

Bounding the effect of penguin diagrams in $a_{CP}(B^0 \rightarrow \pi^+\pi^-)$

Yuval Grossman and Helen R. Quinn

*Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309*

Abstract

A clean determination of the angle α of the unitary triangle from $B \rightarrow \pi\pi$ decays requires an isospin analysis. If the $B \rightarrow \pi^0\pi^0$ and $\bar{B} \rightarrow \pi^0\pi^0$ decay rates are small it may be hard to carry out this analysis. Here we show that an upper bound on the error on $\sin 2\alpha$ due to penguin diagram effects can be obtained using only the measured rate $\text{BR}(B^\pm \rightarrow \pi^\pm\pi^0)$ and an upper bound on the combined rate $\text{BR}(B \rightarrow \pi^0\pi^0) + \text{BR}(\bar{B} \rightarrow \pi^0\pi^0)$. Since no b flavor tagging is needed to measure this combined rate, the bound that can be achieved may be significantly better than any approach which requires separate flavor-tagged neutral pion information.

Submitted to Phys. Rev. D

I. INTRODUCTION

The extraction of the angle α of the unitary triangle from a measurement of the time dependence CP asymmetry in $B \rightarrow \pi^+\pi^-$ is plagued with uncertainty due to penguin diagrams [1]. This problem can in principle be solved, up to certain discrete ambiguities, by the Gronau-London isospin analysis [2], which require the measurement of all charge and neutral $B \rightarrow \pi\pi$ decays rates. In practice, however, this theoretically clean determination of α may be difficult to achieve. The major problem is expected to be the measurements of $\text{BR}(B \rightarrow \pi^0\pi^0)$ and $\text{BR}(\bar{B} \rightarrow \pi^0\pi^0)$: The two neutral pion final state is harder to detect and reconstruct than states with charged pions; furthermore, arguments base on color suppression [3] predict a smaller branching ratio for this channel than for the two-charged-pion channel.

Many ways were proposed to disentangle the penguin pollution in the determination of α [2,4]. In this note we explain how to set a bound on the error in α induced by the penguin diagram contribution to the CP asymmetry in $B \rightarrow \pi^+\pi^-$. This bound requires the measurement of $\text{BR}(B^\pm \rightarrow \pi^\pm\pi^0)$ and only an upper bound on the combined rate $\text{BR}(B \rightarrow \pi^0\pi^0) + \text{BR}(\bar{B} \rightarrow \pi^0\pi^0)$ in addition to the CP-asymmetry. The fact that we use only the average of the B^0 and \bar{B}^0 rates removes the need for tagging of these low rate events, making this measurement simpler than the measurements of each of the rate separately. The error in α decreases when the upper bound on the combined rate decreases. If penguin contributions are large then they may enhance the $B \rightarrow \pi^0\pi^0$ decay rate. If this is so the full isospin analysis can, hopefully, be carried out. Conversely, if these decay rates are small, the isospin analysis is difficult, but, as we will show, the uncertainty due to penguin effects in the determination of α is small, since the bound on the penguin diagram contribution is stronger. In both cases, one can get a meaningful improvement in the knowledge of α , compared to measuring the charged pion asymmetry alone.

II. DERIVATION OF BOUNDS

We start with definitions. The time-dependent CP asymmetry in B decays into a final CP even state f is defined as [1]

$$a_f(t) \equiv \frac{\Gamma[B^0(t) \rightarrow f] - \Gamma[\bar{B}^0(t) \rightarrow f]}{\Gamma[B^0(t) \rightarrow f] + \Gamma[\bar{B}^0(t) \rightarrow f]}, \quad (2.1)$$

and is given by

$$a_f(t) = a_f^{\cos} \cos(\Delta Mt) + a_f^{\sin} \sin(\Delta Mt), \quad (2.2)$$

with

$$a_f^{\cos} \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad a_f^{\sin} \equiv \frac{-2 \operatorname{Im} \lambda_f}{1 + |\lambda_f|^2}, \quad \lambda_f \equiv \frac{q \bar{A}_f}{p A_f}, \quad (2.3)$$

where p and q are the components of the interaction eigenstates in the mass eigenstates, $A_f(\bar{A}_f)$ is the $B_d(\bar{B}_d) \rightarrow f$ transition amplitude, and we will use $|q/p| = 1$ [1]. The time-dependent measurement can separately extract a_f^{\cos} and a_f^{\sin} . In particular,

$$\sin[\arg(\lambda_f)] = \frac{\operatorname{Im} \lambda_f}{|\lambda_f|}, \quad (2.4)$$

can be determined. Notice, however, that

$$a_f^{\sin} = \frac{-2 \operatorname{Im} \lambda_f}{1 + |\lambda_f|^2} = -\sin[\arg(\lambda_f)] \sqrt{1 - (a_f^{\cos})^2}. \quad (2.5)$$

For $f = \pi^+ \pi^-$, and in the absence of penguin diagrams, $\lambda = e^{2i\alpha}$ and $a_{+-}^{\cos} = 0$ (we use $\lambda \equiv \lambda_{+-}$). Thus, we expect a_{+-}^{\cos} to be small, and difficult to determine accurately. However this quantity only enters quadratically in the correction between $\sin[\arg(\lambda)]$ and a_{+-}^{\sin} . Thus the error in the value of $\sin 2\alpha$ that comes from neglecting this quantity is small. Furthermore, we will show that, for any value of the tree and penguin contributions, the difference between $\sin 2\alpha$ and a_{+-}^{\sin} is maximized for $|\lambda| = 1$ ($a_{+-}^{\cos} = 0$). Hence, our bound is obtained without any dependence on a_{+-}^{\cos} .

We further define

$$\begin{aligned}
A^{+-} &\equiv A(B^0 \rightarrow \pi^+\pi^-), & \bar{A}^{+-} &\equiv A(\bar{B}^0 \rightarrow \pi^+\pi^-), \\
A^{00} &\equiv A(B^0 \rightarrow \pi^0\pi^0), & \bar{A}^{00} &\equiv A(\bar{B}^0 \rightarrow \pi^0\pi^0), \\
A^{+0} &\equiv A(B^+ \rightarrow \pi^+\pi^0), & \bar{A}^{-0} &\equiv A(B^- \rightarrow \pi^-\pi^0).
\end{aligned}
\tag{2.6}$$

Isospin symmetry relates these amplitudes

$$\frac{1}{\sqrt{2}}A^{+-} + A^{00} = A^{+0}, \quad \frac{1}{\sqrt{2}}\bar{A}^{+-} + \bar{A}^{00} = \bar{A}^{-0}, \quad |A^{+0}| = |\bar{A}^{-0}|.
\tag{2.7}$$

These equations can be represented by two triangles with unknown orientation in the complex plane. Moreover, since the charged final state of two pions $\pi^+\pi^0$ is a pure isospin 2 channel and thus receives no gluon penguin diagram contributions, the CP-conjugate amplitudes A^{+0} and \bar{A}^{-0} are equal in magnitude. (Here we neglect the contribution of electroweak penguins, since these are at most at the few percent level [5].) With this approximation we can draw the two triangles with a common base $|A^{+0}|$, see Fig 1. This is the Gronau-London construction.

We note in passing that a test for the size of electroweak penguin effects can be made by looking for direct CP violation in the $B^\pm \rightarrow \pi^\pm\pi^0$ channel since these can only occur due to interference between tree and electroweak penguin terms. While a null effect could be due to vanishing relative strong phase between the tree and electroweak penguin terms, any non-zero effect would be evidence for enhanced electroweak penguin effects, or possible beyond standard-model contributions. Hence it is interesting to search for direct CP violation in this channel, precisely because it is expected to be small in the Standard Model.

Returning to the Gronau-London construction, we further remark that, with the common side A^{+0} for the two triangles, the angle between the sides proportional to $|A^{+-}|$ and $|\bar{A}^{+-}|$ is the difference between $\arg(\lambda)$ and 2α (see Fig. 1). (This is a simple way of stating how the Gronau-London construction allows extraction of a corrected value of 2α .) Measurement of this angle and of the direct CP violating asymmetry a_{+-}^{\cos} in addition to the asymmetry a_{+-}^{\sin} are sufficient to obtain α correctly, independent of the size of the penguin effects. Likewise, it is now a straightforward matter to investigate what constraints on this angle can be obtained from a bound on the sum of the $\pi^0\pi^0$ rates for B^0 and \bar{B}^0 .

To make this explicit we rewrite Eq. (2.5) as

$$a_{+-}^{\sin} = -\sin 2(\alpha + \delta)\sqrt{1 - (a_{+-}^{\cos})^2}, \quad (2.8)$$

where we define 2δ as the angle between $e^{2i\alpha}$ and λ , namely the angle between the $+-$ sides of the two triangles with a common base $|A^{+0}|$ (see Fig. 1). A fourfold ambiguity in 2δ arises because we can flip the orientation of either of the two triangles about the common side. For any set of values of the rates, the larger value of $|2\delta|$, and thus the largest correction to α , occurs when the two triangles are on the opposite sides of the base; in our subsequent derivation of a bound on the correction we will consider only this orientation. Flipping both triangles about their common side reverses the sign of the correction, so our bound will be on the magnitude of the correction, with either sign possible.

We define the combined rate, and the rate ratio

$$\langle \text{BR} \rangle^{00} \equiv \frac{\text{BR}(B^0 \rightarrow \pi^0 \pi^0) + \text{BR}(\bar{B}^0 \rightarrow \pi^0 \pi^0)}{2}, \quad B^{00} \equiv \frac{\langle \text{BR} \rangle^{00}}{\text{BR}(B^+ \rightarrow \pi^+ \pi^0)}. \quad (2.9)$$

We consider what we can learn if a_{+-}^{\sin} and $\text{BR}(B^+ \rightarrow \pi^+ \pi^0)$ are measured, and an upper bound on B^{00} is established. Our goal is to set an upper bound on $|\delta|$. We emphasize that in what follows we always assume that $|\delta|$ is small. Actually, the proofs are correct as long as $|\delta| < \pi/2$. In practice, of course, we hope to get a much tighter bound.

Let us define the angles within the triangles by the labels of the sides that are opposite them, thus ϕ_{00} is the angle opposite to the side of length $|A^{00}|$, etc. Then, $2\delta = \phi_{00} - \bar{\phi}_{00}$. (We use the pion charges as the labels for the sides, and denote angles in the \bar{B} rate triangle by a bar over the name.) We use the sine theorem to write

$$\sin \phi_{00} = \frac{|A_{00}| \sin \phi_{+0}}{|A_{+0}|}, \quad \sin \bar{\phi}_{00} = \frac{|\bar{A}_{00}| \sin \bar{\phi}_{+0}}{|A_{+0}|}. \quad (2.10)$$

First we note that, for a given upper bound on B^{00} , $|\delta|$ has a maximum. Geometrically, the maximum is reached when the two isospin triangle have opposite orientation and they are right triangles: $|\phi^{+0}| = |\bar{\phi}^{-0}| = \pi/2$, so that the sine terms in the right side of Eq. (2.10) are maximal (see Fig. 2). This maximum applies with no knowledge about the values of $|A^{+-}|$ and $|\bar{A}^{+-}|$. Using Eq. (2.10) we get

$$|\sin \phi^{00}| \leq \frac{|A^{00}|}{|A^{+0}|}, \quad |\sin \bar{\phi}^{00}| \leq \frac{|\bar{A}^{00}|}{|A^{+0}|}. \quad (2.11)$$

Using the fact that for fixed $x^2 + y^2$, the maximum value of $|x| + |y|$ occurs for $x = y$, and the definition of B^{00} in Eq. (2.9) we see that $|\delta|$ is maximized when $|\delta| = |\phi_{+0}| = |\bar{\phi}_{+0}|$, and obtain

$$\sin^2 \delta \leq B^{00}. \quad (2.12)$$

This is a general bound on $\sin \delta$. Note also that in this situation the two triangles are congruent, which means that there is no direct CP violation, when this bound is saturated and thus that the bound on δ is achieved when $a_{+-}^{\cos} = 0$, as stated above. The known values of A^{+-} and \bar{A}^{+-} may constrain the correction to be slightly less than this generic maximum, but they cannot make it larger.

While we found the maximum value for δ , we still have to show that the absolute value of

$$\Delta \equiv \sin 2\alpha + a_{+-}^{\sin} = \sin 2\alpha - \sin 2(\alpha + \delta) \sqrt{1 - |a_{+-}^{\cos}|^2}, \quad (2.13)$$

has a maximum at $|\lambda| = 1$. Note that $|\Delta|$ is symmetric for positive and negative a_{+-}^{\sin} . Moreover, since $\sqrt{1 - |a_{+-}^{\cos}|^2} \leq 1$ it is clear that for $|\sin 2(\alpha + \delta)| > |\sin 2\alpha|$ the effect of $|\lambda| \neq 1$ is to reduce $|\Delta|$. Thus, we have to check only the case in which $|\sin 2(\alpha + \delta)| < |\sin 2\alpha|$. In particular, it is enough to check only for $0 \leq \alpha \leq \pi/4$ and $-\alpha \leq \delta \leq 0$. We differentiate Δ with respect to $|\lambda|$, taking into account how our bound on $|\delta|$ is decreased as $|\lambda|$ moves away from 1. We find, keeping $|A^{00}| = |\bar{A}^{00}|$ and using the geometry of the triangle construction, that, near $|\lambda| = 1$, we can write

$$\frac{d\Delta}{d|\lambda|} = (|\lambda| - 1) \frac{\cos(2\alpha + \delta)}{\sin \delta}, \quad \left. \frac{d^2\Delta}{d|\lambda|^2} \right|_{|\lambda|=1} = \frac{\cos(2\alpha + \delta)}{\sin \delta}. \quad (2.14)$$

(In the above we kept $|\lambda| \neq 1$ only when it appears in the combination $|\lambda| - 1$.) Thus, we see that the shift in $\sin 2\alpha$ is extremal at $|\lambda| = 1$. Since we are concerned with the $\sin \delta < 0$ and $\cos(2\alpha + \delta) > 0$ case, it is also clear that this is indeed a maximum.

Eq. (2.12) is the main result of this note. Small improvements to this result can sometimes be made if the actual values of A^{+-} , \bar{A}^{+-} and A^{+0} are inconsistent with the congruent right triangles possibility. From the cosine law for the two triangles, with a little algebra and calculus, one can show a more general bound

$$\sin^2 \delta \leq \frac{(2 - \kappa - \bar{\kappa})(2B^{00} - \kappa^2 - \bar{\kappa}^2)}{4(1 - \kappa)(1 - \bar{\kappa})}, \quad (2.15)$$

where we have defined the ratios

$$1 - \kappa = \frac{|A_{+-}|}{\sqrt{2}|A_{+0}|} \quad 1 - \bar{\kappa} = \frac{|\bar{A}_{+-}|}{\sqrt{2}|A_{+0}|}, \quad (2.16)$$

and thus $|\lambda| = (1 - \bar{\kappa})/(1 - \kappa)$. If the neutral rates are too small to measure this is unlikely to be a significant improvement over the simpler bound stated above. However, it is a completely general result, and if κ or $\bar{\kappa}$ are negative it may provide a slight improvement over Eq. (2.12).

III. CONCLUSION

We wish to stress a few points in our argument leading to Eq. (2.12)

1. Since the general bound was saturated for $|\lambda| = 1$ it does not require a measurement of a_{+-}^{\cos} .
2. Since the general bound was obtained for the congruent right triangles case it does not require measurement of the actual B^0 and \bar{B}^0 to charged pion decay rates, but only the asymmetry a_{+-}^{\sin} , which reduces sensitivity to errors from cuts that remove backgrounds in the B^0 decay channels.
3. Finally, since the bound depends only on the sum of the B^0 and \bar{B}^0 decays to neutral pions it can be determined from untagged data in this channel.

With all these advantages, it is clear that the bound Eq. (2.12) can significantly limit the error on the value of α if a bound on $B^{00} \lesssim 0.1$ can be achieved.

In conclusion, we have shown that a measurements of $\text{BR}(B^+ \rightarrow \pi^+\pi^0)$ and an upper bound on the combined $B^0 \rightarrow \pi^0\pi^0$ and $\bar{B}^0 \rightarrow \pi^0\pi^0$ decay rate can used to bound the penguin diagram induced error on the extraction of $\sin 2\alpha$ from the CP asymmetry, a_{+-}^{\sin} , measurement. The bound takes the simple form

$$a_{+-}^{\sin} = -\sin 2(\alpha + \delta), \quad \sin^2 \delta \leq \frac{\text{BR}(B^0 \rightarrow \pi^0\pi^0) + \text{BR}(\bar{B}^0 \rightarrow \pi^0\pi^0)}{\text{BR}(B^+ \rightarrow \pi^+\pi^0) + \text{BR}(B^- \rightarrow \pi^-\pi^0)}. \quad (3.1)$$

If the B into neutral pion decay rates are too small to be measured, then this bound will provide a determination of the theoretical uncertainty in the value of α extracted from the asymmetry in two-charged-pion modes without the assumption of small penguin diagrams effects.

ACKNOWLEDGMENTS

We thank Zoltan Ligeti and Mihir Worah for helpful discussions.

REFERENCES

- [1] For a review see e.g., Y. Nir, Lectures presented in the 20th SLAC Summer Institute, SLAC-PUB-5874 (1992); Y. Nir and H.R. Quinn, *Ann. Rev. Nucl. Part. Sci.* **42** (1992) 211.
- [2] M. Gronau and D. London, *Phys. Rev. Lett.* **65** (1990) 3381.
- [3] G. Kramer and W.F. Palmer, *Phys. Rev. D* **52** (1995) 6411.
- [4] H.J. Lipkin, Y. Nir, H.R. Quinn and A.E. Snyder, *Phys. Rev. D* **44** (1991) 1454; M. Gronau, *Phys. Lett. B* **265** (1991) 389; A.E. Snyder and H.R. Quinn, *Phys. Rev. D* **48** (1993) 2139; J.P. Silva and L. Wolfenstein, *Phys. Rev. D* **49** (1994) 1151; R. Fleischer and T. Mannel, *Phys. Lett. B* **397** (1997) 269.
- [5] N.G. Deshpande and X.-G. He, *Phys. Rev. Lett.* **74** (1995) 26; 4099(E). M. Gronau, O.F. Hernández, D. London and J.L. Rosner, *Phys. Rev. D* **52** (1995) 6374.

FIGURES

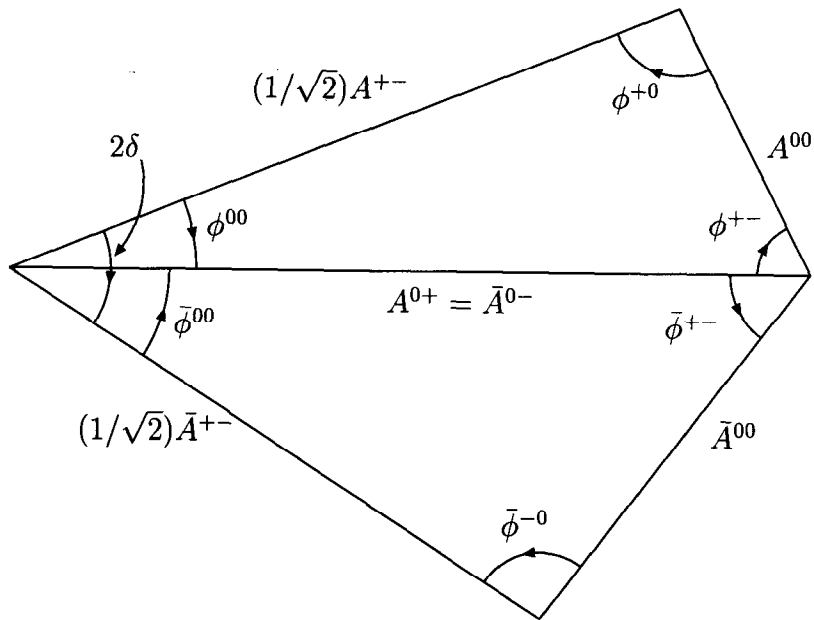


FIG. 1. The isospin triangles of Eq. (2.7).

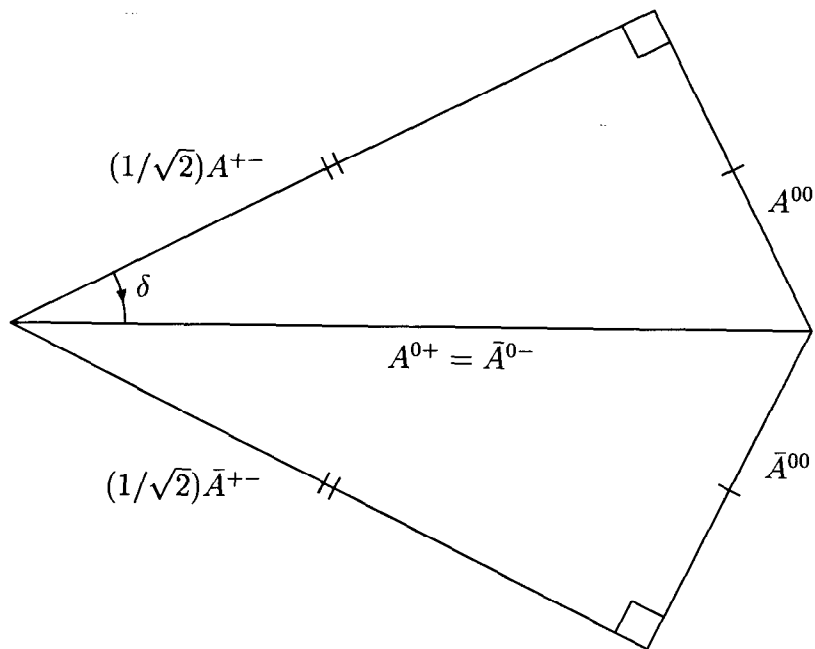


FIG. 2. The isospin triangles in the maximum penguin contribution case.