SLAC-PUB-7739
January 1998

Chiral symmetry breaking and effective lagrangians for softly broken supersymmetric QCD

Stephen P. Martin[a] and James D. Wells[b]

[a] *Physics Department, University of California, Santa Cruz CA 95064*

[b] *Stanford Linear Accelerator Center, Stanford University, Stanford CA 94309*

Submitted to *Physical Review D*

Work supported by Department of Energy contract DE–AC03–76SF00515.

hep-th/9801157
SLAC-PUB-7739

Chiral symmetry breaking and effective lagrangians for softly broken supersymmetric QCD

Stephen P. Martin[a] and James D. Wells[b†]

[a]*Physics Department, University of California, Santa Cruz CA 95064*
[b]*Stanford Linear Accelerator Center, Stanford CA 94309*

Abstract

We study supersymmetric QCD with $N_f < N_c$ in the limit of small supersymmetry-breaking masses and smaller quark masses using the weak-coupling Kähler potential. We calculate the full spectrum of this theory, which manifests a chiral symmetry breaking pattern similar to that caused by the strong interactions of the standard model. We derive the chiral effective lagrangian for the pion degrees of freedom, and discuss the behavior in the formal limit of large squark and gluino masses and for large N_c. We show that the resulting scalings of the pion decay constant and pion masses in these limits differ from those expected in ordinary nonsupersymmetric QCD. Although there is no weak coupling expansion with $N_f = N_c$, we extend our results to this case by constructing a superfield quantum modified constraint in the presence of supersymmetry breaking.

(Submitted to Phys. Rev. D)

[†]Work supported by the Department of Energy under contract DE-AC03-76SF00515.

1 Introduction

Solving strongly coupled field theories is hard. Quantum chromodynamics (QCD) provides a good example of the difficulties. In the weak coupling regime ($Q^2 \gg \Lambda^2_{\rm QCD}$) we have gained much confidence over the last twenty years that QCD describes the interactions between real quarks and gluons. However, in the strong coupling regime ($Q^2 \lesssim \Lambda^2_{\rm QCD}$) the correspondence between QCD and nature becomes more murky. That is, QCD cannot be solved entirely in this energy domain, in the sense that one cannot calculate analytically from first principles the particle spectra, the masses, and the dynamics of the low energy theory in strong coupling. Experimentally, the independent left-right global chiral symmetries of (nearly) massless QCD appear to be spontaneously broken to the vector subgroup by quark condensates. Chiral perturbation theory incorporates these symmetries and produces an effective lagrangian which accurately describes interactions of the low-lying Nambu-Goldstone bosons (pions) of chiral symmetry breaking. However, the effective chiral lagrangian is a phenomenological model not easily understood from first principles. Although much work continues to elucidate our understanding of ordinary QCD in the strong coupling region, no entirely satisfactory mapping of the fundamental QCD lagrangian onto low energy hadron physics has yet to be demonstrated.

In contrast to non-supersymmetric QCD, supersymmetric QCD (SQCD) holds out promise of understanding strong coupling more thoroughly. A long struggle has culminated in an impressive understanding of this supersymmetric theory. The successes include knowledge of the exact β function for the gauge coupling [1], the non-perturbative superpotential [2, 3, 4, 5] and a self-consistent description of the vacuum structure [6, 7]. The hope continues to be that we will gain important new insights of field theory in general and QCD in particular from the more controllable SQCD. One step toward this goal is to introduce supersymmetry breaking operators into SQCD [8, 9] and analyze the resulting theory. This has been done previously using canonical Kähler potentials and small supersymmetry-breaking masses [10, 11]. The spectrum of massless states was calculated and shown to be in qualitative agreement with QCD. Work continues in an effort to find correspondence with QCD [12, 13, 14, 15, 16].

In this paper we will build upon previous analyses and calculate the full mass spectra of supersymmetric QCD with $N_f \leq N_c$ using the weakly coupled Kähler potential (for $N_f < N_c$), squark and gluino soft supersymmetry-breaking masses ($m_{\tilde{Q}}$ and $m_{\tilde{g}}$, or $m_{\rm soft}$ to represent either), and non-zero supersymmetry-preserving quark masses (M_q). We will then construct the effective chiral lagrangian of the theory, and study the behavior of the pion masses and pion decay constant for $m_{\rm soft} \gg M_q$. Furthermore, we will consider the problem of the decoupling of the squarks and gluino by taking the formal limit $m_{\rm soft} \gg \Lambda$ in these results.

2 Softly-broken supersymmetric QCD with $N_f < N_c$

Supersymmetric QCD is defined to be a supersymmetric theory with an $SU(N_c)$ gauge group with N_f quarks in the fundamental (Q) and anti-fundamental ($\overline{Q}$) representations. The chiral symmetry of SQCD with no explicit masses is $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_{R'}$, where $U(1)_B$ is baryon number and $U(1)_{R'}$ is a non-anomalous combination of the axial symmetry for the superfields and an R-symmetry under which the gaugino fields transform non-trivially. In this section we will be concerned with the $N_f < N_c$ case. We denote the gauge-invariant meson field as $T_i^j = Q_i \overline{Q}^j$. The non-perturbative superpotential for such a theory was worked out by Affleck, Dine and Seiberg (ADS) [2] to be

$$W_{\rm ADS} = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det T} \right)^{\frac{1}{N_c - N_f}} . \tag{2.1}$$

This superpotential preserves the chiral symmetries mentioned above, and also respects the anomalous R-symmetry if Λ is taken to transform in the appropriate way [7].

The vacuum of this theory corresponds to $\langle T \rangle$ running to infinity. Since $SU(N_c)$ with $N_f < N_c$ is asymptotically free, we know that the weak-coupling Kähler potential is exact for this theory for large T along the $SU(N_c)$ flat directions. The Kähler potential is then

$$K_T = 2{\rm tr}[\sqrt{T^\dagger T}]. \tag{2.2}$$

There are two operational definitions of this Kähler potential. First, one can write it as

$$K_T = 2 \sum_{i=1}^{N_f} \sqrt{\lambda_i}, \tag{2.3}$$

where λ_i are the eigenvalues of the matrix $T^\dagger T$, written in terms of the N_f independent quantities invariant under $SU(N_f)_L \times SU(N_f)_R$ transformations: $\det T^\dagger T$ and ${\rm tr}[(T^\dagger T)^j]$ for $j = 1 \ldots N_f - 1$. For example, for $N_f = 1$, the meson T is just a single chiral superfield and the Kähler potential is simply $K_T = 2\sqrt{T^* T}$. For $N_f = 2$, the Kähler potential can be written explicitly as $K_T = 2(\sqrt{\lambda_+} + \sqrt{\lambda_-})$, where

$$\lambda_\pm = \frac{1}{2} \left({\rm tr}[T^\dagger T] \pm \sqrt{({\rm tr}[T^\dagger T])^2 - 4\det T^\dagger T} \right) . \tag{2.4}$$

However, for general N_f it is simpler in practice to write the Kähler potential eq. (2.2) as a power series expansion around the suspected minimum of the scalar potential. This can be done in terms of a new chiral superfield Z, by writing $T_i^j = t_0(\delta_i^j + Z_i^j)$, so that

$$K = \frac{t_0}{2}\Big(\mathrm{tr}[Z^\dagger Z] - \frac{1}{4}\mathrm{tr}[Z^\dagger Z^2 + Z^{\dagger 2} Z] + \frac{1}{16}\mathrm{tr}[2Z^{\dagger 2}Z^2 - Z^\dagger Z Z^\dagger Z + 2Z^\dagger Z^3 + 2Z^{\dagger 3}Z] + \cdots\Big) \quad (2.5)$$

(neglecting holomorphic pieces, which do not contribute to the lagrangian), and

$$\mathrm{det}T = t_0^{N_f}\left(1 + \mathrm{tr}[Z] + \frac{1}{2}\mathrm{tr}[Z]^2 - \frac{1}{2}\mathrm{tr}[Z^2] + \cdots\right) \quad (2.6)$$

for use in the superpotential.

We also want to incorporate soft masses into the lagrangian. Introducing soft squark masses $m_{\tilde{Q}}^2(|\tilde{Q}|^2 + |\tilde{\overline{Q}}|^2)$ is straightforward and corresponds to adding $2m_{\tilde{Q}}^2\mathrm{tr}[\sqrt{t^\dagger t}]$ to the scalar potential, where t_i^j is the scalar component of T_i^j. (For simplicity we take all of the $2N_f$ squark masses to be equal.) This quantity can once again be defined either in terms of the square roots of the eigenvalues of the matrix $t^\dagger t$ or by its power series expansion:

$$\mathrm{tr}[\sqrt{t^\dagger t}] = t_0\left(N_c + \frac{1}{2}\mathrm{tr}[z + z^\dagger] - \frac{1}{8}\mathrm{tr}[(z - z^\dagger)^2] + \cdots\right), \quad (2.7)$$

where z_i^j is the scalar component of the superfield Z_i^j. In order to include a gluino mass it is useful to include the chiral superfield associated with the gluino bilinear: $S = \frac{g^2}{32\pi^2}\lambda^\alpha\lambda_\alpha + \cdots$. The superpotential with the S field is [3]

$$W_S = S\left[\log\left(\frac{\Lambda^{3N_c - N_f}}{S^{N_c - N_f}\det T}\right) + N_c - N_f\right]. \quad (2.8)$$

Integrating out S yields W_{ADS} and is equivalent to replacing $S = dW_{\mathrm{ADS}}/d\log\Lambda^{3N_c - N_f}$ [5]. Adding a gluino soft mass is accomplished simply by adding a term proportional to $m_{\tilde{g}}s$, where s is the scalar component of S, before integrating out S.

In our discussion, we will also be considering a small amount of explicit chiral symmetry breaking in the form of quark masses. This can be accomplished with an additional term in the superpotential, $W = W_{\mathrm{ADS}} + \mathrm{tr}[M_q T]$. Throughout the following discussion we will assume that the eigenvalues of the quark mass matrix M_q are all much smaller than the other scales in the problem, Λ and m_{soft}, so that M_q can be treated as a perturbation.

Incorporating all the elements described above, the full lagrangian under consideration in the weak-coupling domain is

$$\mathcal{L} = 2\int d^4\theta\,\mathrm{tr}[\sqrt{T^\dagger T}] + \left(\int d^2\theta\, W + \mathrm{c.c.}\right) - V_{\mathrm{soft}}; \quad (2.9)$$

$$V_{\rm soft} = -c_T m_{\tilde g}\left(\frac{\Lambda^{3N_c-N_f}}{\det t}\right)^{\frac{1}{N_c-N_f}} + {\rm c.c.} + 2m^2_{\tilde Q}{\rm tr}[\sqrt{t^\dagger t}], \tag{2.10}$$

where $c_T = 32\pi^2/g^2$. Note that in the large N_c limit [18] one should think of $g^2 N_c =$ constant and therefore $c_T \propto N_c$. Now it is straightforward to solve for the minimum of the potential and quadratic fluctuations around it using the expansions in eqs. (2.5)-(2.7).

In the limit of small M_q, the minimum of the scalar potential occurs at $\langle t_i^j\rangle = t_0\delta_i^j$, where

$$\left(\frac{\Lambda^2}{t_0}\right)^{\frac{N_c}{N_c-N_f}} = \frac{1}{2(N_c+N_f)\Lambda}\left[c_T m_{\tilde g} + \sqrt{c_T^2 m_{\tilde g}^2 + 4(N_c^2-N_f^2)m^2_{\tilde Q}}\right]. \tag{2.11}$$

For convenience we make the following definition

$$\tilde m \equiv \Lambda\left(\frac{\Lambda^2}{t_0}\right)^{\frac{N_c}{N_c-N_f}} = \begin{cases} m_{\tilde g}\frac{c_T}{N_c+N_f}, & \text{if } m_{\tilde g} \gg m_{\tilde Q}; \\ m_{\tilde Q}\sqrt{\frac{N_c-N_f}{N_c+N_f}}, & \text{if } m_{\tilde Q} \gg m_{\tilde g}, \end{cases} \tag{2.12}$$

so that $\tilde m \sim m_{\rm soft}$. The F-term component of T_i^j has dimensions of (mass)^3 and gets a VEV equal to $F_T\delta_i^j$, where

$$F_T = 2\tilde m^{N_f/N_c}\Lambda^{3-N_f/N_c}. \tag{2.13}$$

The meson matrix can be parameterized as

$$t = t_0 \exp[(x+iy)/\sqrt{2N_f} + \lambda^a(x_a+iy_a)] \tag{2.14}$$

where $\langle t_i^j\rangle = t_0\delta_i^j$ is the meson VEV given above, and λ^a (with $a = 1,\dots,N_f^2-1$) are the generators of $SU(N_f)$, normalized to ${\rm tr}[\lambda^a\lambda^b] = \frac{1}{2}\delta^{ab}$. The fields x, y, x_a and y_a are dimensionless. The corresponding meson fields with canonically-normalized kinetic terms, V, A, V_a and π_a, are obtained by multiplying each of these fields by F_π, given by

$$F_\pi^2 = t_0/2. \tag{2.15}$$

The full spectrum of the model contains the flavor-adjoint pions (π_a), the heavy scalars (V, V_a), heavy pseudo-scalar (A), and heavy fermions (ψ_0 and ψ_a). For the masses of the heavy particles, we find

$$m_V^2 = \frac{4N_c}{N_c-N_f}m^2_{\tilde Q} + \frac{2N_c}{(N_c-N_f)^2}c_T m_{\tilde g}\tilde m, \tag{2.16}$$

$$m_{V_a}^2 = \frac{4N_c}{N_c+N_f}m^2_{\tilde Q} + \frac{2}{N_c+N_f}c_T m_{\tilde g}\tilde m, \tag{2.17}$$

$$m_A^2 = \frac{2N_f}{(N_c - N_f)^2} c_T m_{\tilde{g}} \tilde{m}, \tag{2.18}$$

$$m_{\psi_0} = \frac{N_c + N_f}{N_c - N_f} \tilde{m}, \tag{2.19}$$

$$m_{\psi_a} = \tilde{m}, \tag{2.20}$$

again neglecting $M_q \ll \Lambda, m_{\rm soft}$. The pions π_a are massless when $M_q = 0$, and (as in ordinary QCD) obtain a (mass)^2 matrix which is linear in the quark mass matrix M_q. We find

$$(m_\pi^2)_{ab} = 8\mathrm{tr}[M_q \lambda^a \lambda^b]\,\tilde{m}. \tag{2.21}$$

In the limit $M_q \to 0$ there is an interesting correspondence with ordinary QCD as originally pointed out in [10]. Light pions are indicative of the Nambu-Goldstone modes of the $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry spontaneously breaking to its vector subgroup. Also, when the gluino mass is zero there is an extra Nambu-Goldstone mode associated with the spontaneous breaking of the non-anomalous $U(1)_{R'}$ symmetry. This massless state is A, and it only gains mass when $U(1)_{R'}$ is explicitly broken by a soft gluino mass. All of the masses scale like N_c^0 at large N_c, except m_A^2 which scales like $1/N_c$ (recall that $c_T \propto N_c$). This is because in the large N_c limit [18] the chiral symmetries of SQCD are promoted to $U(N_f)_L \times U(N_f)_R$, leaving an extra Nambu-Goldstone boson in the spectrum when it is spontaneously broken to $U(N_f)_V$. Thus, the A field in supersymmetric QCD can be identified with the η' analog in ordinary QCD.

3 The chiral effective lagrangian and scaling behavior towards the decoupling limit

In the weakly-coupled limit of supersymmetric QCD discussed in the previous section, we find that the chiral effective lagrangian for the pion degrees of freedom in the limit $M_q \ll m_{\rm soft}$ can be written as

$$\mathcal{L} = F_\pi^2 \mathrm{tr}[\partial^\mu U^\dagger \partial_\mu U] + F_T(\mathrm{tr}[U M_q] + \mathrm{c.c.}) \tag{3.1}$$

where F_π and F_T are given in eqs. (2.15) and (2.13), and $U = \exp[i\lambda^a \pi_a / F_\pi]$. This form is familiar as the usual chiral effective lagrangian used to describe the effective low-energy dynamics of pseudo-scalar mesons in ordinary QCD. Thus it is clear that F_π should be interpreted as the pion decay constant. The pion (mass)^2 matrix is given simply in terms of F_T and F_π by

$$(m_\pi^2)_{ab} = 2\mathrm{tr}[M_q \lambda^a \lambda^b] \frac{F_T}{F_\pi^2} \tag{3.2}$$

[cf. eqs. (2.13), (2.15) and (2.21)]. It is tempting to associate F_T with the quark bilinear condensate $\langle\psi_Q\psi_{\overline{Q}}\rangle$ [10] in accordance with our picture of chiral symmetry breaking in ordinary QCD. Because the confining phase and the Higgs phase of the theory are smoothly connected to each other [17], this correspondence suggests that the chiral effective lagrangian for the pion degrees of freedom might be obtained as a sensible limit of the weak-coupling SQCD lagrangian. However, since $F_T = -\psi_Q\psi_{\overline{Q}} + QF_{\overline{Q}} + \overline{Q}F_Q$ in terms of the original quark fields, it is not entirely clear that $\langle F_T\rangle$ can be identified solely as a condensate of the original quark fermions in the weak-coupling regime, and in the strong-coupling regime the calculation with the Kähler potential eq. (2.2) is unreliable.

To investigate this correspondence further, consider the scaling of the chiral effective lagrangian parameters F_π and F_T with $m_{\rm soft}$. In the regime $\tilde{m} \sim m_{\rm soft} \ll \Lambda$ where weak coupling makes our calculations reliable, we find from the results of the previous section that

$$F_\pi^2 \propto \left(\frac{\tilde{m}}{\Lambda}\right)^{-\frac{(N_c-N_f)}{N_c}} \Lambda^2, \tag{3.3}$$

$$F_T \propto \left(\frac{\tilde{m}}{\Lambda}\right)^{\frac{N_f}{N_c}} \Lambda^3. \tag{3.4}$$

Note that as $m_{\rm soft}$ increases, F_π^2 decreases but F_T and m_π^2 grow.

To compare the scaling behavior of these quantities in the formal limit of large $m_{\rm soft}$ with the behavior expected in ordinary QCD, we must determine how the SQCD scale Λ is related to the effective QCD scale $\Lambda_{\rm eff}$ after the squarks and gauginos are decoupled. Using one-loop renormalization group matching, one finds that

$$\Lambda_{\rm eff} = \Lambda \left(\frac{m_{\tilde{g}}}{\Lambda}\right)^{\frac{2N_c}{11N_c-2N_f}} \left(\frac{m_{\tilde{Q}}}{\Lambda}\right)^{\frac{N_f}{11N_c-2N_f}} \tag{3.5}$$

when $m_{\tilde{g}}, m_{\tilde{Q}} \gg \Lambda$, or

$$\Lambda_{\rm eff} = \Lambda \left(\frac{m_{\tilde{g}}}{\Lambda}\right)^{\frac{2N_c}{11N_c-N_f}} \tag{3.6}$$

if only the gluino is heavy compared to Λ, or

$$\Lambda_{\rm eff} = \Lambda \left(\frac{m_{\tilde{Q}}}{\Lambda}\right)^{\frac{N_f}{9N_c-2N_f}} \tag{3.7}$$

if only the squarks are heavy compared to Λ. Since the chiral effective lagrangian parameters for ordinary QCD (in the limit that the squarks and gluinos decouple) should only depend on the soft masses through the scale $\Lambda_{\rm eff}$, one expects that the pion dynamics would be governed by a lagrangian with the same form as eq. (3.1), but with

$$F_\pi^2 \propto \Lambda_{\rm eff}^2 \tag{3.8}$$

$$F_T \propto \Lambda_{\rm eff}^3 \tag{3.9}$$

so that $m_\pi^2 \propto \Lambda_{\rm eff} M_q$. Unfortunately, it is easy to check that eqs. (3.8) and (3.9) do not have the same scaling behavior with $m_{\rm soft}$ in the formal decoupling limit as the weak-coupling results eqs. (3.3) and (3.4), no matter which of eqs. (3.5), (3.6) or (3.7) applies. Furthermore, the large N_c scaling of the large-$m_{\rm soft}$ chiral lagrangian does not conform with expectations from ordinary QCD. Non-supersymmetric chiral perturbation theory implies [18] that $F_\pi^2 \propto N_c$ and $F_T \propto N_c$, but the formal decoupling limit of the weakly-coupled SQCD chiral lagrangian scales as $F_\pi^2 \propto N_c^0$ and $F_T \propto N_c^0$.

The failure of the weak-coupling results found in section 2 to agree with ordinary QCD is not unexpected. For one thing, one should probably allow for the possibility of a much more general scaling of the Kähler potential with T as one approaches the large-$m_{\rm soft}$ regime. The lack of non-renormalization theorems for the Kähler potential means that other contributions, over which we have little control, likely will dominate for the small T region relevant for the decoupling to ordinary QCD. One such contribution can be seen by considering the original Kähler potential for the composite glueball chiral superfield S, of the form $K_S \sim (S^* S)^{1/3}$ [3]. Integrating out S using this Kähler potential and the superpotential eq. (2.8), one finds an additional contribution to the Kähler potential for the remaining T fields of the form

$$K_S \sim (S^* S)^{1/3} \sim \Lambda^2 \left(\frac{\Lambda^{4N_f}}{\det[T^\dagger T]} \right)^{1/3(N_c - N_f)} . \tag{3.10}$$

In weak coupling the neglect of this term in the Kähler potential is justified because $K_T = 2{\rm tr}[\sqrt{T^\dagger T}]$ always dominates for large T. However, as we move towards strong coupling this term becomes important, as do presumably many other contributions which are singular for small T. Moreover, there are additional unknown contributions coming from integrating out the entire spectrum of composite massive hadronic degrees of freedom. Likewise, the soft terms can be more complicated as well in the strong coupling region [11, 15], leading to different scaling behavior of the chiral lagrangian.

In ref. [10], a canonical Kähler potential and soft terms were used, with arbitrary prefactors which parameterize our lack of knowledge of the effects just mentioned. Alternatively, we can introduce such renormalization constants for the Kähler potential, soft terms and quark masses in the weak-coupling lagrangian:

$$K = 2a_T \mathrm{tr}[\sqrt{T^\dagger T}] \tag{3.11}$$

instead of eq. (2.2);

$$V_{\mathrm{soft}} = -c_T m_{\tilde{g}} \left(\frac{\Lambda^{3N_c - N_f}}{\det t}\right)^{\frac{1}{N_c - N_f}} + \mathrm{c.c.} + 2a_m m_{\tilde{Q}}^2 \mathrm{tr}[\sqrt{t^\dagger t}] \tag{3.12}$$

instead of eq. (2.10). In the presence of supersymmetry breaking, one must also consider a similar renormalization of the quark mass terms $M_q \to a_M M_q$. Here a_T, a_m, c_T and a_M are dimensionless quantities which can *a priori* have an arbitrary dependence on Λ and the soft terms in the underlying lagrangian, which in an effective lagrangian approach can also mimic any dependence of these terms on the dynamical fields. The limits $a_T, a_m, a_M \to 1$ can of course be used to recover the weak coupling results.

Using this as a toy model, one can show that the VEV occurs for $\langle t_i^j \rangle = t_0 \delta_i^j$, with t_0 satisfying

$$\left(\frac{\Lambda^2}{t_0}\right)^{\frac{N_c}{N_c - N_f}} = \frac{a_T}{2(N_c + N_f)\Lambda}\left[c_T m_{\tilde{g}} + \sqrt{c_T^2 m_{\tilde{g}}^2 + 4(N_c^2 - N_f^2) a_m m_{\tilde{Q}}^2 / a_T}\right] \equiv \frac{\tilde{m}'}{\Lambda} \tag{3.13}$$

in the limit of small M_q. In terms of $\tilde{m}'$, the masses of the heavy mesons and fermions are given in this case by eqs. (2.16)-(2.20) with the replacements $m_{\tilde{Q}}^2 \to a_m m_{\tilde{Q}}^2 / a_T$ and $\tilde{m} \to \tilde{m}'/a_T$. The pion masses and interactions in the low energy effective theory are given by a lagrangian of exactly the same form as eq. (3.1), but now with $M_q \to a_M M_q$ and

$$F_\pi^2 = \frac{a_T}{2}\left(\frac{\tilde{m}'}{\Lambda}\right)^{-\frac{(N_c - N_f)}{N_c}} \Lambda^2 \tag{3.14}$$

$$F_T = \frac{2}{a_T}\left(\frac{\tilde{m}'}{\Lambda}\right)^{\frac{N_f}{N_c}} \Lambda^3. \tag{3.15}$$

If one attempts to make these results agree with the expected behavior in the decoupling limit eqs. (3.5)-(3.9), then one finds that

$$a_T \sim \left(\frac{\Lambda_{\mathrm{eff}}}{\Lambda}\right)^2 \left(\frac{\tilde{m}'}{\Lambda}\right)^{\frac{N_c - N_f}{N_c}} \tag{3.16}$$

$$a_M \sim \left(\frac{\Lambda_{\mathrm{eff}}}{\Lambda}\right)^5 \left(\frac{\tilde{m}'}{\Lambda}\right)^{\frac{N_c - 2N_f}{N_c}}. \tag{3.17}$$

However, it is hard to imagine a sound origin for these assignments, in view of our lack of knowledge of the lagrangian for small T. Matching on to a correct quantitative description of the decoupling of superpartners requires the inclusion of non-trivial Kähler potential terms and terms containing higher superderivatives (giving rise to operators which contain more than two powers of the auxiliary fields) over which we do not seem to have even qualitative control. Thus it appears that the difficulty of finding a softly broken supersymmetric QCD lagrangian which gives the correct chiral lagrangian is perhaps equally as difficult as solving ordinary QCD. The language of supersymmetry does not seem to have made this task easier.

4 Supersymmetric QCD with $N_c = N_f$

When the number of flavors is equal to or greater than the number of colors there are non-trivial gauge-invariant baryonic degrees of freedom. In the $N_c = N_f$ case, two such baryon fields (implicitly antisymmetric in color) are allowed:

$$B = Q_1 Q_2 \cdots Q_{N_c}, \tag{4.1}$$

$$\overline{B} = \overline{Q}_1 \overline{Q}_2 \cdots \overline{Q}_{N_c}. \tag{4.2}$$

Decoupling flavors to form an $N_f < N_c$ theory consistent with the ADS superpotential requires that the meson fields and the baryon fields satisfy a quantum modified constraint [6]:

$$\mathrm{det}T - B\overline{B} = \Lambda^{2N_c}. \tag{4.3}$$

Any vacuum which resembles ordinary QCD requires $\langle B \rangle = \langle \overline{B} \rangle = 0$ to preserve baryon number and $\langle T_i^j \rangle \propto \delta_i^j$ to preserve the vectorial $SU(N_f)$. This implies $\langle T_i^j \rangle = \delta_i^j \Lambda^2$ from the quantum modified constraint.

An unfortunate corollary to the quantum modified constraint is the inability to have even approximate control over the Kähler potential for any values of the soft terms. In the $N_f < N_c$ case we were able to have confidence in the small supersymmetry breaking results because the theory was still at weak coupling. However, now the vacuum at $\langle T_i^j \rangle = \delta_i^j \Lambda^2$ corresponds to strong coupling. With no guiding principle for the Kähler potential we choose the weak-coupling Kähler potential for T_i^j and a canonical Kähler potential for B and $\overline{B}$:

$$K = 2a_T \mathrm{tr}[\sqrt{T^\dagger T}] + \frac{a_B}{\Lambda^{2N_c-2}}(B^* B + \overline{B}^* \overline{B}). \tag{4.4}$$

(Other Kähler potentials, including a canonical Kähler potential for T, give qualitatively similar results.) We also choose soft terms of the form

$$V_{\mathrm{soft}} = -(c_T m_{\tilde{g}} s + \mathrm{c.c.}) + 2a_m m_{\tilde{Q}}^2 \mathrm{tr}[\sqrt{t^\dagger t}] + \frac{m_{\tilde{B}}^2}{\Lambda^{2N_c-2}}(|b|^2 + |\overline{b}|^2). \tag{4.5}$$

where $m_{\tilde{B}}^2$ originates from the soft terms in the underlying lagrangian in some undetermined way, and b and $\overline{b}$ are the scalar components of B and $\overline{B}$. Based on the constituent squark description of T, B and $\bar{B}$, one might naively expect that $m_{\tilde{B}}^2 \simeq N_c^2 m_{\tilde{Q}}^2$. However, soft terms for the baryons in the confining description have no direct analog to the soft terms of squarks in the Higgs description. Therefore, if the $N_c = N_f$ case had a self-consistent weak coupling expansion then it would appear that the baryons should have zero soft masses. Since no weak coupling domain exists for this theory, and given the above ambiguities it is best to treat $m_{\tilde{B}}$ as a free parameter.

The S-dependent superpotential can be written as [3]

$$W_S = S \log\left(\frac{\mathrm{det}T - B\overline{B}}{\Lambda^{2N_c}}\right) + a_M \mathrm{tr}[M_q T]. \tag{4.6}$$

We ignore the Kähler potential for the S term since it would induce more complicated Kähler terms for T, B and $\overline{B}$ after it is integrated out; these can hopefully be absorbed into the definitions of a_T and a_B in an effective field theory approach. So, treating S as a Lagrange multiplier, the superpotential can be written simply as $W = a_M \mathrm{tr}[M_q T]$ subject to the new superfield constraint

$$\mathrm{det}T - B\overline{B} = \Lambda^{2N_c}(1 - \theta\theta c_T m_{\tilde{g}}). \tag{4.7}$$

Integrating out the auxiliary fields for T_i^j, B and $\overline{B}$ subject to this constraint produces a scalar potential (neglecting M_q for now):

$$\begin{aligned} V \;=\; & -\frac{a_B a_T c_T^2 m_{\tilde{g}}^2 \Lambda^{4N_c}}{2a_B \prod_i |t_i|^2 \sum_j |t_j|^{-1} \;+\; a_T \Lambda^{2N_c-2}(|b|^2 + |\overline{b}|^2)} \\ & + 2a_m m_{\tilde{Q}}^2 \sum_i |t_i| + \frac{m_{\tilde{B}}^2}{\Lambda^{2N_c-2}}(|b|^2 + |\overline{b}|^2) \end{aligned} \tag{4.8}$$

where t_i are the eigenvalues of t_i^j, subject to the scalar field constraint $\prod_i t_i = b\overline{b} + \Lambda^{2N_c}$. The minimum of this potential always occurs for $t_i^j = t_0 \delta_i^j$ and $b = -\overline{b}$.

A vacuum like that of ordinary QCD with conserved baryon number and chiral symmetry breaking corresponds to $t_0 = \Lambda^2$ and $b = \overline{b} = 0$. This is a local minimum of the scalar potential if

$$\frac{c_T^2 m_{\tilde{g}}^2}{N_c^2}\left(a_T - (2N_c - 1)a_B\right) > 4\frac{a_B}{a_T}(a_m m_{\tilde{Q}}^2 - m_{\tilde{B}}^2). \tag{4.9}$$

Note that in the limit $a_B \ll a_T$, this condition is always satisfied. A necessary but not sufficient condition for this to also be a global minimum of the potential is

$$\frac{c_T^2 m_{\tilde{g}}^2}{N_c^2}(a_T - N_c a_B) > 4(a_m m_{\tilde{Q}}^2 - \frac{m_{\tilde{B}}^2}{N_c}). \tag{4.10}$$

This is because there is always a local minimum at $t_0 = 0$ with $b = -\overline{b} = \Lambda^{N_c}$, which has lower energy than the $b = \overline{b} = 0$ minimum if eq. (4.10) is not satisfied. Note also that for some values of the parameters, the global minimum of the potential can occur for both $b = -\overline{b}$ and t_0 non-zero, if the $m_{\tilde{g}}^2$ term dominates and $a_T < a_B(2N_c - 1)$. If so, then both baryon number and chiral symmetry will be broken in the vacuum state. However, one must remember that since the description of the theory in terms of the parameters a_T, a_m, a_M, $m_{\tilde{B}}^2$ and c_T is only an effective one, it is not clear that the global properties of the potential are significant. Therefore, presumably only the condition eq. (4.9) should be considered as a constraint on the parameters of the model, even if baryon number is assumed or required to be unbroken.

The spectrum of this model in the baryon number conserving vacuum contains heavy flavor-adjoint scalars V_a with

$$m_{V_a}^2 = \frac{c_T^2 m_{\tilde{g}}^2}{2N_c^2} + \frac{2a_m m_{\tilde{Q}}^2}{a_T} \tag{4.11}$$

and heavy flavor-adjoint fermions with

$$m_{\psi_a} = \frac{c_T m_{\tilde{g}}}{2N_c}. \tag{4.12}$$

Note that these results agree with those for the corresponding states in the $N_f < N_c$ case as found above, if one takes the $N_f \to N_c$ limit. The flavor-singlet components of T and ψ_T are removed by the scalar and fermionic components of the superfield constraint (4.7). (The latter constraint takes the form $\det T\, \mathrm{tr}[T^{-1}\psi_T] - B\psi_{\overline{B}} - \overline{B}\psi_B = 0$.) This corresponds to the singularity in eqs. (2.16), (2.18) and (2.19) as $N_f \to N_c$. Instead, there are some additional degrees of freedom corresponding to the baryons in the $N_c = N_f$ case which have no direct analog in the $N_f < N_c$ theory. The complex scalars B and $\overline{B}$ mix to form two degenerate pairs of real scalar mass eigenstates, with

$$m_{B\overline{B}}^2 = \left(\frac{a_T^2 c_T^2 m_{\tilde{g}}^2}{4N_c^2 a_B^2} + \frac{m_{\tilde{B}}^2}{a_B}\right) \pm \left(\frac{a_T c_T^2 m_{\tilde{g}}^2 (2N_c - 1)}{4N_c^2 a_B} + \frac{a_m m_{\tilde{Q}}^2}{a_B}\right). \tag{4.13}$$

The two heavy fermionic partners of B and $\overline{B}$ pair up to get a Dirac mass,

$$m_{\psi_B \psi_{\overline{B}}} = \frac{a_T c_T m_{\tilde{g}}}{4N_c a_B}. \tag{4.14}$$

(In all of the preceding, we have neglected $M_q \ll m_{\rm soft}$ as before.)

Of particular interest is the chiral lagrangian governing the pion masses. It has the same form as eq. (3.1), with now $M_q \to a_M M_q$ and

$$F_\pi^2 = \frac{a_T \Lambda^2}{2}, \tag{4.15}$$

$$F_T = \frac{c_T m_{\tilde{g}} \Lambda^2}{N_c}, \tag{4.16}$$

and

$$(m_\pi^2)_{ab} = 2a_M {\rm tr}[M_q \lambda^a \lambda^b] \frac{F_T}{F_\pi^2} = 4{\rm tr}[M_q \lambda^a \lambda^b] \frac{a_M c_T m_{\tilde{g}}}{a_T N_c}. \tag{4.17}$$

If $m_{\tilde{g}}$ vanishes, then the pions are massless in linear order in M_q. This can be understood easily from the fact that with $m_{\tilde{g}} = 0$, the scalar potential has only a quadratic dependence on M_q. Equations (4.15)-(4.17) again agree exactly with the $N_f \to N_c$ limit of the corresponding results found above for $N_f < N_c$. If one could make a weak-coupling expansion with $a_T, a_m, a_M \to 1$, then the $N_c = N_f$ case would experience the same similarities (chiral symmetry breaking pattern) and the same dissimilarities ($m_{\rm soft}$ and N_c scaling in the chiral lagrangian) as the $N_f < N_c$ case when comparisons are made to ordinary nonsupersymmetric QCD. It is possible to choose values for the unknown parameters so that the expected scalings for the decoupling limit of ordinary QCD are realized; however, as noted for $N_f < N_c$, these assignments are hard to justify with actual calculations.

5 Discussion

We have analyzed the spectroscopy and chiral lagrangian of supersymmetric QCD with $N_f \leq N_c$ in an attempt to learn more about these theories with small explicit chiral symmetry breaking and small supersymmetry breaking. For $N_f < N_c$ the supersymmetric theory is weakly coupled in this limit, and the chiral symmetry breaking and spectroscopy are reliably calculated. Our calculations have been carried out using confining phase degrees of freedom. We can compare our results with those of ref. [11] which worked with Higgs-phase degrees of freedom in the limit of $M_q = 0$ and $m_{\tilde{g}} = 0$. The particle spectrum we find in this limit from the lagrangian of eq. (2.9) agrees with their spectrum, thus demonstrating equivalence when the appropriate weak-coupling Kähler potential and soft terms are used in both approaches. We have presented results also for the chiral effective lagrangian parameters in this theory.

For $N_f \geq N_c$, the situation becomes less controlled, because the theory has no reliable weak coupling limit with the same gauge group for small supersymmetry breaking masses. Nevertheless, we have calculated the potential, chiral lagrangian and spectrum assuming a modified weak-coupling Kähler potential for $N_f = N_c$, where one can at least obtain the ordinary QCD-like chiral symmetry breaking pattern. We also noted the possible existence of an exotic phase with both broken chiral symmetry and broken baryon number, which may exist for finite squark and gluino masses, in addition to the possible vacuum with spontaneously broken baryon number and conserved chiral symmetry as argued in [10]. Unfortunately, the effective lagrangian approach is quite incapable of answering whether the true softly broken SQCD theory can actually have these vacuum states. If they do exist at all, they cannot occur for arbitrarily large squark masses, because QCD with only vectorlike fermions cannot spontaneously break baryon number [19].

We found that attempts to match the chiral effective lagrangian for softly-broken SQCD to ordinary QCD by merely extrapolating the weak coupling results to the formal limit of large squark and gluino masses do not yield the correct scaling behavior for the pion decay constant and pion masses. Furthermore the behavior of these quantities with large N_c does not agree with the expectation from non-supersymmetric QCD. This outcome is not unexpected, since we have no reliable information about the Kähler potential and higher superderivative terms in the effective action for supersymmetric QCD beyond weak coupling. As our results illustrate, this lacking information is crucial for any quantitative attempts to understand ordinary pion dynamics in supersymmetric language. Although it may be possible to mimic the correct decoupling behavior by appropriate rescalings of the terms in the weak-coupling lagrangian, it is quite problematic to justify the necessary terms. Thus the apparent success associated with finding the correct pattern of chiral symmetry breaking is of limited direct applicability. However, one could use the exact supersymmetric results even in the presence of supersymmetry breaking as a laboratory to test calculational methods of strongly coupled theories [13]. Furthermore, lattice calculations can be compared with well-controlled supersymmetric theories at strong coupling to gain insights into both [10, 20].

Supersymmetric QCD studies have resulted in a much improved understanding of all supersymmetric field theories. These advances have centered mainly on the properties of the superpotential. Together with the assumption that the Kähler potential is non-singular in some region of field space, this allows us to understand the location of possible vacua and massless fields, but does not yield complete information about massive states and interactions. To learn more about strongly coupled theories with large supersymmetry breaking will require further insights and knowledge not only about the Kähler potential but also about interactions with higher superderivatives, which contribute terms to the potential with more than two powers of the auxiliary fields. The fact that we have almost no control over such contributions in $N = 1$ theories in four dimensions seemingly forbids progress in this direction. Perhaps a more useful starting point may be with a higher number of supersymmetries and/or dimensions [21] where the full lagrangian can be better controlled.

Acknowledgements. We thank N. Arkani-Hamed, M. Dine and M. Peskin for helpful discussions. The work of SPM was supported in part by the US Department of Energy.

References

[1] V. Novikov, M. Shifman, A. Vainshtein, V. Zakharov, Nucl. Phys. **B229**, 381 (1983).

[2] I. Affleck, M. Dine, N. Seiberg, Nucl. Phys. **B241**, 493 (1984); Nucl. Phys. **B256**, 557 (1985).

[3] G. Veneziano, and S. Yankielowicz, Phys. Lett. **B113**, 321 (1983); T.R. Taylor, G. Veneziano, and S. Yankielowicz, Nucl. Phys. **B218**, 493 (1983).

[4] D. Amati *et al.*, Phys. Rep. **162**, 169 (1988).

[5] K. Intriligator, R. Leigh, N. Seiberg, Phys. Rev. **D50**, 1092 (1994).

[6] N. Seiberg, Nucl. Phys. **B435**, 129 (1995).

[7] For reviews, see K. Intriligator and N. Seiberg, hep-th/9509066; M.E. Peskin, hep-th/9702094; M. Shifman, hep-th/9704114.

[8] A. Masiero and G. Veneziano, Nucl. Phys. **B249**, 593 (1985).

[9] N. Evans, S. Hsu, M. Schwetz, Nucl. Phys. **B456**, 205 (1995); Phys. Lett. **B404**, 77 (1997).

[10] O. Aharony, J. Sonnenschein, M.E. Peskin, and S. Yankielowicz, Phys. Rev. **D52**, 6157 (1995).

[11] E. D'Hoker, Y. Mimura, and N. Sakai, Phys. Rev. **D54**, 7724 (1996); hep-ph/9611458.

[12] L. Álvarez-Gaumé, J. Distler, C. Kounnas, M. Mariño, Int. J. Mod. Phys. **A11**, 4745 (1996).

[13] A. Kaiser, S. Selipsky, hep-th/9708087; T. Appelquist, A. Nyffeler, S. Selipsky, hep-th/9709177.

[14] G.R. Farrar, G. Gabadadze, M. Schwetz, hep-th/9711166.

[15] F. Sannino, J. Schechter, hep-th/9708113; S.D.H. Hsu, F. Sannino, J. Schechter, hep-th/9801097.

[16] H.-C. Cheng and Y. Shadmi, hep-th/9801146.

[17] E. Fradkin and S. Shenker, Phys. Rev. **D19**, 3682 (1979); T. Banks and E. Rabinovici, Nucl. Phys. **B160**, 349 (1979); S. Dimopoulos, S. Raby and L. Susskind, Nucl. Phys. **B173**, 208 (1980).

[18] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974); G. Veneziano, Nucl. Phys. **B117**, 519 (1974); E. Witten, Nucl. Phys. **B160**, 57 (1979); S. Coleman, E. Witten, Phys. Rev. Lett. **45**, 100 (1980); P. Di Vecchia and G. Veneziano, Nucl. Phys. **B171**, 253 (1980); C. Rosenzweig, J. Schechter and T. Trahern, Phys. Rev. **D21**, 3388 (1980); P. Nath and R. Arnowitt, Phys. Rev. **D23**, 473 (1981); H. Leutwyler, Phys. Lett. **B374**, 163 (1996); P. Herrera-Siklody, J.I. Latorre, P. Pascual and J. Taron, hep-ph/9610549.

[19] C. Vafa and E. Witten, Nucl. Phys. **B234**, 173 (1984).

[20] G. Curci, G. Veneziano, Nucl. Phys. **B292**, 555 (1987); I. Montvay, Nucl. Phys. **B466**, 259 (1996); G. Koutsoumbas, I. Montvay, Phys. Lett. **B398**, 130 (1997); J. Nishimura, Phys. Lett. **B406**, 215 (1997); N. Maru, J. Nishimura, hep-th/9705152; N. Evans, S. Hsu, M. Schwetz, hep-th/9707260; A. Donini, M. Guagnelli, P. Hernandez, A.Vladikas, hep-lat/9710065.

[21] See, for example, discussion in M. Shifman [7], and E. Witten, Nucl. Phys. **B507**, 658 (1997).