Probing the Flavor and CP Structure of Supersymmetric Models with $K \to \pi\nu\bar{\nu}$ Decays

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Abstract

We study the implications of various supersymmetric models on the rare $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ decays. Although large effects are possible in generic supersymmetric models, most of the known supersymmetric flavor models lead to negligible effects. Thus, it is likely that one can get information about CKM matrix elements from these decays even in the presence of supersymmetry. Moreover, the possibility of large contributions to $K \to \pi\nu\bar{\nu}$ in generic supersymmetry models can be constrained by improved bounds on $D - \bar{D}$ mixing. We show that it may be possible to distinguish between different supersymmetric flavor models by combining the information from the $K \to \pi\nu\bar{\nu}$ decays with that from $B - \bar{B}$ and $D - \bar{D}$ mixing.

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I. INTRODUCTION

It has been realized in recent years that the flavor and CP structure of supersymmetric theories might be very rich. Measurements of flavor and CP violating processes may become a sensitive probe of the soft supersymmetry breaking parameters and, consequently, of the mechanism of dynamical supersymmetry breaking. For example, various supersymmetric flavor models give different predictions for the electric dipole moment of the neutron d_N, for CP asymmetries in B decays (e.g. $a_{\psi K_S}$, the CP asymmetry in $B \to \psi K_S$), for CP violation in $D - \bar{D}$ mixing, as well as the $D - \bar{D}$ mass difference (Δm_D) [1]. In this work we study in detail the implications of various classes of supersymmetric flavor models for the rare $K \to \pi \nu \bar{\nu}$ decays. In particular, $\mathrm{BR}(K^+ \to \pi^+ \nu \bar{\nu})$ depends on the flavor structure of the model, while the ratio

$$a_{\pi \nu \bar{\nu}} \equiv \frac{\Gamma(K_L \to \pi^0 \nu \bar{\nu})}{\Gamma(K^+ \to \pi^+ \nu \bar{\nu})} \tag{1.1}$$

depends also on the mechanism of CP violation [2]. We show that combining the predictions for $a_{\pi \nu \bar{\nu}}$ with $a_{\psi K_S}$ and Δm_D allows us to potentially discriminate between the various supersymmetric flavor models.

The $K \to \pi \nu \bar{\nu}$ decay is generated by four fermion operators of the form $\bar{s} d \nu \nu$. The calculation is essentially clean of hadronic uncertainties [3–5]. Defining θ to be the relative phase between the $s \to d \bar{\nu} \nu$ decay amplitude and the $K - \bar{K}$ mixing amplitude, then $a_{\pi \nu \bar{\nu}} = \sin^2 \theta$ [6]. In the Standard Model, the $K - \bar{K}$ mixing amplitude is dominated by box diagrams with intermediate charm and up quarks. The $s \to d \bar{\nu} \nu$ decay amplitude gets significant contributions from both Z penguins and box diagrams. The dominant contribution in the Standard Model is proportional to m_t^2, coming from diagrams with top quarks in the loops. Had these been the only important contributions, we would have $a_{\pi \nu \bar{\nu}} = \sin^2 \beta$, where β is the CKM angle, $\beta \equiv \arg\left[-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right]$. However, there is also a smaller but non-negligible charm-loop contribution proportional to m_c^2, with the larger CKM matrix elements present there compensating for the m_c^2 / m_t^2 suppression. (Both the charm and the top quark amplitudes

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contribute to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ rate, whereas the CP violating decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ only gets a contribution from the dominant top quark amplitude.) The relation between $a_{\pi \nu \bar{\nu}}$ and $\sin^2 \beta$ is modified, but one still gets a clean determination of the CKM angle β [7].

The $K \rightarrow \pi \nu \bar{\nu}$ decays in supersymmetric models have been studied before [8–11]. Our analysis has a different emphasis from the previous analyses:

(i) Most of these studies have analyzed models of exact universality [8,9,11]. (In [10], they allowed an arbitrary flavor structure; however they did not study the contribution from chargino penguins, which we find to be the potentially dominant one.) Recently, various predictive and viable mechanisms for naturally suppressing the supersymmetric flavor violation have been suggested which do not assume exact universality. Our main focus is put on these recent models that have a much richer flavor structure.

(ii) All previous studies have only studied the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay. We study also the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay. This is important since the latter is also sensitive to the supersymmetric CP violation. Furthermore, various experimental proposals were recently made to measure this challenging mode and it is not unlikely that the rates of both decays will be known in the future.*

(iii) We emphasize the theoretical strength of combining the information from $a_{\pi \nu \bar{\nu}}$ and $a_{\psi K_S}$ [13]. Within the Standard Model, these two quantities are strongly correlated because they both depend on β (see Fig. 1). In most classes of supersymmetric flavor models, this relation is violated. Moreover, looking for deviations in the patterns of flavor physics like comparing $a_{\pi \nu \bar{\nu}}$ with $a_{\psi K_S}$ is independent of the hadronic uncertainties that enter the constraints on β and, furthermore, of the effects that new physics might have on determining the CKM parameters, unlike the predictions for the rates which sensitively depend on them.

We find that most of the known supersymmetric flavor models have a negligible effect on $a_{\pi \nu \bar{\nu}}$. Therefore, $a_{\pi \nu \bar{\nu}}$ is likely to give us a clean measurement of β even in the presence

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*Recently, first evidence for the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay was presented by the E787 Collaboration [12].

of supersymmetry. In contrast, many supersymmetric flavor models generally give large contributions to $B_d - \bar{B}_d$ mixing, resulting in the fact that we no longer have $a_{\psi K_S} = \sin 2\beta$. While this new contribution may be hard to detect from just a comparison of $a_{\psi K_S}$ to the presently allowed range for $\sin 2\beta$ [14], it is likely to be signalled by comparing to the range that will be allowed by a measurement of $a_{\pi\nu\bar{\nu}}$.

We further note that it is possible to generate large corrections to the $K \to \pi\nu\bar{\nu}$ decay rates and to $a_{\pi\nu\bar{\nu}}$ in R parity conserving models if one allows an arbitrary flavor structure in the supersymmetric sector. However, in this case, large contributions to the $K \to \pi\nu\bar{\nu}$ decays are often accompanied by large, detectable contributions to $D - \bar{D}$ mixing. Thus, improved bounds on $D - \bar{D}$ mixing would further constrain the possibility of large supersymmetric contributions to the $K \to \pi\nu\bar{\nu}$ decays. (For the future prospects of searching for $D - \bar{D}$ mixing, see *e.g.* [15].)

II. SUPERSYMMETRIC CONTRIBUTIONS TO $K \to \pi\nu\bar{\nu}$

A. General Considerations

The dominant new contributions to the $K \to \pi\nu\bar{\nu}$ decays in supersymmetric models come from Z-mediated penguin diagrams, with supersymmetric particles inducing the effective $\bar{s}dZ$ coupling. Integrating out the Z, leads to the relevant $\bar{s}d\bar{\nu}\nu$ four fermion operator.

The analysis is simplified by noting the following points.

1. The effect is always proportional to $SU(2)_L$ breaking. In the absence of $SU(2)_L$ breaking, the corrections to the $\overline{s_L}d_L Z$ ($\overline{s_R}d_R Z$) coupling are proportional to the corrections to the $\overline{s_L}d_L\gamma$ ($\overline{s_R}d_R\gamma$) coupling, which vanish at $s = 0$, where s is the four-momentum squared of the intermediate boson. This is the source of the m_t^2 factor in the dominant Standard Model contribution to $K \to \pi\nu\bar{\nu}$. In the supersymmetric framework, wino-higgsino mixing or $\tilde{q}_L - \tilde{q}_R$ mixing can provide the necessary $SU(2)_L$ breaking insertion. Note that the $SU(2)_L$ breaking contributions generate corrections that do not decouple when the masses

of the fermions in the loop are much larger than $\sqrt{s}$ [16].

2. Magnetic moment type couplings of the form $\overline{s_L} d_R Z$ are proportional to the masses of the fermions on the external legs, and therefore are unimportant.

A closely related calculation is that of the supersymmetric contributions to the effective $b\bar{b}Z$ coupling. Theoretically, both vertices have exactly the same structure (modulo their dependence on different flavor mixing matrix elements), one difference being that the effective $b\bar{b}Z$ vertex for R_b is evaluated at $s = M_Z^2$, whereas the $s\bar{d}Z$ vertex relevant for $K \to \pi \nu \bar{\nu}$ is to be evaluated at $s = 0$. Thus, it may be possible in some models to constrain possible new contributions to one from the other observable.

These observations lead to the following expectations:

(i) There is a potentially non-negligible contribution from the charged Higgs – top loop [17]. This contribution is proportional to the top quark mass squared. It can reach $\sim 15\%$ of the Standard Model amplitude for $\tan\beta \sim 1$, and a charged Higgs mass $m_{H_+} \sim 300$ GeV. This contribution can be enhanced by either smaller values of $\tan\beta$ or a smaller charged Higgs mass. However, such an enhancement is disallowed by its correlated contribution to the $\overline{b_L} b_L Z$ vertex resulting in a large negative correction to R_b [18,19]. It is also disfavored by constraints from the $b \to s\gamma$ decay rate [20]. Although both of these constraints can be avoided by invoking cancellations with chargino-squark diagrams, significant cancellations are not generic and require fine-tuning. This contribution scales like $1/\tan^2\beta$ and, consequently, its significance decreases rapidly for larger values of $\tan\beta$. Note that the charged Higgs amplitude and the Standard Model top amplitude are in phase and interfere constructively. Therefore, in the parts of parameter space where the charged Higgs contribution is significant and, furthermore, there are no other significant new contributions, the importance of the charm contribution is weakened and, consequently, $a_{\pi\nu\bar{\nu}}$ is predicted to lie between the Standard Model prediction and $\sin^2\beta$.

(ii) Gluino – down squark penguins are negligible. To get a non-decoupling contribution

to the $\overline{s_L}d_L Z$ or $\overline{s_R}d_R Z$ vertex, two A_d-insertions are required.[†] Since left-right mixing in the down squark sector is proportional to down quark masses, the contribution is very small.

(iii) The chargino – up squark penguins generate only the $\overline{s_L}d_L Z$ vertex since both the wino coupling and the large top Yukawa enhanced higgsino coupling (for small $\tan\beta$) are only to left-handed down-type quarks. The relevant $SU(2)_L$ breaking insertion in the loop involves either A_t, namely $\tilde{t}_L - \tilde{t}_R$ mixing, or $\tilde{w}^+ - \tilde{h}^+$ mixing. The size of this contribution depends sensitively on the flavor structure of the model. In models where the squark mass matrix is diagonal (up to $\tilde{t}_L - \tilde{t}_R$ mixing) in the Super-CKM basis, the contribution is small, typically $\lesssim 5\%$ of the Standard Model amplitude for super-partner masses $\gtrsim 200$ GeV. Furthermore, it is in phase with the Standard Model top quark contribution and can partially cancel the charged Higgs contribution. Allowing super partner masses to be as small as possible consistent with data could slightly enhance this contribution.

Introducing flavor violation into the (RR) squark mass-squared matrix does not enhance the contribution since the higgsino couplings of the first and second generation squarks are suppressed by factors of m_u and m_c respectively. However, in supersymmetric models with misalignment between the (LL) squark and quark mass matrices, large effects are possible, in principle, with new CP violating phases (see [11] for a short discussion of this point).

Thus we learn that:

(a) Without SUSY flavor violation but with light ($\lesssim 100$ GeV) super partner masses, the SUSY contribution can reach 10% of Standard Model and it is always in phase with the Standard Model top penguin.

(b) With SUSY flavor violation in mass-squared matrices for 'left-handed' up-type squarks, the SUSY contribution could be as large as the Standard Model contribution and with an arbitrary phase.

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[†]We disagree with the calculation of Ref. [10], where non-zero effects from gluino penguins with one flavor changing (RR) mass insertion are found.

B. Phenomenological Constraints

The relevant flavor violating couplings could also affect other FCNC processes such as neutral meson mixing [21], and $b \to s\gamma$ [22]. Since, as argued above, a potentially dominant contribution to the $K \to \pi\nu\bar{\nu}$ decays can come from chargino – up squark penguins, we study flavor violation arising from the mass-squared matrix $m^2_{U_L}$ for the 'left-handed' up squarks. An important point is that the effective second family to first family $(2 \to 1)$ flavor changing transition in the up-type squark mass matrix required for the $K \to \pi\nu\bar{\nu}$ decay in many cases also leads to large $D - \bar{D}$ mixing from box diagrams with gluinos and the up-type squarks.

In our calculations we use the results of [18] (appropriately generalized to allow for arbitrary flavor structure) to calculate the effective $\bar{s}dZ$ coupling. We have checked that our results for the Wilson coefficients of the $\bar{s}d\bar{\nu}\nu$ four fermion operator agree with those presented in [20]. To get a feel for the size of the flavor violations allowed, we use the following parameters: m_0, M_3, M_2, μ, $A \sim 200 - 300$ GeV and $\tan\beta = 2$.[‡] The interesting diagrams are those where the $SU(2)_L$ breaking is introduced by higgsino – wino mixing while the flavor violation is introduced by one of the following four options:

(i) $(m^2_{U_L})_{12}$ insertion: This gives a large contribution to $K - \bar{K}$ mixing, and one obtains the limit $\frac{|(m^2_{U_L})_{12}|}{\tilde{m}^2} \lesssim 0.1$. Values close to this bound lead to large $D - \bar{D}$ mixing by gluino box diagrams [23,24]. Moreover, one can obtain contributions to the $K \to \pi\nu\bar{\nu}$ amplitude of the order of the Standard Model contribution consistent with this bound. Similarly large contributions to the $K_L \to \mu^+\mu^-$ decay are obtained. Note that $\text{Im}[(m^2_{U_L})_{12}]^2$ is constrained

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[‡]A mass insertion analysis applied to chargino diagrams requires some care. Besides the obvious dependence on $\tan\beta$, in special cases, large off-diagonal mass terms in the squark mass matrix can be cancelled by factors of CKM matrix elements that are present in the $d_L - \tilde{u}_L - \tilde{w}$ vertex in the Super-KM basis. An example of this arises in certain U(1) models of alignment [23,24] in which there is a basis where all the mass matrices except the up quark mass matrix are almost exactly diagonal.

to be vanishingly small by ε_K.

(ii) $(m_{U_L}^2)_{13}$ insertion and V_{ts} factor in the wino coupling: The strongest constraint is from $B_d - \bar{B}_d$ mixing, leading to $\frac{|(m_{U_L}^2)_{13}|}{\tilde{m}^2} \lesssim 0.3$. This does not contribute to $D - \bar{D}$ mixing (since the supersymmetric contribution to the latter involves gluinos). One obtains supersymmetric contributions to $K \to \pi \nu \bar{\nu}$ of order 10%-20% of the Standard Model amplitude.

(iii) $(m_{U_L}^2)_{23}$ insertion and V_{td} factor in the wino coupling: This is constrained by the $b \to s\gamma$ rate: $\frac{|(m_{U_L}^2)_{23}|}{\tilde{m}^2} \lesssim 0.7$. This again does not contribute to $D - \bar{D}$ mixing but can, however, lead to 30%-50% effects in the $K \to \pi \nu \bar{\nu}$ decay amplitude.

(iv) $(m_{U_L}^2)_{13}(m_{U_L}^2)_{23}$ insertion: This gives a large contribution to $D - \bar{D}$ mixing. The experimental bound on Δm_D leads to the bound $\sqrt{\frac{|(m_{U_L}^2)_{13}|}{\tilde{m}^2} \frac{|(m_{U_L}^2)_{23}|}{\tilde{m}^2}} \lesssim 0.3$. We can obtain contributions to the $K \to \pi \nu \bar{\nu}$ amplitude of 50% the Standard Model contribution consistent with this bound. Again, ε_K constrains $\text{Im}[(m_{U_L}^2)_{13}(m_{U_L}^2)_{23}]$ to be vanishingly small.

We collect the results above in Table 1. The constraints quoted in this table are imposed by demanding the supersymmetric contribution to measured quantities to be less than (a factor of two) the Standard Model contribution and to quantities where only an upper bound exists to be less than this bound.

Before concluding this section let us emphasize that the considerations above generically apply to the coefficient of the $\overline{s_L} d_L \overline{\nu_L} \nu_L$ four fermion operator. Although one can trivially obtain from these the corrections to the $K^+ \to \pi^+ \nu \bar{\nu}$ decay rate (i.e. it is proportional to the absolute value squared), the rate for the $K_L \to \pi^0 \nu \bar{\nu}$ decay depends also on the CP structure of the model (i.e. it is proportional to the imaginary part squared). In particular, the ratio $a_{\pi \nu \bar{\nu}}$, defined in Eq. (1.1), is sensitive to the supersymmetric CP violation and, in many cases, only weakly dependent on the flavor violation.

III. SUPERSYMMETRIC FLAVOR MODELS

In this section, we apply the above general analysis to specific classes of models. The classification of models is explained in detail in ref. [1].

Insertion	Constraint	Limit	$\frac{\Delta m_D}{2\times10^{-13}\,GeV}$	$\frac{A(K^+\to\pi^+\nu\bar\nu)^{\text{SUSY}}}{A(K^+\to\pi^+\nu\bar\nu)^{\text{SM}}}$
$(m^2_{U_L})_{12}$	Δm_K	0.1	0.1	1
$(m^2_{U_L})_{13}$	Δm_{B_d}	0.3	0	0.2
$(m^2_{U_L})_{23}$	$b\to s\gamma$	0.7	0	0.5
$\sqrt{(m^2_{U_L})_{13}(m^2_{U_L})_{23}}$	Δm_D	0.3	1	0.5

TABLE I. Constraints on the flavor violation in the "left-handed" up-type squark mass-squared matrix, and the largest possible contributions to Δm_D and $K \to \pi\nu\bar\nu$ consistent with these constraints. We have used $\tan\beta = 2$, a universal scalar mass of 200 GeV, and chargino and gluino masses in the same range. The quoted limits are on the ratio between $(m^2_{U_L})_{ij}$ and the typical supersymmetric mass scale $\tilde{m}^2$. The bounds from neutral meson mixing scale like $\times \left(\frac{\tilde{m}}{200\ GeV}\right)$. The bound from $b \to s\gamma$ scales like $\times \left(\frac{\tilde{m}}{200\ GeV}\right)^2$. $A(K^+ \to \pi^+\nu\bar\nu)^{\text{SUSY}}$ scales like $\times \left(\frac{200\ GeV}{\tilde{m}}\right)^2$.

(i) Exact Universality: At some high energy scale, all squark masses are universal and the A terms are proportional to the corresponding Yukawa couplings. Then the Yukawa matrices represent the only source of flavor (and possibly of CP) violation which is relevant in low energy physics. There exists the supersymmetric analogue of the GIM mechanism which operates in the Standard Model. Flavor violations can feed into the soft terms via renormalization group evolution. The only significant supersymmetric effect is $\tilde{t}_L - \tilde{t}_R$ mixing. This was discussed early in the paper. The contribution to the $K \to \pi\nu\bar\nu$ amplitudes could reach 10% for very light super particle masses. There is no significant effect on any of the other observables.

(ii) Approximate CP: The supersymmetric CP problems are solved if CP is an approximate symmetry broken by a small parameter of order 10^{-3}. The flavor structure of this class of models is not well defined. If we just assume that the flavor violations could saturate the upper bounds from FCNC processes, then all of the observables above could get significant contributions. In particular, there could be a strong enhancement of BR($K^+ \to \pi^+\nu\bar\nu$).

However, since all CP violation is of order $\varepsilon_K \sim 10^{-3}$, BR($K_L \to \pi^0 \nu\bar\nu$) will not be similarly enhanced. Instead, we expect $a_{\pi\nu\bar\nu} \sim 10^{-3}$. CP violation in B and D decays is also expected to be of order 10^{-3}.

(iii) Abelian H (alignment): The squark mass-squared matrices have a structure, but they have a reason to be approximately diagonal in the basis set by the quark mass matrix. This is achieved in models of Abelian horizontal symmetries [23–25]. In these models $(m^2_{U_L})_{12}$ is required to be $\sim \theta_C$ (in the super-CKM basis), necessarily leading to large $D - \bar{D}$ mixing. However, in processes involving external down-type quarks, there is a cancellation by factors of CKM matrix elements that are present in the $d_L - \tilde{u}_L - \tilde{w}$ vertex in the Super-KM basis. If this cancellation is the weakest consistent with the Δm_K constraint, that is, in the interaction basis

$$\frac{(m^2_{U_L})_{12}}{\tilde{m}^2} \sim 0.05, \tag{3.1}$$

then a large contribution to the $K^+ \to \pi^+ \nu\bar\nu$ rate is predicted. This could lead to either enhancement or suppression, depending on the sign of the supersymmetric amplitude. However, in such a case, the new contribution is essentially in phase with the $K - \bar{K}$ mixing amplitude (to satisfy the ε_K constraint). Consequently, $K_L \to \pi^0 \nu\bar\nu$ will *not* be affected: it will get contributions from the Standard Model diagrams only. In all existing models of alignment, the situation is different. The ε_K problem is solved by an almost exact cancellation between the $(m^2_{U_L})_{12}$ insertion and the flavor changing $d_L - \tilde{u}_L - \tilde{w}$ vertex insertion. This requires $\frac{(m^2_{U_L})_{12}}{\tilde{m}^2} \leq 0.004$, but in some explicit examples the suppression is even stronger ($\sim 3 \times 10^{-5}$). Under such circumstances, the supersymmetric contributions to the $K \to \pi\nu\bar\nu$ rates are negligibly small. Moreover couplings of the third family via (13) or (23) insertions are too small to have any effect.

(iv) Non-Abelian H (approximate universality): A non-Abelian horizontal symmetry is imposed, where quarks of the light two families fit into an irreducible doublet. The resulting splitting among the squarks of these families is very small, leading to essentially no flavor violation in the $(LL)_{12}$ sector. The third family supermultiplets are, however, in singlets of

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the horizontal symmetry allowing for much larger flavor violating effects. We have examined several models of this type [26–28]. We find that, similarly to the models of Abelian horizontal symmetries discussed above, the mixing between the third generation and the first two is not of order one but rather of the order of the corresponding CKM elements:

$$\frac{(m_{U_L}^2)_{13}}{\tilde{m}^2} \sim |V_{ub}|, \quad \frac{(m_{U_L}^2)_{23}}{\tilde{m}^2} \sim |V_{cb}|. \tag{3.2}$$

Such insertions are too small to lead to significant effects in either $K \to \pi\nu\bar{\nu}$ or $D - \bar{D}$ mixing.

(v) Heavy squarks: The flavor problems can be solved or, at least, relaxed if the masses of the first and second generation squarks m_i are larger than the other soft masses, $m_i^2 \sim 100\tilde{m}^2$ [29–32]. This does not necessarily lead to naturalness problems, since these two generations are almost decoupled from the Higgs sector. A detailed study of the implications on flavor and CP violation is given in ref. [33]. In the mass basis, the gluino interaction mixing angles Z_{ij}^q are constrained by naturalness:

$$|Z_{13}^u| \le \max\left(\frac{m_{\tilde{Q}_3}}{\tilde{M}}, |V_{ub}|\right), \quad |Z_{23}^u| \le \max\left(\frac{m_{\tilde{Q}_3}}{\tilde{M}}, |V_{cb}|\right). \tag{3.3}$$

The ratio $m_{\tilde{Q}_3}/\tilde{M}$ is of order 1/20 in these models. The mass of the third generation squarks is of order 1 TeV. Then diagrams with third generation squarks give only small contributions. The situation regarding Z_{12}^u is less clear. Some mechanism to suppress FCNC in the first two generations (beyond the large squark masses) is necessary in order to satisfy the Δm_K constraint (not to mention the ε_K constraint). Even if we assume only mild alignment, so that $Z_{12}^u \sim \sin\theta_C$ then, for squark mass of order 20 TeV, the contribution is small. (One can get order 20% contributions to $K \to \pi\nu\bar{\nu}$ and large $D - \bar{D}$ mixing if there is not only large mixing between the first two generation squarks, but also their mass scale is lower than ~ 4 TeV. But then some extra ingredients are required to explain the smallness of ε_K in $K - \bar{K}$ mixing.)

IV. DISCUSSION AND COMMENTS

The situation of $K \to \pi \nu \bar{\nu}$ decays in supersymmetric models is very interesting. It is possible to construct models where there are significant new contributions to these modes. However, these models have in general a rather contrived flavor structure and, moreover, fine-tuned CP violating phases. Consequently, in most supersymmetric models where the flavor and CP problems are solved in a *natural* way, the $K \to \pi \nu \bar{\nu}$ modes get only small new contributions (of order 5% or less of the Standard Model amplitude). In many cases, these new contributions are in phase with the dominant Standard Model amplitude, the top quark penguin and box diagrams. Thus, $a_{\pi \nu \bar{\nu}}$ provides a measurement of the angle β of the unitarity triangle to an accuracy of order 5% even in the presence of supersymmetry. The possible exceptions to this statement are models of approximate CP, where $a_{\pi \nu \bar{\nu}} \sim 10^{-3}$.

The combination of measurements of $K \to \pi \nu \bar{\nu}$ decay rates with CP violation in neutral B decays (and in $D - \bar{D}$ mixing) may provide a particularly sensitive probe of supersymmetry. The interesting point is that, within the Standard Model, the CP asymmetry in $B \to \psi K_S$, $a_{\psi K_S}$, and the CP asymmetry in $K \to \pi \nu \bar{\nu}$, that is $a_{\pi \nu \bar{\nu}}$ defined in ref. (1.1), are both devoid of hadronic uncertainties, and related to the angle β of the unitarity triangle. Therefore, the Standard Model predicts a well-defined relation between the two. Furthermore, $D - \bar{D}$ mixing is predicted to be vanishingly small in the Standard Model. In some classes of supersymmetric flavor models, these predictions do not necessarily hold:

(i) Exact universality: The contributions to $B - \bar{B}$ mixing are small (up to $O(0.2)$ of the Standard Model) and with no new phases, so there is no effect on $a_{\psi K_S}$. Similarly, the contributions to $K \to \pi \nu \bar{\nu}$ are small (of $O(0.1)$ of the Standard Model) and in phase with the top amplitude, so there is no effect on $a_{\pi \nu \bar{\nu}}$. Also $D - \bar{D}$ mixing is not affected.

(ii) Approximate CP: CP violating phases are small. Therefore we have $a_{\psi K_S} \lesssim 10^{-3}$, independent of whether SUSY contributions to the mixing are large or not. Similarly, we have $a_{\pi \nu \bar{\nu}} \lesssim 10^{-3}$ independent of whether $K^+ \to \pi^+ \nu \bar{\nu}$ gets a significant contribution or not.

(iii) Alignment: If the relevant squark masses (say the sbottom) are $\leq O(300\ GeV)$, then the SUSY contribution to $B - \bar{B}$ mixing can be $O(1)$. There are arbitrary new CP phases, so $a_{\psi K_S}$ could differ significantly from the standard model. Moreover, large contributions to $D - \bar{D}$ mixing are generic in these models. In existing models, the contributions to $K \to \pi \nu \bar{\nu}$ decays are small and $a_{\pi \nu \bar{\nu}}$ is not affected. (However, it may be possible to construct alignment models with a larger $(m^2_{U_L})_{12}$, leading to a large contribution to $K \to \pi \nu \bar{\nu}$. The ε_K constraint requires that it is in phase with the Standard Model charm contribution, thus changing the overall phase of the amplitude, and the Standard Model expectation for $a_{\pi \nu \bar{\nu}}$.)

(iv) Approximate universality: The situation with regard to $a_{\psi K_S}$ is similar to models of alignment. There is no effect on $a_{\pi \nu \bar{\nu}}$ or on $D - \bar{D}$ mixing.

(v) Heavy squarks: Third generation squark masses are $O(1\ TeV)$ and the first two generations are $O(20\ TeV)$. But the mixing angles in the gaugino couplings to (s)quarks can be large, so SUSY contributions to FCNC processes are potentially significant. In particular, if CP phases are large, then large effects in $a_{\psi K_S}$ are possible. But if the ε_K problem is solved by small phases, then a situation similar to models of approximate CP might arise. The effect on the $K \to \pi \nu \bar{\nu}$ decays is generally small.

Based on this discussion, we list in Table 2 possible flavor physics signals that could help us to distinguish between the different SUSY flavor models. Thus we see that combining $a_{\pi \nu \bar{\nu}}$ with $a_{\psi K_S}$ and Δm_D, allows us to partially distinguish between the various supersymmetric flavor models. Moreover, a clean determination of β from $a_{\pi \nu \bar{\nu}}$ may allow us to unambiguously detect new contributions to $B - \bar{B}$ mixing.

Finally, we stress that in an arbitrary supersymmetric model, where one just tunes all flavor violating parameters to be consistent with the phenomenological constraints, it is possible to obtain deviations from the Standard Model predictions that are different from those that appear in any of the classes of models discussed above. In particular, $a_{\pi \nu \bar{\nu}}$ itself can be strongly modified even if CP is not an approximate symmetry. The absence of large $D - \bar{D}$ mixing could serve to rule out this possibility. Also, in this case, the pattern of CP asymmetries in B_s decays might deviate significantly from both the Standard Model and the

Model	$a_{\pi\nu\bar\nu} = a_{\psi K_S}$	$\Delta m_D > 10^{-14} GeV$
Standard Model	Yes	No
Universality	Yes	No
Approximate CP	0	——
Alignment	No	Yes
Approx. Universality	No	No
Heavy Squarks	No	No
Arbitrary	——	——

TABLE II. Possible outcomes for flavor and CP violation in different models. By $a_{\psi K_S} = a_{\pi\nu\bar\nu}$ we do not mean that they are equal but rather that they are consistent with the same value of β, namely within the allowed range of Fig. 1. A blank entry implies that there is no specific prediction.

existing supersymmetric flavor models. Therefore, measurements of FCNC and CP violating processes may not only exclude the Standard Model, but also require that we examine again the flavor structure of supersymmetry.

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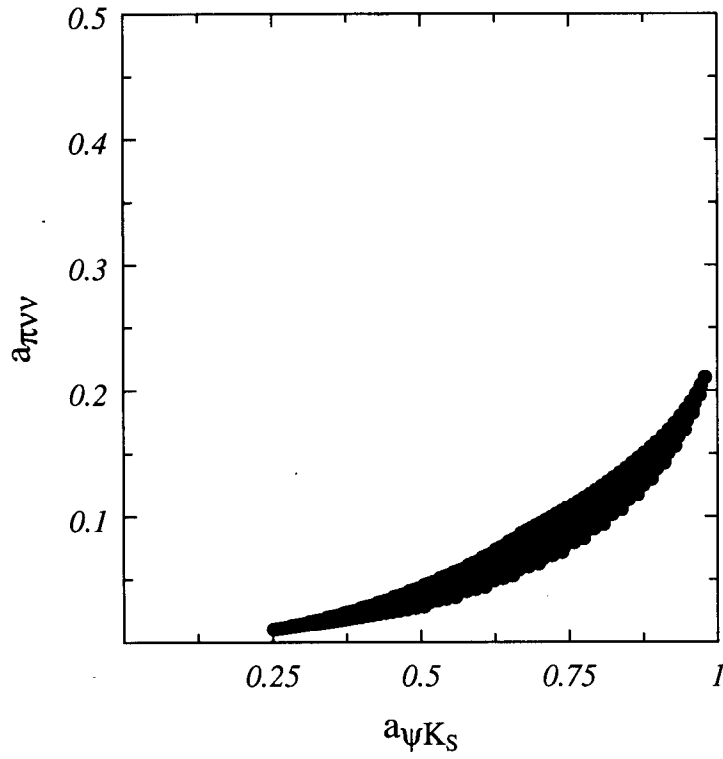[33] A.G. Cohen, D.B. Kaplan, F. Lepeintre and A.E. Nelson, Phys. Rev. Lett. **78**, 2300 (1997), hep-ph/9610252.

Figure 1. The Standard Model allowed region in the $a_{\psi K_S}$- $a_{\pi\nu\bar\nu}$ plane. We have used $-0.25 \leq \rho \leq 0.40$, $0.16 \leq \eta \leq 0.50$ [13].