformal symmetry in the field theory, in the non-supersymmetric case as well we expect a fixed line (at least to all orders in perturbation theory).

This nontrivial prediction is born out by explicit study of the orbifolded D3 brane field theories along the lines of [9,10]. In the orbifolds which preserve $\mathcal{N} = 1, 2$ supersymmetry, we can study the exact beta functions for the couplings in the field theory. This allows us to directly verify the existence of the predicted conformal fixed lines, and identify marginal operators with spacetime moduli like the IIB string coupling.

In the $\mathcal{N} = 0$ case, though the exact beta function is not known, we compute the one-loop beta function for the gauge coupling and find that it vanishes! Because of the geometrical setup which preserves the AdS symmetry in the spacetime, we expect the beta functions to vanish order by order in g . If so, then we obtain in this way the first example of a nonsupersymmetric string vacuum which exists for arbitrary values of the coupling. For appropriate choice of N , keeping g nontrivial, one can then tune the curvature to be as small as desired. Then we would like to argue that since the CFT should exist for all g , the spacetime background should be stable for all g , so a g -dependent vacuum energy should not be generated.¹

Another class of interesting non-supersymmetric models to consider is those for which there exists a fixed point at large N , as in non-supersymmetric QCD with an appropriate number of flavors [12]. In this type of model, which can also be realized in branes [13], one has the intriguing possibility of fixing g in terms of N , and therefore getting constraints on the coupling as a function of the curvature.

In §2 we study field theories with $\mathcal{N} = 2$ supersymmetry and in §3 we study theories with $\mathcal{N} = 1$ supersymmetry. In §4 we present a nonsupersymmetric orbifold example and its 1-loop beta function, and in §5 we indulge in a discussion of the predictions of this duality for the vacuum energy and field theory beta functions in the nonsupersymmetric case.

¹ Further computations to test these arguments are underway [11].

2. $\mathcal{N} = 2$ Supersymmetric Theories

Let us consider the theory on N Type IIB D3 branes at the \mathbb{Z}_k orbifold singularity of an A_{k-1} ALE space. Following [9], we can construct the worldvolume theory by taking kN D3 branes on the covering space and doing a \mathbb{Z}_k projection on both the worldvolume fields and the Chan-Paton factors.

In the picture of [2], this translates into a \mathbb{Z}_k orbifold of the $AdS_5 \times S^5$ IIB supergravity solution. The \mathbb{Z}_k acts only on the S^5 while leaving the AdS_5 untouched. The resulting spacetime still has an AdS_5 factor. Before taking the scaling limit [2], the threebranes sit at a point in the transverse \mathbb{R}^6 . The orbifold fixes a plane in this \mathbb{R}^6 , which intersects the S^5 in an S^1 . So on the S^5 there is an S^1 (a great circle) fixed by the orbifold action. Although this means that the supergravity solution for this background is singular, the IIB string theory is sensible. The spacetime supersymmetry is broken from 32 to 16 supercharges by the orbifold.

The presence of the AdS_5 factor in spacetime means that we still expect the D3 brane theory to have an $SO(4, 2)$ conformal invariance *after* orbifolding. In fact, we find a theory with $U(N)^k$ gauge group and matter in the

$$(N, \bar{N}, 1, \dots, 1) \oplus (1, N, \bar{N}, \dots, 1) \oplus \dots \oplus (\bar{N}, 1, \dots, N) \quad (2.1)$$

Therefore, each of the k $U(N)$ gauge factors effectively has $2N$ fundamental hypermultiplets. The $\mathcal{N} = 2$ gauge theory with this matter spectrum is known to be superconformal, and in fact to have a fixed line parametrized by the gauge coupling.² There are k different gauge couplings, so we in fact expect a fixed surface of dimension k . If we keep all of the gauge couplings equal, then g_{YM}^2 simply maps to the spacetime string coupling as in [2].

Since there are a total of k spacetime moduli ($k-1$ blow up modes of the \mathbb{Z}_k singularity, and the string coupling g), it is natural to associate the other $k-1$ marginal operators along the fixed surface to the blow up modes. This identification of field theory parameters with spacetime moduli is significantly different from the one that occurs in the theory of D3 brane probes at an orbifold singularity, where the blow-up modes map to FI terms in the worldvolume action [9]. However, it is motivated by the following. The \mathbb{Z}_k action on $AdS_5 \times S^5$ does not act on the AdS space. In perturbative Type IIB string theory there

² In this and later cases, we ignore the $U(1)$ factors in the $U(N)$ gauge groups which decouple from the interacting conformal theory in the infrared.

is a product of two 2d conformal field theories representing this vacuum, one for the AdS space and one for the S^5 . Since the \mathbb{Z}_k acts on only the S^5 CFT, we expect the blow up modes will also only affect the S^5 CFT. Then, since the AdS piece is untouched, we still expect a 4d conformal field theory on the D3 branes after blowing up. The FI terms break the conformal invariance, while the $k - 1$ marginal operators we have found of course do not. The marginal operators therefore correspond to the blow-up modes of the type IIB theory on $AdS_5 \times (S^5/\mathbb{Z}_k)$.

This distinction between the near-horizon theory and the original bulk theory after orbifolding will become even sharper in the $\mathcal{N} = 1$ case. There the orbifold will act freely on the S^5 factor, while in the full theory before the scaling of [2] the orbifold has a fixed point at the origin of the transverse \mathbb{R}^6 . Again we will find agreement between this geometry and the number of marginal directions in the field theory.

3. $\mathcal{N} = 1$ Supersymmetric Theories

We can find $\mathcal{N} = 1$ field theories by studying D3 branes at orbifold singularities of the form \mathbb{R}^6/Γ (with Γ a finite abelian group) as in [10]. Again, we can translate the Γ action to an action on $AdS_5 \times S^5$, and since it acts orthogonally to the D3 branes, it will preserve the AdS structure. So in these cases we expect the worldvolume theory on N D3 branes to be an $\mathcal{N} = 1$ superconformal field theory.

Here we only present the simplest example. More general cases work out similarly. Consider the \mathbb{Z}_3 orbifold

$$X^{1,2,3} \rightarrow e^{\frac{2\pi i}{3}} X^{1,2,3} \tag{3.1}$$

where $X^{1,2,3}$ are the three (complex) spacetime coordinates transverse to the N parallel D3 branes. In the full theory, before the scaling [2], the orbifold (3.1) has a fixed point at the origin ($X^\mu = 0$, i.e. $r = 0$ in polar coordinates). In polar coordinates, the original space can be thought of as a sphere whose size varies as a function of the radial coordinate r ; the orbifold acts by rotations on the sphere and the fixed point in this case occurs where the sphere shrinks to zero at $r = 0$. After the scaling limit [2], the space is a product of the sphere times the AdS space; that is, the sphere has constant volume over the AdS space and never shrinks to zero. So the orbifold action in this case is free. Since there are no orbifold fixed points, we expect no blowup moduli. So in the field theory we should find

a single marginal direction, corresponding to the string coupling. This is borne out in the following analysis.

We can construct the D3 brane gauge theory at the orbifold singularity by considering $3N$ D3 branes on the covering space with suitable projections. The result is a $U(N)^3$ gauge theory with chiral multiplets in the

$$3 \times \{(N, \bar{N}, 1) \oplus (1, N, \bar{N}) \oplus (\bar{N}, 1, N)\} \quad (3.2)$$

Each of the $U(N)$ gauge groups therefore has the equivalent of $N_F = 3N$ flavors of quarks. Let us call the three types of charged matter fields U^μ, V^μ, W^μ with $\mu = 1, 2, 3$.

There is also a superpotential

$$W = h_{123}U^1V^2W^3 + h_{312}V^1W^2U^3 + h_{231}W^1U^2V^3 - h_{213}U^2V^1W^3 - h_{321}V^2W^1U^3 - h_{132}W^2U^1V^3 \quad (3.3)$$

with all the couplings $h_{abc} \equiv h$ equal for the field theory obtained from the orbifold [10]. We are predicting that this theory has a conformal fixed line. Following the analysis of [14], we can check this conjecture. In their notation, the scaling coefficient for a gauge coupling g is

$$A_g = -[3C_2(G) - \sum_i T(R_i) + \sum_i T(R_i)\gamma_i] \quad (3.4)$$

where i runs over charged matter fields in representation R_i and γ_i are the anomalous dimensions. Since we have $3N$ flavors of quarks for each of the three $U(N)$ s in our theory, the first two terms in (3.4) cancel for each group, leaving a contribution to each gauge coupling β -function which is a linear combination of anomalous dimensions γ_i .

We wish to determine whether there is a family of fixed points in this theory as a function of couplings, and to determine its dimension. Let us first consider directions in coupling space which respect the S_3 global symmetry between the three complex coordinates. In such directions,

$$\gamma^{U^1} = \gamma^{U^2} = \gamma^{U^3} \equiv \gamma^U \quad (3.5)$$

$$\gamma^{V^1} = \gamma^{V^2} = \gamma^{V^3} \equiv \gamma^V \quad (3.6)$$

$$\gamma^{W^1} = \gamma^{W^2} = \gamma^{W^3} \equiv \gamma^W \quad (3.7)$$

so we have three independent γ functions.

The overall coupling h in (3.3) is the only superpotential coupling respecting the S_3 global symmetry. In addition we have three gauge couplings $g_i, i = 1, \dots, 3$. The various β functions satisfy:

$$\beta_h \propto \gamma^U + \gamma^V + \gamma^W \quad (3.8)$$

$$\beta_{g_1} \propto \gamma^U + \gamma^V \quad (3.9)$$

$$\beta_{g_2} \propto \gamma^V + \gamma^W \quad (3.10)$$

$$\beta_{g_3} \propto \gamma^U + \gamma^W \quad (3.11)$$

There is one linear relation between the four β functions, so that setting them to zero gives us 3 conditions on the four couplings. This generically yields a fixed line.

So far we have found one marginal direction. To check whether there are any others, we can now relax the condition forcing the couplings to preserve the global symmetry. If we preserve permutations of the $\mu = 1, 2$ directions, we get six independent γ functions. Having relaxed the S_3 constraint to \mathbb{Z}_2 , there are three independent superpotential couplings ($h_{123} = h_{213}$, $h_{312} = h_{321}$, and $h_{231} = h_{132}$). Combining these with the gauge couplings $g_{1,2,3}$ gives six independent couplings. So here one generically does not expect a linear relation, and these symmetry-breaking deformations are not marginal. Similarly, relaxing the symmetry completely gives nine independent γ functions and nine couplings (3 gauge couplings and 6 superpotential couplings).

So we indeed find a single marginal direction. In the type IIB string theory this direction corresponds to the string coupling. There are other examples of finite $\mathcal{N} = 1$ theories on D3-branes in IIB string theory (at orientifold fixed points) [15], whose near-horizon geometry presumably has a similar structure.

4. Nonsupersymmetric Theories

We will now turn to orbifolds preserving no supersymmetry. One of the simplest examples satisfying level-matching is to take a \mathbb{Z}_5 action on two complex variables:

$$(X^1, X^2, X^3) \rightarrow (\alpha X^1, \alpha^3 X^2, X^3) \quad (4.1)$$

where $\alpha = e^{\frac{2\pi i}{5}}$ and we are using the notation of §3. This model is tachyon-free.

The projection yields a nonsupersymmetric $U(N)^5$ gauge theory with the following matter content. Let us denote the five $U(N)$ factors by $U(N_i)$, with a cyclic index $i =$

1, \dots, 5. The projection on X^3 yields one complex adjoint scalar in each $U(N_i)$. From X^1 we get complex scalars in the (N_i, \bar{N}_{i+1}) representation, and from X^2 we get complex scalars in the (N_i, \bar{N}_{i+2}) . We get fermions in the $(N_i, \bar{N}_{i\pm 2})$ and in the $(N_i, \bar{N}_{i\pm 1})$. Note that this theory is not supersymmetric at the level of the spectrum, even before considering interactions. It has matter self-interactions inherited from the $\mathcal{N} = 4$ theory at tree level (whose superpotential in $\mathcal{N} = 1$ language is $\text{Tr}[X^1, X^2]X^3$). That is, as in the $\mathcal{N} = 1$ case discussed above [10], the terms in the $\mathcal{N} = 4$ superpotential involving only the fields surviving the orbifold projection constitute the tree-level interactions of our matter fields. This gives us a set of Yukawa couplings and quartic scalar couplings.

Consider the 1-loop β -function for the gauge coupling g_i of $U(N_i)$. The vector bosons contribute $-\frac{g_i^3}{16\pi^2}(\frac{11N}{3})$. The complex adjoint scalar X^3 contributes $-\frac{g_i^3}{16\pi^2}(-\frac{N}{3})$ while the fundamentals and antifundamentals from X^1 and X^2 contribute $-\frac{g_i^3}{16\pi^2}(-\frac{2N}{3})$. The fermions contribute $-\frac{g_i^3}{16\pi^2}(-\frac{8N}{3})$. So the total β_{g_i} vanish at one loop. It is important to check this for the Yukawa couplings and quartic scalar couplings also, and also to check explicitly that this persists to higher orders in the loop expansion [11]. For now we note that at the orbifold point, although we do not have an $\mathcal{N} = 4$ spectrum, we have interactions which at tree-level are given in terms of the gauge coupling g , since the tree-level interactions are inherited from the $\mathcal{N} = 4$ theory. So we are on a special locus of measure zero in the coupling space of the general non-supersymmetric model.

Our preliminary check bears out our expectations from the product structure of the metric, even in the non-supersymmetric case. Again in this case, our orbifold acts only on the S^5 factor of the product spacetime $AdS_5 \times S^5$. As long as the product structure remains, the $SO(4, 2)$ AdS/conformal symmetry should remain, which means the field theory should have a vanishing β -function. In spacetime, we know that the product structure remains at least to all orders in perturbation theory, since the IIB worldsheet conformal field theory factorizes into a product of conformal field theories corresponding to the AdS_5 and S^5/\mathbf{Z}_5 respectively.

Another case in which we should find concrete checks is in orbifolding the proposals of [2] for describing compactifications. For example, we can consider the \mathbf{Z}_5 orbifold (4.1) acting on a T^4 instead of \mathbb{R}^4 in type IIB string theory. The proposal in [2] for the unorbifolded theory is a 1 + 1-dimensional SCFT living on wrapped D5-branes bound to D1-branes. Our projection will reduce this to a nonsupersymmetric CFT. Because again the orbifold does not act on the AdS space, we should have a conformal field theory for all values of the string coupling. Therefore we require the 1 + 1 dimensional CFT to have a marginal direction corresponding to the string coupling. This is again a nontrivial prediction which should be accessible to standard 2d CFT methods.

5. Discussion

We have explained that by orbifolding transversely to D3-branes in type IIB string theory, we should automatically produce field theories with conformal fixed lines. We checked this exactly for the $\mathcal{N} = 2, 1$ cases and so far at the level of the 1-loop gauge coupling β -functions in the $\mathcal{N} = 0$ case. There are examples in field theory where the coupling space has marginal directions which break some supersymmetry. It is interesting to consider in this setup whether such models can be related to spacetime backgrounds in M theory. Then transitions would occur that change the number of spacetime supersymmetries.

The non-supersymmetric case is of particular interest as a potential formulation of non-supersymmetric string theory well-defined at non-zero but tunable coupling.³ There are clearly many similar non-supersymmetric models one can consider, in addition to checking further properties of our examples. In general these models give us a concrete opportunity to study the cosmological constant. There are many promising features of this setup.

Let us first discuss the curvature of the $AdS \times S^n$. If g indeed turns out to correspond to a fixed line in the D3-brane CFT, then we have a conformal field theory and therefore a well-defined spacetime theory for all values of N and g . The curvature in string units takes the form

$$R\alpha' \sim \frac{1}{gN}(1 + a_1g + a_2g^2 + \dots) . \quad (5.1)$$

Presumably we can then tune N to make the overall spacetime curvature as small as desired while retaining a nonzero string coupling g .

As mentioned before, another class of models would come for example from brane configurations corresponding to large- N QCD with the right number of flavors. These will have a fixed point instead of a fixed line [12], and will yield a prediction for the curvature in terms of the coupling.

Let us now discuss the cosmological constant. At one loop in the string theory, the cosmological constant is proportional to a tadpole for the dilaton, and in general quantum corrections produce a g -dependent vacuum energy. However, a potential for the dilaton would suggest that the theory does not make sense for arbitrary coupling. But on the other hand, the AdS symmetry group is expected to persist order by order in perturbation

³ There have been earlier studies of non-supersymmetric string backgrounds such as the $O(16)$ String [16], which generates a dilaton tadpole at one loop and rolls to weak coupling. A more recent attempt [17] yields models at fixed coupling of order one which are unfortunately difficult to study.

theory. This in turn implied a vanishing beta function order by order, so that from the point of view of the CFT defining the background it does make sense for arbitrary coupling (at least at the level of perturbation theory). In particular, the quantum field theory is well defined and therefore by the duality there is no apparent instability in the spacetime background. By this logic, the dilaton potential and therefore the cosmological constant should not be generated in the first place. We can check this concretely by computing perturbative corrections to the beta function and the spacetime cosmological constant [11].

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