

SLAC-PUB-7710  
September 1997

## **Closing the Generation Gap**

Eva Silverstein

`evas@slac.stanford.edu`

*Stanford Linear Accelerator Center,  
Stanford University, Stanford, CA 94309*

Presented at the Strings '97 Meeting, 6/19/97—6/21/97, Amsterdam, The Netherlands

---

Work supported by Department of Energy contract DE-AC03-76SF00515.

Chiral fermions play a large role in low-energy particle physics. The fermion mass term in a Lagrangian is given by

$$\mathcal{L}_m = -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L). \quad (1)$$

This ensures that fermions in complex representations of the gauge group (as in the Standard Model) do not have gauge and Lorentz invariant mass terms, as long as the gauge group which distinguishes the left and right-handed fermions remains unbroken.

In string theory, or more generally M theory, chirality is related to the topology of the space on which the strings propagate. For example in the limit of weakly coupled  $E_8 \times E_8$  Heterotic strings, chiral matter in four dimensions is obtained in the following way [1]. To obtain a four-dimensional vacuum, we take the strings to propagate on a spacetime of the form

$$M_4 \times (X_6, V) \quad (2)$$

where  $M_4$  is four-dimensional Minkowski space, and  $(X_6, V)$  is a manifold  $X_6$  with a vector bundle  $V$  satisfying the string equations of motion. For example let us take the components  $A_i$  of the ten-dimensional gauge bosons along  $X_6$  to have vacuum expectation values in  $SU(3)$ . Then the adjoint of  $E_8$  (the representation under which the ten-dimensional gauge fields transform) decomposes under the surviving unbroken gauge group  $E_6$  times the broken  $SU(3)$  as

$$248 \rightarrow (\mathbf{1}, \mathbf{8}) + (\mathbf{78}, \mathbf{1}) + (\mathbf{27}, \mathbf{3}) + (\overline{\mathbf{27}}, \overline{\mathbf{3}}). \quad (3)$$

So the net number of generations of  $E_6$  is

$$N_{gen} = n_{27}^R - n_{27}^L = n_{\mathbf{3}}^+ - n_{\mathbf{3}}^- = Index_{\mathbf{3}}(\mathcal{D}_{X_6}). \quad (4)$$

where  $\mathcal{D}_{X_6}$  is the gauge-covariant Dirac operator on  $X_6$ . In other words, the chirality in the four-dimensional Minkowski space is related to the chirality on the compactification manifold. Now  $N_{gen}$  is a topological invariant of  $(X_6, V)$ . Therefore it cannot change under smooth deformations of  $(X_6, V)$ .

This leads to two apparent (related) obstructions to unifying M-theory vacua, which in general have different net generation numbers: (i) We have seen that in order to change  $N_{gen}$  the compactification must become singular, since the Dirac index must change. More seriously perhaps, (ii) From (1) we saw that chirality change cannot happen in ordinary low-energy field theory (i.e. weakly coupled Lagrangian field theory), where chiral matter cannot get lifted.

Recent progress has uncovered sensible physical resolutions to singularities in compactifications, so (i) is not a problem in and of itself. Indeed, we now know how to interpret

many examples of singular geometries (i.e. singular solutions to classical general relativity). There are examples [2][3][4] of topology change at the classical level in string theory. This is possible because point particle classical spacetime geometry is at best approximately valid in the large-radius limit. One can achieve topology change by going through regions in the moduli space of the  $2d$  CFT defining the string vacuum where  $\alpha'$  corrections and worldsheet instanton effects are large so classical geometry breaks down. The singularity can be avoided in going between phases of the compactification with different topology at large radius.

More similar to our situation are cases where in order to effect topology change one must grapple with a singularity which remains after all classical stringy corrections are included. The lesson has been that physics is nonsingular as long as all light degrees of freedom associated with the singularity are taken into account. This has been a central principle in analysis of supersymmetric gauge theory dynamics [5][6]. The phenomenon has been similarly observed in string theory compactifications with  $N = 4$  [7],  $N = 2$  [8], and  $N = 1$  [9] supersymmetry in four dimensions. All of these examples of singularity resolution involved conventional low-energy quantum field theories realized in various string compactifications.

As for problem (ii), the impossibility of lifting chiral matter in Lagrangian field theory, the loophole is to consider regions of moduli space in which the low-energy effective quantum field theory is not weakly coupled. Many nontrivial interacting fixed point quantum field theories have been found in the moduli space of various compactifications of M-theory. We find [10] examples in which the net number of generations changes upon going through a locus in the moduli space where the effective theory is a nontrivial RG fixed point.

## 1. Review of chirality change in six dimensions

Indeed, something similar occurs in six dimensional (1,0) supersymmetric theories [11][12] obtained by considering M theory compactified on  $S^1/Z_2 \times K3$ . At the end of the interval  $S^1/Z_2$  live ten-dimensional  $E_8$  gauge bosons [13]. In general their components along  $K3$  comprise the connection of a holomorphic vector bundle  $\tilde{V}$  over  $K3$ , in other words a configuration of instantons living on  $K3$ . In order to have a perturbative heterotic description, we need 24 instantons on  $K3$  at the end of the interval. Let us take all the instantons to lie in an  $SU(2)$  subgroup of  $E_8$ , leaving an unbroken  $E_7$  gauge group in spacetime.

The spectrum consists of  $20 \frac{1}{2} \mathbf{56}$ 's of  $E_7$  and 65 singlet moduli (45 moduli of the instanton bundle and 20 moduli of  $K3$ ). Each instanton has a scale size modulus. Let

us consider shrinking a collection of instantons to zero size. As explained in [11][12] the shrunk instantons correspond to M-theory fivebranes at the end of the interval. There is then another phase in which the M-theory fivebranes move off into the eleven-dimensional bulk.

The spectrum in this second phase includes for each fivebrane a  $(1,0)$  tensor multiplet, whose real scalar parameterizes the distance of the fivebrane from the end of the interval, as well as a hypermultiplet whose four scalars give its position on  $K3$ . In order to shrink an instanton and move into this second phase, one loses the other collective coordinates of the Yang-Mills instanton: one hypermultiplet containing the scale size and one  $\frac{1}{2}\mathbf{56}$  of  $E_7$ . This is in accord with the anomaly condition which requires the addition of one tensor to be compensated by losing 29 hypermultiplets. As explained in [12], this transition changing the number of tensor multiplets cannot happen in weakly coupled Lagrangian field theory. For the four-dimensional application we are interested in here, the main result we will need from six dimensions is the loss of charged matter (one  $\frac{1}{2}\mathbf{56}$  per instanton).

## 2. Changing $N_{gen}$

We can return to four-dimensional physics by considering M-theory on  $S^1/Z_2 \times (X_6, V)$  where  $X_6$  is taken to be a  $K3$  fibration, and  $V$  as above is taken to be an  $SU(3)$  bundle. In other words,  $(X_6, V)$  has the structure of a family of  $(K3, \tilde{V})$ 's varying over a  $\mathbf{CP}^1$  base. The four-dimensional theory is a sort of "twisted" dimensional reduction of the six dimensional theory studied in §2 down to four dimensions. The massless spectrum is obtained by finding zero eigenstates of the Dirac operator on the  $\mathbf{CP}^1$  base, where the Dirac operator includes contributions arising from the variation of the fiber  $(K3, \tilde{V})$  over the base.

In [9][10] we studied a set of  $K3$  fibrations using the linear sigma model approach [3] for the heterotic compactifications. In particular, in [10] we found the following pattern in the four dimensional spectrum. Each  $\frac{1}{2}\mathbf{56}$  in the six dimensional theory descends to chiral matter (either  $\mathbf{27}$ 's of  $E_6$  or a  $\overline{\mathbf{27}}$  of  $E_6$ ). For more details on the analysis of the spectrum, see the paper [10]. In addition, there is charged matter that does not descend from the six dimensional theory, in other words matter that is associated to the singular fibers.

Now we can shrink one of the instantons in the generic fiber theory  $(K3, \tilde{V})$ , and move it off the end of the interval as an M-theory fivebrane wrapped on the base  $\mathbf{CP}^1$ . Recall from §2 that in the corresponding six dimensional theory we lost a  $\frac{1}{2}\mathbf{56}$  in the transition. This means that in four dimensions we lose the *chiral* matter that descends from the  $\frac{1}{2}\mathbf{56}$ .

At the origin between the two branches there is, as in six dimensions, a nontrivial interacting fixed point theory. In six dimensions, the presence of stringlike BPS states

coming down to zero tension at the origin signals the presence of a CFT there. This remains true after our fibration down to four dimensions: in particular, the base  $\mathbf{CP}^1$  does not admit string winding modes so the unwrapped strings remain the lightest degrees of freedom as we approach the origin.

One can identify the charged matter that gets removed in the transition using the linear sigma model description of singularities [3]. At the singularity in the vector bundle that we studied above, the linear sigma model target space becomes noncompact, developing a long tube. The vertex operators can be identified [14], and those which are supported down this tube are chiral (i.e. either  $\mathbf{27}$ 's or  $\overline{\mathbf{27}}$ 's, but not both). The throat carries the information about the singularity and its nonperturbative resolution by a nontrivial interacting conformal field theory. So only the states which are supported down the throat are involved in the transition.

The singular fibers also do not appear to change this result, for the following reasons. Generically, the singularities in the singular fibers are at points on the  $K3$ . These points are generically separated from the singularity induced by shrinking the instanton. Furthermore an F-theory analysis of the  $6d$  theory shows that removing the small instanton from the end of the interval does not introduce additional singularities.

So the conclusion is that the net number of generations  $N_{gen}$  changes in the transition!

### 3. Instanton Effects

So far we have been doing essentially a Kaluza-Klein analysis, i.e. looking at scales below  $1/R$  where  $R$  is the radius of the base  $\mathbf{CP}^1$ , but ignoring instanton effects. There are ordinary heterotic string instantons arising from fundamental heterotic string worldsheets wrapping the base  $\mathbf{CP}^1$   $B$ . In eleven dimensions, this corresponds to the membrane world-volume stretching between the ends of the interval and wrapping  $B$ . With the fivebrane in the middle of the interval, we have two additional types of strings: those arising from the membrane stretched between the fivebrane and either end of the interval.

The fundamental string instanton effects go like  $e^{-\frac{R^2}{\alpha'}}$ . The others go like  $e^{-R^2\phi}$  and  $e^{-\frac{R^2}{\alpha'}+R\phi}$ , where  $\phi$  is the dimensional reduction of the scalar in the  $6d$  tensor multiplet. Instanton contributions depend on zero-mode counting, which depends on how  $V$  restricts to  $B$ . There are cases for which instanton effects do not contribute to the superpotential for singlet moduli. In these cases the phase transition proceeds at zero energy as described above.

There are other cases for which there are nontrivial instanton effects. These can be understood by considering a T-dual  $SO(32)$  heterotic description in three dimensions (after

compactifying the above on a circle of radius  $r$ ). There the small instanton singularity is nonperturbatively resolved by a gauge symmetry enhancement. Each small instanton carries an  $SU(2)$  gauge group with matter in the **(32, 2)**. The number of surviving doublets in four dimensions depends on how  $V$  restricts to  $B$  [9].

In the case where two flavors survive the superpotential was determined in [15] to be

$$W = W_{tree} - Y P f V + e^{-\frac{R^2}{\alpha'}} Y. \quad (3.1)$$

where  $Y = e^{R^2\phi}$  for large  $R^2\phi$ , and  $V_{ij}$  are gauge-invariant coordinates on the moduli space. The  $3d$  and  $4d$  gauge couplings are related by  $\frac{1}{r g_3^2} = \frac{1}{g_4^2} = \frac{R^2}{\alpha'}$ . We are interested in the limit  $r \rightarrow 0$ ,  $\frac{R^2}{\alpha'}$  fixed, in order to return to the four-dimensional compactification of the  $E_8 \times E_8$  heterotic string that we have been discussing. As in [9], with an appropriate  $W_{tree}$  one can reproduce the pole in the Yukawa couplings which occurs for these compactifications.

In this case there is a term  $|P f V - e^{-\frac{R^2}{\alpha'}}|^2$  in the potential energy. In order to shrink an instanton (which here corresponds to taking  $V \rightarrow 0$ ) there is a cost in energy proportional to  $\frac{1}{\sqrt{\alpha'}} e^{-2\frac{R^2}{\alpha'}}$ . So in this case the transition can occur, but only by going over a small energy barrier.

## 4. Conclusions

We have explained how the naive obstructions to unifying vacua with different net generation numbers can be overcome, and we gave a class of examples of chirality-changing phase transitions in four dimensions (see also the recent examples of [16] in orbifold models). The phenomenon exhibited here presumably occurs quite generally; it would be very interesting to understand whether in fact one can connect all vacua with  $N \leq 1$  supersymmetry at low energies. For this an F-theory analysis could be quite instructive, once that approach is developed more fully for four-folds (see e.g. [17][18]).

Perhaps the next “in principle” question to ask along these lines is whether there could be any physical process which changes the number of supersymmetries. At low energies, any such transition would necessarily involve gravity, since the number of gravitinos would have to change in the transition. The moduli space of theories with  $N \geq 4$  supersymmetry is so constrained that there does not seem to be any room for such a phase transition at low energies (the singularities in these moduli spaces are all accounted for in weakly coupled Lagrangian field theory at low energies). This pushes the question to high energies, where we probably need a background-independent formulation of M-theory to really address it; perhaps the Matrix theory [19] can provide some clues.

## Acknowledgements

This talk was based on a paper [10] with S. Kachru. I am also indebted to O. Aharony, P. Aspinwall, T. Banks, P. Candelas, J. Distler, M. Douglas, O. Ganor, A. Hanany, J. Louis, N. Seiberg, S. Shenker, and E. Witten for discussions on this and related topics. I would also like to thank the organizers of Strings '97 for a very stimulating conference.

## References

- [1] M. Green, J. Schwarz, and E. Witten, *Superstring Theory*, v. 2, Cambridge University Press (1987).
- [2] P. Aspinwall, B. Greene, and D. Morrison, "Calabi-Yau Moduli Space, Mirror Manifolds and Spacetime Topology Change in String Theory", Nucl.Phys. B416 (1994) 414-480, hep-th/9309097.
- [3] E. Witten, "Phases of N=2 Theories in Two Dimensions", Nucl.Phys. B403 (1993) 159-222, hep-th/9301042.
- [4] J. Distler and S. Kachru, "(0,2) Landau-Ginsburg Theory", Nucl. Phys. B413 (1994) 213, hep-th/9309110.
- [5] K. Intriligator and N. Seiberg, "Lectures on supersymmetric gauge theories and electric-magnetic duality", Nucl.Phys.Proc.Suppl. 45BC (1996) 1-28, hep-th/9509066.
- [6] N. Seiberg and E. Witten, "Electric-Magnetic Duality, Monopole Condensation, and Confinement in N=2 Supersymmetric Yang-Mills Theory", Nucl.Phys. B426 (1994) 19-52, hep-th/9407087.
- [7] see e.g. E. Witten, "String Theory Dynamics in Various Dimensions", Nucl.Phys. B443 (1995) 85-126, hep-th/9503124.
- [8] see e.g. P. Green and T. Hubsch, "Phase Transitions among (Many of) Calabi-Yau Compactifications", Phys. Rev. Lett. 61 (1988) 1163; P. Candelas, P. Green, and T. Hubsch, "Finite Distances between Distinct Calabi-Yau Vacua", Phys. Rev. Lett. 62 (1989) 1956; A. Strominger, "Massless Black Holes and Conifolds in String Theory", Nucl. Phys. B451 (1995) 96, hep-th/9504090; B. Green, D. Morrison, and A. Strominger, "Black Hole Condensation and the Unification of String Vacua", Nucl. Phys. B451 (1995) 109, hep-th/9504145.
- [9] see e.g. S. Kachru, N. Seiberg, and E. Silverstein, "SUSY Gauge Dynamics and Singularities of 4d N=1 String Vacua", Nucl.Phys. B480 (1996) 170-184, hep-th/9605036; Matthias Klein and Jan Louis, "Singularities in d=4, N=1 Heterotic String Vacua", hep-th/9707047.
- [10] S. Kachru and E. Silverstein, "Chirality Changing Phase Transitions in 4d String Vacua", hep-th/9704185.
- [11] O. Ganor and A. Hanany, "Small  $E_8$  Instantons and Tensionless Non-critical Strings", Nucl.Phys. B474 (1996) 122-140, hep-th/9602120.
- [12] N. Seiberg and E. Witten, "Comment on String Dynamics in Six Dimensions", Nucl.Phys. B471 (1996) 121-134, hep-th/9603003.
- [13] P. Horava and E. Witten, "Heterotic and Type I String Dynamics from Eleven Dimensions", Nucl.Phys. B460 (1996) 506-524, hep-th/9510209.
- [14] E. Silverstein and E. Witten, "Criteria for Conformal Invariance of (0,2) Models", B444 (1995) 161, hep-th/9503212.



- [15] O. Aharony, A. Hanany, K. Intriligator, N. Seiberg, M.J. Strassler, "Aspects of N=2 Supersymmetric Gauge Theories in Three Dimensions", Nucl.Phys. B499 (1997) 67-99, hep-th/9703110.
- [16] G. Aldazabal, A. Font, L.E. Ibanez, A.M. Uranga, G. Violero, "Non-Perturbative Heterotic D=6,4 Orbifold Vacua", hep-th/9706158.
- [17] Robert Friedman, John Morgan, and Edward Witten, "Vector Bundles And F Theory", Commun.Math.Phys. 187 (1997) 679-743, hep-th/9701162.
- [18] Michael Bershadsky, Andrei Johansen, Tony Pantev and Vladimir Sadov, "On Four-Dimensional Compactifications of F-Theory", hep-th/9701165.
- [19] T. Banks, W. Fischler, S. Shenker, and L. Susskind, "M Theory As A Matrix Model: A Conjecture", Phys.Rev. D55 (1997) 5112-5128, hep-th/9610043.