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We report on longitudinal beam-transfer function (BTF) measurements at the Advanced Light Source (ALS) for large phase-modulation amplitudes. As the amplitude of the swept-frequency excitation is increased, the dipole beam response around the synchrotron frequency develops a notch in the amplitude response with a corresponding sharp jump in the phase response. These observations are explained by a splitting of the beam into two beamlets created by the nonlinearity of the forced synchrotron oscillations. This is demonstrated with measurements of the longitudinal bunch profile using a streak camera. We also present measurements of the diffusion rate from one stable island to the other for a fixed modulation frequency and compare the measured rates with those expected from Touschek scattering. Finally, we comment on BTF measurements for sub- and superharmonic resonances.

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ABSTRACT

We report on longitudinal beam-transfer function (BTF) measurements at the Advanced Light Source (ALS) for large phase-modulation amplitudes [1, 2]. As the amplitude of the swept-frequency excitation is increased, the dipole beam response around the synchrotron frequency develops a notch in the amplitude response with a corresponding sharp jump in the phase response. These observations are explained by a splitting of the beam into two beamlets created by the nonlinearity of the forced synchrotron oscillations. This is demonstrated with measurements of the longitudinal bunch profile using a streak camera. We also present measurements of the diffusion rate from one stable island to the other for a fixed modulation frequency and compare the measured rates with those expected from Touschek scattering. Finally, we comment on BTF measurements for sub- and superharmonic resonances.

## 1 Introduction

A 'beam transfer function' (BTF) describes the response of a stored particle beam to an external harmonic excitation as a function of the excitation frequency. Usually, the excitation frequency is gradually swept across a resonance. In this paper we

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consider the beam phase response to an rf-phase modulation near the synchrotron frequency.

Beam-transfer functions are a common means to measure the stability limit of a particle beam [3, 4, 5]. Coasting-beam measurements for the ISR were reported by Hofmann and Zotter as early as 1977 [6, 7]. Subsequently, longitudinal bunched-beam transfer functions were measured, for example, at SPEAR [8], at CESR [9], at the Indiana University Cyclotron Facility (IUCF) [10, 11], and at the ALS [12]. Many parameters of interest, such as the zero-amplitude synchrotron frequency, the impedance, the beam frequency distribution and even the radiation damping time [12] can be deduced from the BTF. In recent measurements at the SLC damping rings, for large excitation a notch developed in the beam response curve [13]. Similar notches have been observed in BTF measurements at CESR [14] and at the ALS; an example is shown in Fig. 1 (left). The dependence of the notch frequency on the excitation amplitude indicated that the notch was related to a bifurcation in the nonlinear Hamiltonian [13], with the bifurcation frequency $f_b$ being given by [15],

$$\frac{f_b}{f_s} = 1 - \hat{\phi}^{2/3} \left( \frac{3}{8} 2^{1/3} \right), \tag{1}$$

where $f_s$ is the synchrotron frequency and $\hat{\phi}$ the modulation amplitude. The exact reason for the notch remained a mystery however. To understand its origin, we launched an experimental program at the ALS, where, using a streak camera, we measured the shape of the longitudinal beam profile as a function of both excitation frequency and amplitude, thereby extending the classical concept of the BTF. A detailed description of our studies can be found in Ref. [1].

| parameter | symbol | value |
|---|---|---|
| energy | $E$ | 1.5 GeV |
| circumference | $C$ | 196.8 m |
| rf frequency | $f_{rf}$ | 499.658 MHz |
| harmonic number | $h$ | 328 |
| momentum compaction | $\alpha$ | $1.6 \times 10^{-3}$ |
| synchrotron tune | $\nu_s$ | 0.0075 |
| long. rad. damping rate | $\lambda_{rad}$ | $5 \times 10^{-5}$ |
| rms natural bunch length | $\sigma_z$ | 4.5 mm |
| rms rel. energy spread | $\sigma_\delta$ | $7.1 \times 10^{-4}$ |
| norm. hor. emittance | $\epsilon_x$ | $1.2 \times 10^{-5}$ m |
| betatron tunes | $Q_{x,y}$ | 14.28, 8.18 |

Table 1: Nominal ALS parameters.

## 2 Measurement Results

Table 1 lists the nominal ALS machine parameters. The experimental set up for BTF measurements at the ATF was described in Refs. [1, 12]. Figure 1 displays typical measurement results. The two pictures on the left show amplitude and phase of the beam-phase response relative to the excitation; clearly visible for large excitation amplitudes is a notch in the amplitude response accompanied by a sudden 180-degree phase shift. On the right longitudinal profiles are depicted (vs. time), as measured with the streak camera, below, at and above the notch frequency. The wiggle period equals the modulation period. Close to the notch frequency the beam is split into two bunchlets, within the same rf bucket, oscillating 180 degree out of phase. When the dipole moments of the two bunchlets cancel exactly, the measured response approaches zero, giving rise to the 'notch'. The presence of two islands in phase space below a certain bifurcation frequency, Eq. (1), is a well known feature of the driven nonlinear pendulum [15, 16]. Figure 2 (left) shows that the theoretically predicted response amplitudes [1, 17] (2 stable and 1 unstable fixed points) agree well with the island locations extracted from the streak-camera image.

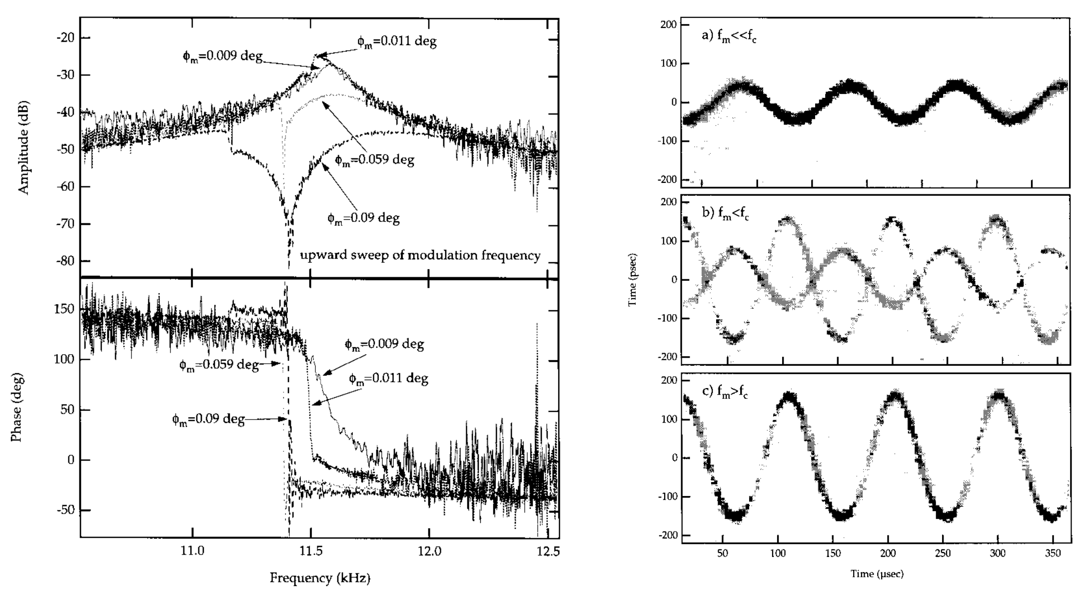


Figure 1: *(Left) amplitude and phase of longitudinal BTF for an upward sweep of the modulation frequency; the different curves correspond to different drive amplitudes; (right) images of the longitudinal beam profile in psec vs. time using the streak camera in dual scan mode: (top right) $f_m$ below frequency of notch; (middle right) $f_m$ at notch frequency; (bottom right) $f_m$ above notch frequency.*

The outer island is populated by a diffusion process, which can be observed

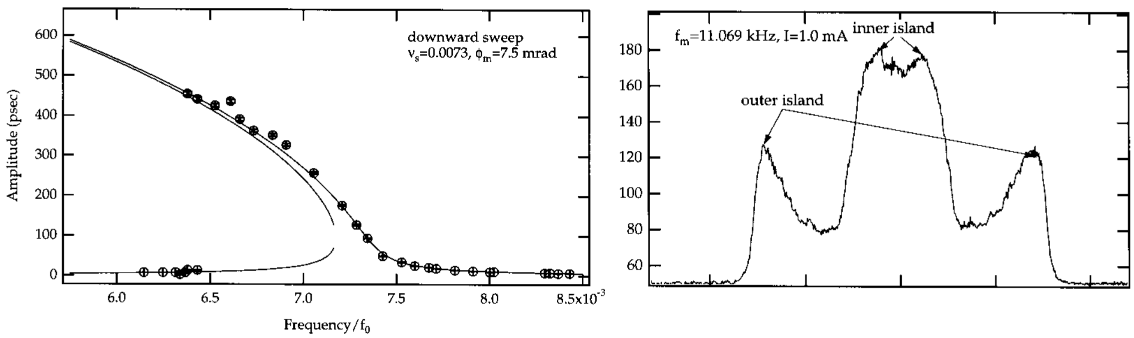


Figure 2: *(Left) response amplitude of driven longitudinal oscillations for 3 excitation amplitudes; the nominal synchrotron frequency $f_0$ is 11.6 kHz. The measured responses are also shown by the circles; (right) longitudinal profile in arb. units averaged over about 200 synchrotron oscillations. The central peaks correspond to the inner island and the two outer peaks to the outer island.*

when the beam is excited at a fixed frequency below the bifurcation. In this case, starting with the entire beam in the inner island, we observe a slow gradual growth in the population of the outer island over a typical time scale of several seconds or minutes. The diffusion is demonstrated in the right picture of Fig. 2, showing the longitudinal profile recorded about 45 s after the phase modulation was initiated. At the start of the modulation the outer island had been empty. We believe that the dominant diffusion mechanism from the inner to the outer island is Touschek scattering. Its magnitude can be estimated by replacing in the standard formula for the Touschek lifetime the rf bucket height with the height of the separatrix between the two islands. Figure 3 compares the prediction so obtained with the measurements. The overall agreement is good; in particular the measured diffusion rate shows a pronounced dependence on the bunch current as expected for intrabeam or Touschek scattering.

Similar measurements have been performed for higher order sub- or superharmonic resonances, *e.g.*, for $\nu_m = 2\nu_s$ and $\nu_m = 3\nu_s$. Also at these resonances bifurcation phenomena are observed, with subharmonic resonances involving a larger number of fixed points. Again, we find a good agreement between the measured island positions and the predicted fixed-point locations.

## 3 Conclusions and Outlook

The nonlinearity of the synchrotron motion has a dramatic effect on longitudinal beam response measurements at large amplitudes. The observed notch in the BTF is

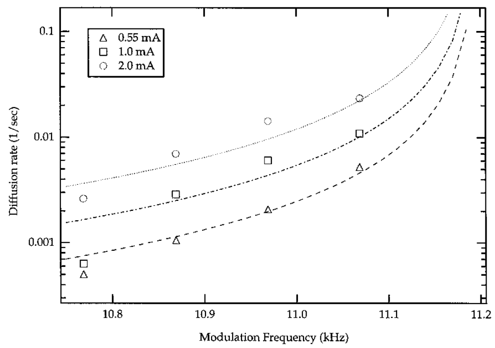


Figure 3: *Measured inter-island diffusion rates (symbols) compared with predicted diffusion due to Touschek scattering (curves) for three different bunch currents, as a function of the modulation frequency.*

caused by two stable islands; the frequency at which the notch occurs depends on the sweep rate, the diffusion rate and the synchrotron frequency. Similar observations were made at the SLC damping rings and at CESR, which suggests that this is a universal phenomenon. Excitation of higher-order resonances at multiples of the synchrotron frequency results in the formation of multiple islands and higher-order bifurcations, which can also be monitored by the streak camera.

In future studies, we intend to use the method described in this report to study effects of higher-order momentum compaction, Touschek and intra-beam scattering, and collective instabilities. In particular, it has been suggested [18] that the observed 2-island formation is intimately related to the longitudinal 'sawtooth' instability in the SLC damping rings. Other promising applications of the swept-frequency excitation include: slow extraction, off-energy injection [19], generation of short bunches [20], and fast energy-aperture measurements.

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