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RESULTS ON α_s AND QCD FROM (AND ABOVE) THE Z^0 *

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ABSTRACT

Measurements of α_s from e^+e^- annihilation experiments are reviewed and compared with measurements from other processes. Highlights are presented of recent QCD studies in e^+e^- annihilation at the Z^0 resonance.

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1 Introduction

In electron-positron annihilation hadronic activity is, by construction, limited to the final state, making the study of hadronic events cleaner and simpler relative to lepton-hadron and hadron-hadron collisions, from both the experimental and theoretical points-of-view. On the experimental side there are no remnants of the beam particles to add confusion to the interpretation of hadronic structures, and, apart from initial and final-state photon radiation effects, the hadronic centre-of-mass frame coincides with the laboratory frame. On the theoretical side the absence of hadrons in the incoming beams removes dependence on the limited knowledge of the parton density functions of hadrons, as well as rendering QCD calculations at a given order of perturbation theory easier to perform because there are generally fewer strong-interaction Feynman diagrams to consider.

To be specific, samples of hadronic events can be selected by experiments at the Z^0 resonance with efficiency and purity of better than 99%. Jet and event-shape observables have been calculated at next-to-leading order, $O(\alpha_s^2)$, and some inclusive observables have been calculated at $O(\alpha_s^3)$. Non-perturbative calculations, in the form of ‘power corrections’ to perturbatively-evaluated observables, have been performed, and there are well-understood models of hadronisation that have been carefully tuned to the data collected over the past 20 years from experiments at the SPEAR, CESR/DORIS, PETRA/PEP, TRISTAN, and SLC/LEP colliders. Electron-positron annihilation thus provides an ideal environment for precise tests of QCD, and has yielded spectacular results which include the first observation of jets at SPEAR, the direct observation of the gluon at PETRA, and the detailed study of jets of different flavour at SLC and LEP.

It is impossible to review this wealth of information in the time allotted here; for a more pedagogical review see Ref. [1]. Instead I have chosen some ‘highlights of the past decade’ that I feel have significantly advanced our understanding of strong-interaction physics in the years since PETRA and PEP. Ten years ago

the world’s highest-energy e^+e^- data sample comprised a few hundred thousand events clustered around a c.m. energy $Q \sim 30$ GeV. Since then roughly 16 million hadronic events have been collected at the Z^0 resonance, and there are, in addition, recent data taken at energies as high as 172 GeV. Furthermore, whereas 10 years ago most QCD studies were ‘flavour blind’, the advent of precise silicon-based vertex detectors has made the separation of light (u,d,s), c and b events, with high efficiency and purity, relatively straightforward today.

I have arbitrarily divided my selection of highlights into the 3-jet and 4-jet sectors. The former includes precise measurements of α_s and study of the running of α_s , tests of the flavour-independence of strong interactions, measurement of quark- and gluon-jet differences, and the search for T_N -odd effects. The latter category includes the 4-jet cross-section and angular correlations among the jets, which are relevant for searches for possible beyond-Standard Model (SM) particles known as light gluinos.

2 Measurements of α_s in e^+e^- Annihilation

A Theoretical Considerations

The theory of strong interactions, Quantum Chromodynamics (QCD), contains in principle only one free parameter, the strong coupling α_s . QCD can hence be tested in a quantitative fashion by measuring α_s in different processes and at different hard scales Q . In practice most QCD calculations of observables are performed using finite-order perturbation theory, and calculations beyond leading order depend on the renormalisation scheme employed. It is conventional to work in the modified minimal subtraction scheme (\overline{MS} scheme) [2], and to use the strong interaction scale $\Lambda_{\overline{MS}}$ for five active quark flavours. If one knows $\Lambda_{\overline{MS}}$ one may calculate the strong coupling $\alpha_s(Q^2)$ from the solution of the QCD renormalisation group equation [3]:

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_{\overline{MS}}^2)} \left\{ 1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln(\ln(Q^2/\Lambda_{\overline{MS}}^2))}{\ln(Q^2/\Lambda_{\overline{MS}}^2)} + \dots \right\} \quad (1)$$

Because of the large data samples taken in e^+e^- annihilation at the Z^0 resonance, it has become conventional to use as a yardstick $\alpha_s(M_Z^2)$, where M_Z is the mass of the Z^0 boson; $M_Z \approx 91.2$ GeV. Tests of QCD can therefore be quantified in terms of the consistency of the values of $\alpha_s(M_Z^2)$ measured in different experiments. Measurements of α_s have been performed in e^+e^- annihilation, hadron-hadron collisions, and deep-inelastic lepton-hadron scattering, covering a range of Q^2 from roughly 1 to 10^5 GeV²; for a recent review see Ref. [4]. Some excitement in this area has been generated in recent years due to claims of an ‘ α_s crisis’, see *eg.* Ref. [5], with potential implications for beyond-Standard-Model physics.

In e^+e^- annihilation $\alpha_s(M_Z^2)$ has been measured from inclusive observables relating to the Z^0 lineshape and to hadronic decays of the τ lepton, as well as from jet-related hadronic event shape observables, and scaling violations in inclusive hadron fragmentation functions.

B R and the Z^0 Lineshape

For the inclusive ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$, the SM electroweak contributions are well understood theoretically and the perturbative QCD series has been calculated up to $O(\alpha_s^3)$ [6] for massless quarks, and up to $O(\alpha_s^2)$ including quark mass effects [7]. Closely-related observables at the Z^0 resonance are: the Z^0 total width, Γ_Z , the pole cross section, $\sigma_h^0 \equiv 12\pi\Gamma_{ee}\Gamma_{had}/M_Z^2\Gamma_Z^2$, and the ratio of hadronic to leptonic Z^0 decay widths $R_l \equiv \Gamma_{had}/\Gamma_l$. These are all related to the Z^0 hadronic width:

$$\Gamma_{had} = 1.671 \left(1 + a_1 \left(\frac{\alpha_s}{\pi} \right) + a_2 \left(\frac{\alpha_s}{\pi} \right)^2 + a_3 \left(\frac{\alpha_s}{\pi} \right)^3 + \dots \right) \quad (2)$$

where: $a_1 = 1$, $a_2 = 0.75$ and $a_3 = -15.3$.

The procedure adopted [8] is to perform a global SM fit to a panoply of electroweak data that includes the W boson and top quark masses as well as the Z^0 lineshape, left-right production asymmetry, branching ratios to heavy quarks, forward-backward asymmetries of final-state fermions, and polarisation of final-state τ s. The free parameters are the Higgs mass, M_{Higgs} , and $\alpha_s(M_Z^2)$. Data presented at the 1996 summer conferences yield the results shown in Fig. 1 [8], updated for the 1997 Winter conferences to yield [9] the positively-correlated results $M_{Higgs} = 127_{-72}^{+127}$ GeV and

$$\alpha_s(M_Z^2) = 0.120 \pm 0.003 \text{ (exp.)} \pm 0.002 \text{ (theor.)} \quad (3)$$

The $\alpha_s(M_Z^2)$ value is lower than the corresponding results presented at the 1995 conferences [10], $\alpha_s(M_Z^2) = 0.123 \pm 0.005$, and at the 1994 conferences [11], $\alpha_s(M_Z^2) = 0.125 \pm 0.005$. The change between 1995 and 1996 is due to a combination of shifts in the values of the Z^0 lineshape parameters, redetermined in light of the recalibration of the LEP beam energy due to the ‘TGV effect’ [8], and a change in the central value of M_{Higgs} at which $\alpha_s(M_Z^2)$ is quoted, from 300 GeV (1995) to the fitted value 127 GeV (1997). Studies of theoretical uncertainties imply [9] that they contribute at the level of ± 0.002 on $\alpha_s(M_Z^2)$. Since data-taking at the Z^0 resonance has now been completed at the LEP collider the precision of this result is not expected to improve further.

C Hadronic τ Decays

An inclusive quantity similar to R is the ratio R_τ of hadronic to leptonic decay branching ratios, B_h and B_l respectively, of the τ lepton:

$$R_\tau \equiv \frac{B_h}{B_l} = \frac{1 - B_e - B_\mu}{B_e} \quad (4)$$

where B_e and B_μ can either be measured directly, or deduced from a measurement of the τ lifetime τ_τ . In addition, a family of observables known as ‘spectral moments’, R_τ^{kl} , of the invariant mass-squared s of the hadronic system has been

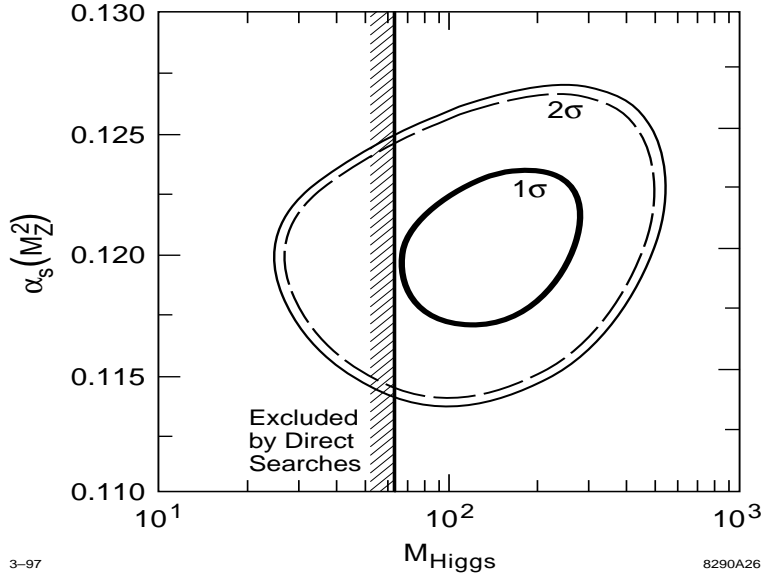


Figure 1: Results of a global fit of the Standard Model to electroweak observables [8]; the 1- and 2-standard deviation contours are shown in the $\alpha_s(M_Z^2)$ vs. M_{Higgs} plane.

proposed [12]. R_τ and R_τ^{kl} have been calculated perturbatively up to $O(\alpha_s^3)$. However, because $M_\tau \sim 1$ GeV one expects (eq. (1)) $\alpha_s(M_\tau) \sim 0.3$ and it is not *a priori* obvious that the perturbative calculation can be expected to be reliable, or that the non-perturbative contributions of $O(1/M_\tau)$ will be small. In recent years a large theoretical effort has been devoted to this subject; see *eg.* Refs. [12, 13, 14].

The ALEPH Collaboration derived R_τ from its measurements of B_e , B_μ , and τ_τ , and also measured the (10), (11), (12), and (13) spectral moments. A combined fit yielded [15] $\alpha_s(M_Z^2) = 0.124 \pm 0.0022 \pm 0.001$, where the first error receives equal contributions from experiment and theory, and the second derives from uncertainties in evolving α_s across the c and b thresholds. The OPAL Collaboration measured R_τ from B_e , B_μ , and τ_τ , and derived [16] $\alpha_s(M_Z^2) = 0.1229^{+0.0016}_{-0.0017}$ (exp.) $^{+0.0025}_{-0.0021}$ (theor.). The CLEO Collaboration measured the same four spectral moments as ALEPH and also derived R_τ using 1994 Particle Data Group values

for B_e , B_μ and τ_τ . A combined fit yielded [17] $\alpha_s(M_Z^2) = 0.114 \pm 0.003$. This central value is slightly lower than the ALEPH and OPAL values. If more recent world average values of B_e and B_μ are used CLEO obtains a higher central $\alpha_s(M_Z^2)$ value [17]. Averaging the second CLEO result and the ALEPH and OPAL results by weighting with the experimental errors, assuming they are uncorrelated, yields:

$$\alpha_s(M_Z^2) = 0.122 \pm 0.001 \text{ (exp.)} \pm 0.002 \text{ (theor.)}. \quad (5)$$

This is nominally a very precise measurement, although recent studies have suggested that additional theoretical uncertainties may be as large as ± 0.006 [18].

D Hadronic Event Shape Observables

The rate of 3-jet production, $R_3 \equiv \sigma_{3\text{-jet}}/\sigma_{had}$, is directly proportional to α_s . More generally one can define other infra-red- and collinear-safe measures of the topology of hadronic final states; for a discussion see *eg.* Ref. [19]. Such observables are constructed to be directly proportional to α_s at leading order, and so are potentially sensitive measures of the strong coupling. The $O(\alpha_s^2)$ QCD prediction for each of these observables X can be written [20]:

$$\frac{1}{\sigma_0} \frac{d\sigma}{dX} = A(X) \left(\frac{\alpha_s}{2\pi} \right) + B(X) \left(\frac{\alpha_s}{2\pi} \right)^2 \quad (6)$$

so that α_s can be determined from each. Though these observables are intrinsically highly correlated, by using many one can attempt to maximise the use of the information in complicated multi-hadron events, and in some sense make a more demanding test of QCD than by using only one or two observables. Moreover, the study of many observables is essential, as it exposes systematic effects. Finally, the α_s determination from hadronic event shape observables is based on the information content within 3-jet-like events, and is essentially uncorrelated with the measurements from the Z^0 lineshape which are based on event-counting of predominantly 2-jet-like final states.

The technology of this approach has been developed over the past 15 years of analysis at the PETRA, PEP, TRISTAN, SLC and LEP colliders, so that the method is considered to be well understood both experimentally and theoretically. Note, however, that before they can be compared with perturbative QCD predictions, it is necessary to correct the measured distributions for any bias effects originating from the detector acceptance, resolution, and inefficiency, as well as for the effects of initial-state radiation and hadronisation, to yield ‘parton-level’ distributions.

A fit of $O(\alpha_s^2)$ perturbative QCD to 15 of these observables is shown in Fig. 2 [19]. It yields the distressing result that the $\alpha_s(M_Z^2)$ values so determined are not internally consistent with one another! A measure of the scatter among the results is given by the r.m.s. deviation of ± 0.008 , which is much larger than the experimental error of ± 0.003 on a typical observable. One can explain the scatter as an artefact of the fact that an $O(\alpha_s^2)$ calculation was employed, and argue that the data tell us that higher-order terms are needed in order to obtain consistent results.

A consensus has arisen among experimentalists that the effect of such missing higher-order terms can be estimated from the dependence of $\alpha_s(M_Z^2)$ on the value of the renormalisation scale μ assumed in fits of the calculations to the data, and a renormalisation scale uncertainty is often quoted. An estimate [19] of the renormalisation scale uncertainty for each observable is shown in Fig. 2b. It is apparent that the scale uncertainty is much larger than the experimental error, and that the $\alpha_s(M_Z^2)$ values are consistent within these uncertainties. Though this is comforting, in that it indicates that QCD is self-consistent, the necessary addition of large theoretical uncertainties to otherwise precise experimental measurements is frustrating.

For the hadronic event shape observables $O(\alpha_s^3)$ contributions have not yet been calculated completely. However, for six observables improved calculations can be formulated that incorporate the resummation [21] of leading and next-to-

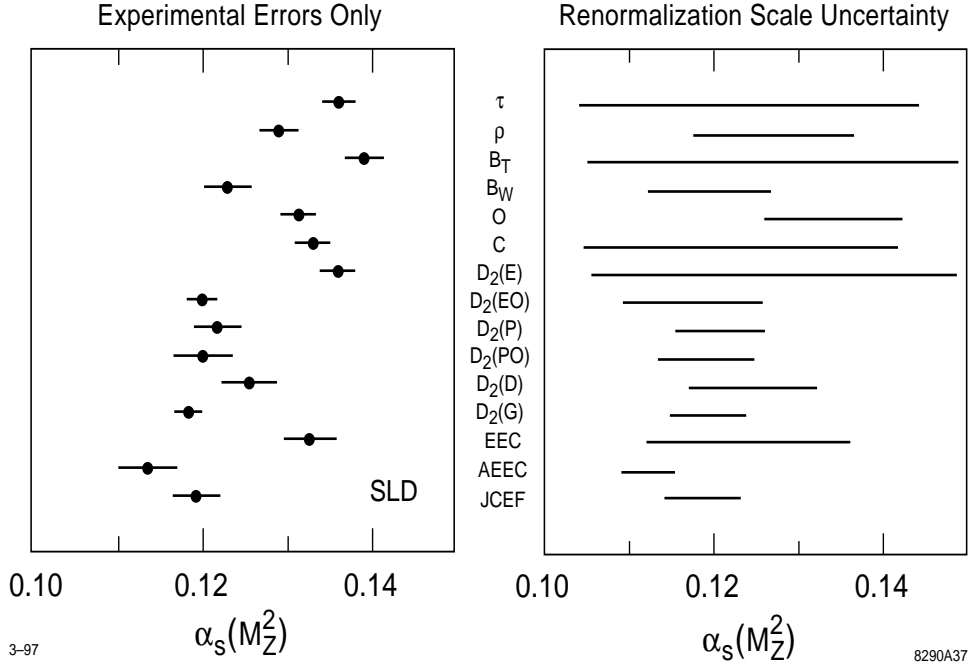


Figure 2: (a) Values of $\alpha_s(M_Z^2)$ determined [19] by fitting $O(\alpha_s^2)$ QCD predictions to 15 hadronic event shape observables using a fixed value of the renormalisation scale $\mu = Q$; the results are clearly inconsistent within the experimental errors. (b) Renormalisation scale uncertainties.

leading logarithmic terms matched to the $O(\alpha_s^2)$ results. The matched calculations are expected *a priori* both to describe the data in a larger region of phase space than the fixed-order results, and to yield a reduced dependence of α_s on the renormalization scale, both of which have indeed been found [19]. Application of other approaches to circumvent the scale ambiguity in α_s measurement, involving the use of ‘optimised’ perturbation theory [22] and Padé Approximants [23], can be found in Refs. [24, 25] respectively.

Hinchliffe has reviewed the various hadronic event shapes-based measurements from experiments performed in the c.m. energy range $10 \leq Q \leq 91$ GeV, utilising both $O(\alpha_s^2)$ and resummed calculations, and quotes an average value of $\alpha_s(M_Z^2) =$

0.122 ± 0.007 [3], where the large error is dominated by the renormalisation scale uncertainty, which far exceeds the experimental error of about ± 0.002 . The LEP experiments have applied similar techniques to determine α_s from their high-energy data samples at $Q = 133, 161$ and 172 GeV; results from OPAL are shown in Fig. 3 [26]. These data points add considerable lever-arm to tests of the running of α_s . Schmelling’s recent compilation of α_s measurements using event shapes includes the 133 GeV results, and yields [27] a global average:

$$\alpha_s(M_Z^2) = 0.121 \pm 0.005,$$

in agreement with Ref. [3], but assuming a more aggressive scale uncertainty.

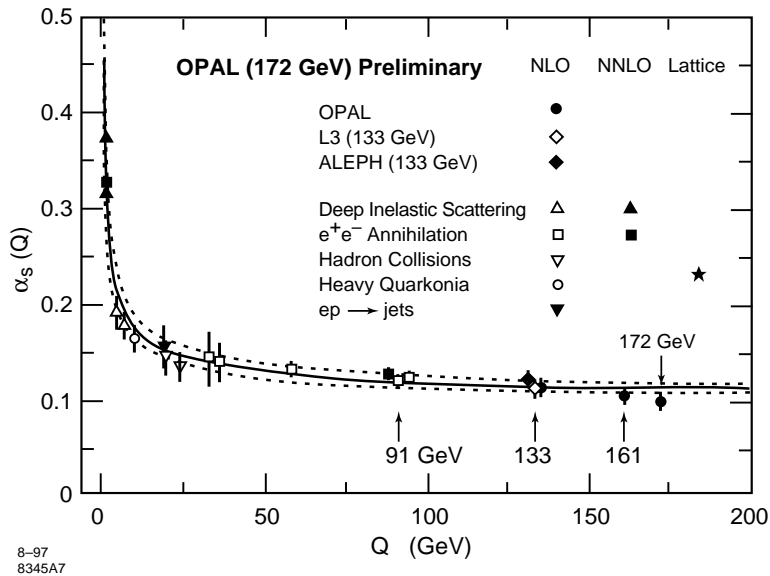


Figure 3: Measurements of α_s as a function of Q , including the latest high-energy data points [26].

Finally, in a recent analysis the L3 Collaboration has utilised Z^0 events with a hard radiated final-state photon, which reduces the effective c.m. energy available to the hadronic system, to examine the Q -evolution of four event shape observables in the range $30 \leq Q \leq 86$ GeV. By comparing with resummed +

$O(\alpha_s^2)$ calculations they derived [28] values of α_s in each energy bin (Figure 4), which are consistent with $\alpha_s(M_Z^2) = 0.121$.

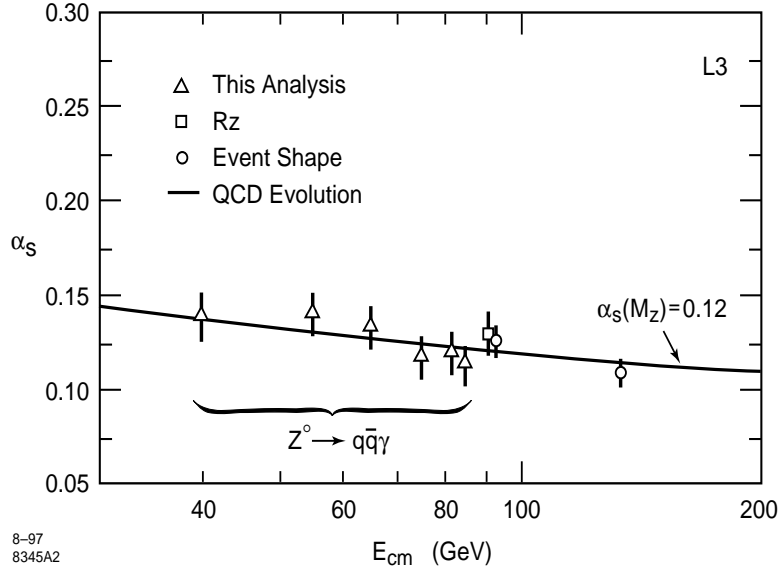


Figure 4: Measurements of α_s as a function of c.m. energy from L3 [28].

E Scaling Violations in Fragmentation Functions

Though distributions of final-state hadrons are not, in general, calculable in perturbative QCD, the Q^2 -evolution of the scaled momentum ($x_p = p/p_{beam}$) distributions of hadrons, or ‘fragmentation functions’, can be calculated and used to determine α_s . In addition to the usual renormalisation scale μ , a *factorisation scale* μ_F must be defined that delineates the boundary between the calculable perturbative, and incalculable non-perturbative, domains. Additional complications arise from the changing composition of the underlying event flavour with Q due to the different Q -dependence of the γ and Z^0 exchange processes. Since B and D hadrons typically carry a large fraction of the beam momentum, and contribute a large multiplicity from their decays, it is necessary to consider the scaling violations separately in b, c, and light quark events, as well as in gluon jet

fragmentation.

The ALEPH Collaboration used its Z^0 data to constrain flavour-dependent effects by tagging event samples enriched in light, c, and b quarks, as well as a sample of gluon jets [29]. The fragmentation functions for the different flavours and the gluon were parametrised at a reference energy, evolved with Q according to the perturbative DGLAP formalism calculated at next-to-leading order [30], in conjunction with a parametrisation proportional to $1/Q$ to represent non-perturbative effects, and fitted to data in the range $22 \leq Q \leq 91$ GeV (Fig. 5). They derived $\alpha_s(M_Z^2) = 0.126 \pm 0.007$ (exp.) ± 0.006 (theor.), where the theoretical uncertainty is dominated by variation of the factorisation scale μ_F in the range $-1 \leq \ln \mu_F^2/Q^2 \leq 1$; variation of the renormalisation scale in the same range contributed only ± 0.002 . DELPHI has recently reported a similar analysis [31] yielding $\alpha_s(M_Z^2) = 0.124_{-0.007}^{+0.006}$ (exp.) ± 0.009 (theor.). Combining the ALEPH and later DELPHI results, assuming uncorrelated experimental errors, yields:

$$\alpha_s(M_Z^2) = 0.125 \pm 0.005(\text{exp.}) \pm 0.009(\text{theor.}) \quad (7)$$

F Comparison with Other Measurements of $\alpha_s(M_Z^2)$

A summary of world α_s measurements, all evolved to $Q = M_Z$, is shown in Fig. 6 [4]. These are drawn from lepton-hadron scattering, hadron-hadron collisions, heavy quarkonia decays and lattice gauge theory, as well as e^+e^- annihilation. In addition to being relatively precise, the e^+e^- results have the invaluable feature that they bracket the Q -range of the experiments, from around 1 GeV to 172 GeV, providing the largest lever-arm for tests of consistency of $\alpha_s(M_Z^2)$ measured at different energy scales. It is clear that, within the uncertainties, all results are consistent with each another.

Taking an average over all 17 measurements *assuming* they are independent, by weighting each by its *total* error, yields $\alpha_s(M_Z^2) = 0.118$ with a χ^2 of 6.4; the low χ^2 value reflects the fact that most of the measurements are theoretical-systematics-

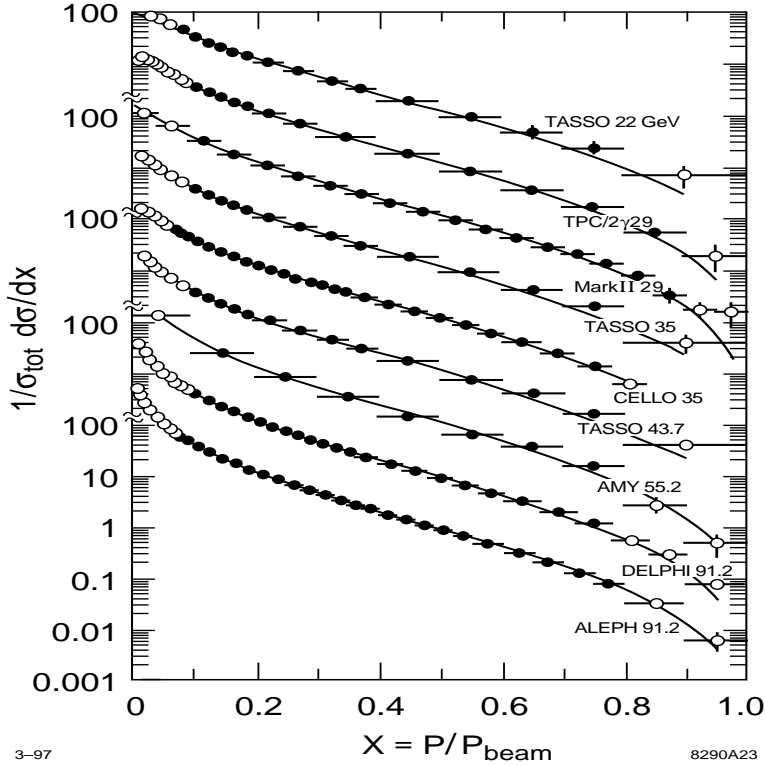


Figure 5: Inclusive charged-particle fragmentation functions used in α_s determination [29].

limited. Taking an unweighted average, which in some sense corresponds to the assumption that all 17 measurements are completely correlated, yields the same result. The r.m.s. deviation of the 17 measurements w.r.t. the average value characterises the dispersion, and is ± 0.005 . In a quantitative sense, therefore, QCD has been tested to a level of about 5%.

3 3-jet Production

A Flavour-Independence of Strong Interactions

A fundamental assumption of QCD is that the strong coupling is independent of quark flavor. This can be tested by measuring α_s in events of the type

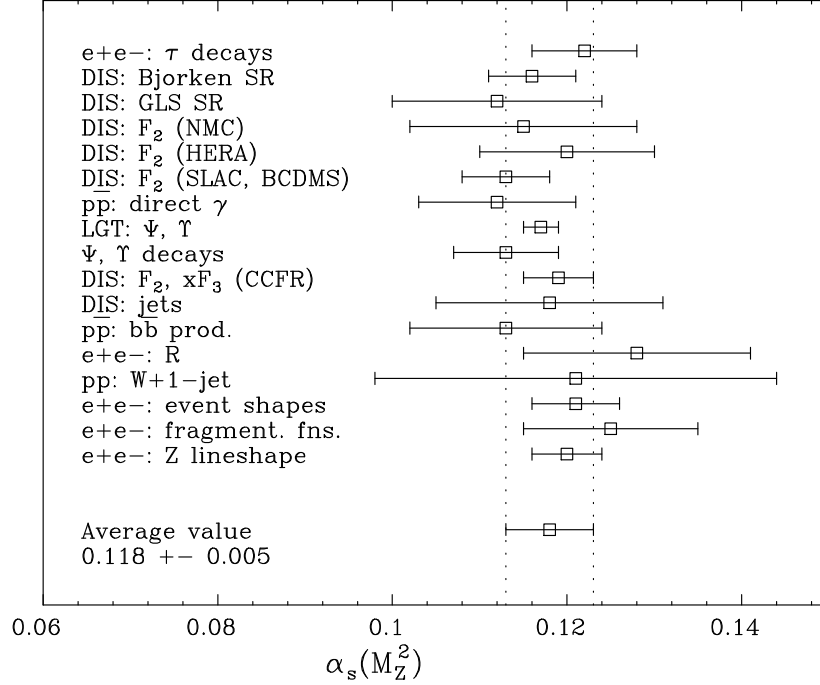


Figure 6: Summary of world $\alpha_s(M_Z^2)$ measurements [4]. The results are ordered vertically in terms of the hard scale Q of the experiment.

$e^+e^- \rightarrow q\bar{q}(g)$ for specific quark flavors q . Although an absolute determination of α_s for each quark flavor would have large theoretical uncertainties, it is possible to test the flavor-independence of QCD precisely by measuring ratios of couplings in which most experimental errors and theoretical uncertainties are expected to cancel. Since a flavor-dependent anomalous quark chromomagnetic moment could modify the probability for the radiation of gluons, comparison of the strong coupling for different quark flavors may also provide information on physics beyond the Standard Model.

The development of precise vertex detectors at e^+e^- colliders has allowed pure samples of b-quark events to be tagged with high efficiency. Measurements made at LEP of $\alpha_s^b/\alpha_s^{uds}$ have reached precisions between ± 0.06 and ± 0.013 (Fig. 7). However, these tests make the simplifying assumption that α_s is independent of flavor for all the non- b quarks, and are insensitive to differences between α_s for

these flavors, especially a different α_s for c quarks compared with either b or light quarks. The OPAL Collaboration has measured $\alpha_s^f/\alpha_s^{all}$ for all five flavors f with no assumption on the relative value of α_s for different flavors to precisions of ± 0.026 for b and ± 0.09 to ± 0.20 for the other flavors [32]. The kinematic signatures of charmed meson and light hadron production that OPAL used to tag c - and light-quark events, respectively, suffer from low efficiency and strong biases, due to preferential tagging of events without hard gluon radiation.

The SLD Collaboration has recently presented an update [33] of its test of the flavor independence of strong interactions, based on selection of $Z^0 \rightarrow b\bar{b}(g)$ and $Z^0 \rightarrow q_l\bar{q}_l(g)$ ($q_l = u, d, s$) events using their quark decay lifetime signatures, with high efficiency and purity, and with low bias against 3-jet events. From a comparison of the 3-jet fractions in each tagged sample with that in the all-flavours sample, and after unfolding for sample purity and tag bias, they obtained [33]:

$$\begin{aligned}\frac{\alpha_s^{uds}}{\alpha_s^{all}} &= 0.997 \pm 0.011 (\text{stat}) \pm 0.011 (\text{syst}) \pm 0.005 (\text{theory}) \\ \frac{\alpha_s^c}{\alpha_s^{all}} &= 0.984 \pm 0.042 (\text{stat}) \pm 0.053 (\text{syst}) \pm 0.022 (\text{theory}) \\ \frac{\alpha_s^b}{\alpha_s^{all}} &= 1.022 \pm 0.019 (\text{stat}) \pm 0.023 (\text{syst}) \pm 0.012 (\text{theory})\end{aligned}$$

A compilation of the Z^0 results is shown in Fig. 7; there is currently no indication of any flavour dependence of strong interactions.

B Quark- and Gluon-jet Differences

Many attempts were made at PETRA, PEP and TRISTAN to investigate possible differences between the properties of quark and gluon jets. These searches were motivated by the observation that the ‘colour charge’ of a gluon in a separating octet gg system is $9/4$ times that of a quark in a separating triplet $q\bar{q}$ system. It then follows from a leading logarithm bremsstrahlung-type calculation [34] that in the asymptotic limit $Q \rightarrow \infty$, the multiplicity of soft gluons in a gluon-initiated

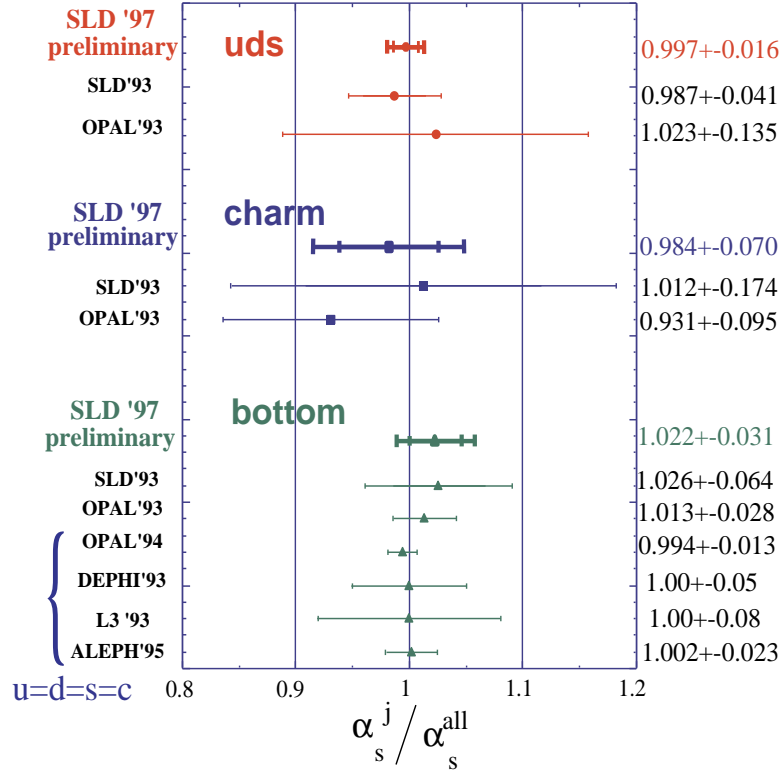


Figure 7: Summary of Z^0 tests of the flavour-independence of strong interactions [33].

jet is $9/4$ times the multiplicity in a quark-initiated jet. Assuming proportionality between the gluon multiplicity and the ensuing hadron multiplicity leads to the prediction that the particle multiplicity in gluon jets should be $r = 9/4$ times that in quark jets, and hence that the former will have a softer fragmentation function by roughly the same factor. Using similar arguments, it was also shown [35] that the angular widths δ of gluon and quark jets are related by: $\delta_g = \delta_q^{4/9}$, *i.e.* that gluon jets should be wider than quark jets.

However, the experimental searches for these effects, see *eg.* Ref. [36], yielded differences in properties significantly smaller than the factor of $9/4$. It is important to note that there are several caveats to the naive theoretical predictions which tend to dilute the factor r . One should consider beyond-leading-order corrections,

finite energy corrections, heavy quark decays, and fragmentation effects. For jets in Z^0 decays, with energy between 30 and 50 GeV, these effects reduce r to the range $1.4 < r < 1.6$; see Ref. [37] and references therein. The differences in particle multiplicity, width and hardness of fragmentation are thus expected to be less apparent than the naive prediction.

Studies at LEP have established such differences at about the expected level [38], but a quantitative comparison with the QCD predictions has been complicated by the difficulty of relating the experimental jet definition and event selection procedures to those assumed in the calculations. In particular, the calculations assume massless separating $q\bar{q}$ and gg systems and are completely inclusive, whereas the experimental studies are based upon the selection of three-jet events using particular jet-finding algorithms, and the results are algorithm-dependent.

Recently a more consistent analysis procedure has been proposed [39], and applied by the OPAL Collaboration. The method involves selecting 3-jet final states in which two heavy-quark jets recoil in the same hemisphere against the third (gluon) jet. After correcting for misidentification, the properties of jets in this gluon sample were then compared with those of a sample of back-to-back two-jet light-quark $q\bar{q}$ events, tagged on the basis of the absence of long-lived (heavy-quark) decay products. Because of the kinematic bias caused by the 3-jet topology the 278 gluon-tagged jets had a mean energy of 39.2 GeV, compared with 45.6 GeV for the sample of roughly 28,000 light-quark jets. This small difference was corrected assuming the QCD energy-dependence of the mean multiplicity, and yielded [37] $r(39\text{GeV}) = 1.552 \pm 0.041$ (stat) ± 0.060 (syst.). This result is shown in Fig. 8 [37], where it is compared with the analytic QCD calculations. The measurement is in good agreement with the next-to-next-to-leading order calculation that includes energy/momentum conservation. This important result helps to resolve the long-standing confusion over the low measured values of r , and gives us further confidence that QCD is able to describe the inclusive properties of hadronic jets.

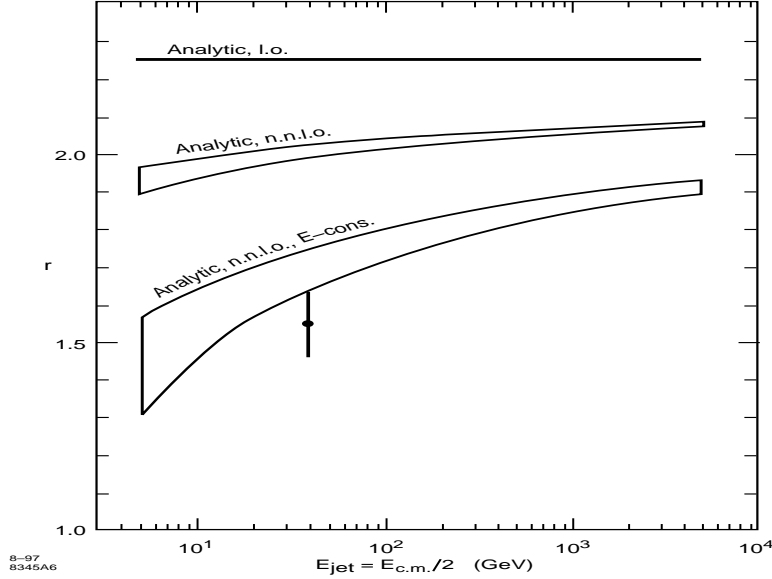


Figure 8: The ratio of particle multiplicity in gluon and quark jets vs. jet energy [37].

C Search for T_N -odd Effects in $Z^0 \rightarrow q\bar{q}g$

The Z^0 bosons produced at SLC using longitudinally polarized electrons have polarization along the beam direction $A_Z = (P_{e^-} - A_e)/(1 - P_{e^-} \cdot A_e)$, where P_{e^-} is the electron beam polarization, defined to be negative (positive) for a left-(right-) handed beam, and $A_e = 2v_e a_e/(v_e^2 + a_e^2)$ with v_e and a_e the electroweak vector and axial vector coupling parameters of the electron, respectively. An electron-beam polarization at the e^+e^- interaction point of approximately 0.77 in magnitude was achieved in the 1994-95 and 1996 runs, yielding $A_Z = -0.82$ ($+0.71$) for $P_{e^-} = -0.77$ ($+0.77$) respectively. For polarized Z^0 decays to three hadronic jets one can define the triple-product: $\vec{S}_Z \cdot (\vec{k}_1 \times \vec{k}_2)$, which correlates the Z^0 boson polarization vector \vec{S}_Z with the normal to the three-jet plane defined by \vec{k}_1 and \vec{k}_2 , the momenta of the highest- and the second-highest-energy jets respectively. The triple-product is even under C and P reversals, and odd under T_N , where T_N reverses momenta and spin-vectors without exchanging initial and final states.

Since T_N is not a true time-reversal operation a non-zero value does not signal CPT violation and is possible in a theory that respects CPT invariance.

The tree-level differential cross section for $e^+e^- \rightarrow q\bar{q}g$ for a longitudinally polarized electron beam and massless quarks may be written [40]:

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \omega} = \frac{9}{16} \left[\left(1 - \frac{1}{3} \cos^2 \omega\right) + \beta A_Z \cos \omega \right], \quad (8)$$

where ω is the polar angle of the vector normal to the jet plane, $\vec{k}_1 \times \vec{k}_2$, w.r.t. the electron beam direction. With $\beta|A_Z|$ representing the magnitude, the second term is proportional to the T_N -odd triple-product, and appears as a forward-backward asymmetry of the jet-plane-normal relative to the Z^0 polarization axis. The sign and magnitude of this term are different for the two beam helicities. Recently Standard Model T_N -odd contributions of this form at the Z^0 resonance have investigated [40]. The triple-product vanishes identically at tree level, but non-zero contributions arise from higher order processes. Due to various cancellations these contributions are found to be very small at the Z^0 resonance and yield $|\beta| \sim 10^{-5}$. Because of this background-free situation, measurement of the cross section (8) is sensitive to physics processes beyond the Standard Model that give $\beta \neq 0$.

The SLD measurement of the ω distribution in all-flavours Z^0 decays is shown in Fig. 9 [41]. A fit of Eq. (8) yields $\beta = 0.008 \pm 0.015$, or $-0.022 < \beta < 0.039$ at 95% c.l. Similar measurements for the potentially more interesting $b\bar{b}g$ system are now in progress [42].

4 4-jet Production

A Total Cross-Section

It has been known since studies performed at PETRA at c.m. energies of approximately 35 GeV, see *eg.* Ref. [43], that the tree-level $O(\alpha_s^2)$ matrix element calculation of the 4-jet cross-section is insufficient to describe the data, Fig. 10.

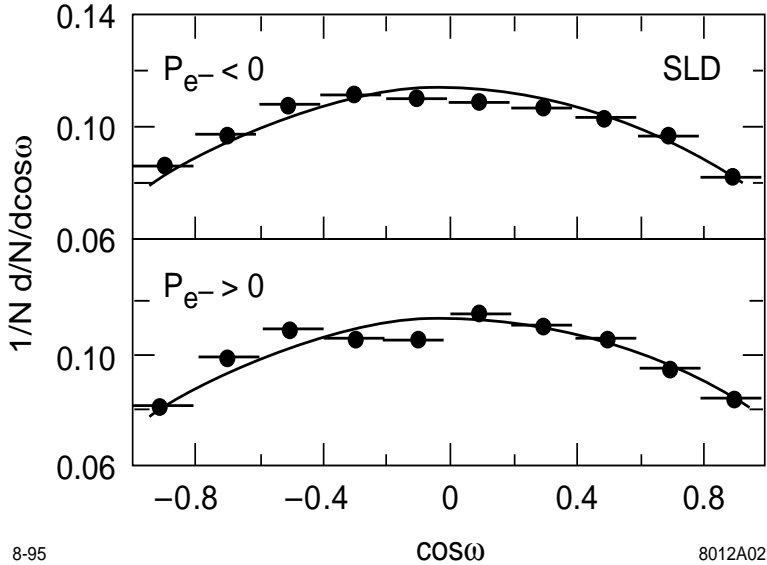


Figure 9: SLD measured distributions of $\cos\omega$; the line is a fit of Eq. (8).

Tremendous progress has been made recently and the full 1-loop calculation has been performed [44]. A comparison of the leading- N_C contributions to the 4-jet rate measured with Z^0 data and a similar jet algorithm as in Fig. 10 is given in Fig. 11. The 1-loop result is roughly a factor of two larger than the tree-level result and describes the data well; sub-leading N_C contributions are typically an order of magnitude smaller [44].

B Angular Correlations

The QCD tree-level couplings contributing to 4-jet events are shown in Fig. 12; they may be classified in terms of the Casimir factors C_F , T_F , and N_C that characterise the $SU(3)_C$ group; see *eg.* Ref. [1]. It is interesting to consider whether the Casimir factors can be measured. Clearly nature does not deliver events corresponding to the tree-level vertices shown in Fig. 12! Instead, one must write down the Feynman amplitudes for the 4-jet event diagrams, add them to those for 2- and 3-jet production at the same order of perturbation theory, and square

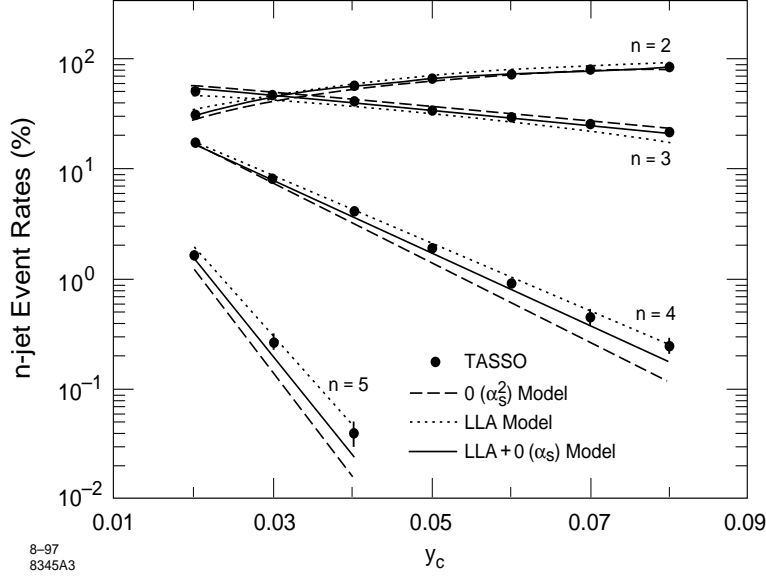


Figure 10: n -jet event rates measured at $Q = 35$ GeV compared with QCD calculations; the $O(\alpha_s^2)$ calculation is unable to describe the 4-jet cross-section [43].

them to derive the total hadronic cross section. The terms corresponding to 4-jet production can then be identified in a gauge-invariant manner, and yield a differential cross section of the form:

$$\begin{aligned}
 \frac{1}{\sigma_0} d\sigma^4 &= \left(\frac{\alpha_s C_F}{\pi} \right)^2 \left[F_A + \left(1 - \frac{1}{2} \frac{N_C}{C_F} \right) F_B + \frac{N_C}{C_F} F_C \right] \\
 &+ \left(\frac{\alpha_s C_F}{\pi} \right)^2 \left[\frac{T_F}{C_F} N_f F_D + \left(1 - \frac{1}{2} \frac{N_C}{C_F} \right) F_E \right] \quad (9)
 \end{aligned}$$

where $F_A \dots F_E$ are kinematic factors. The overall normalisation of the cross-section is proportional to $(\alpha_s C_F)^2$, and the kinematical distribution of the four jets depends on the ratios N_C/C_F and T_F/C_F , which can hence in principle be measured.

A number of 4-jet angular correlation observables that are potentially sensitive to these ratios have been proposed [45]. If one orders and labels the four jets in an event in terms of their momenta (or energies) such that $p_1 > p_2 > p_3 > p_4$ one

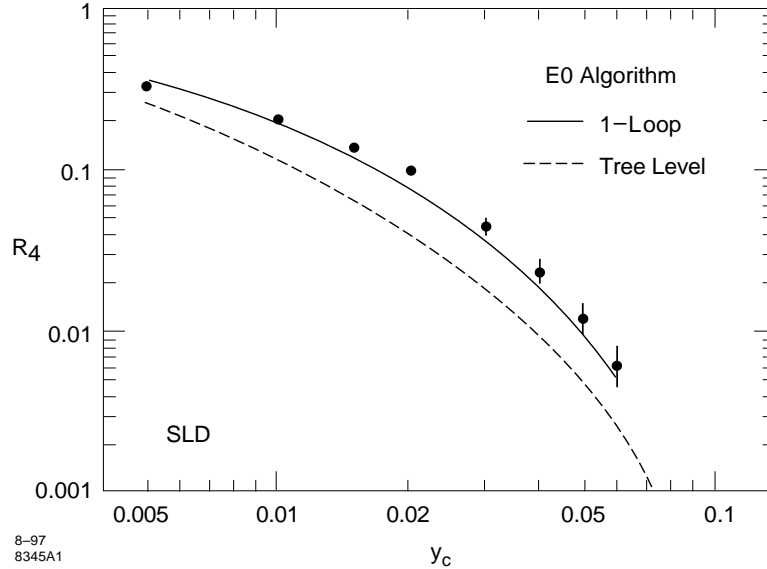


Figure 11: 4-jet event rate measured at $Q = 91$ GeV compared with QCD calculations; the $O(\alpha_s^3)$ calculation describes the data [44].

can define the Bengtsson-Zerwas angle:

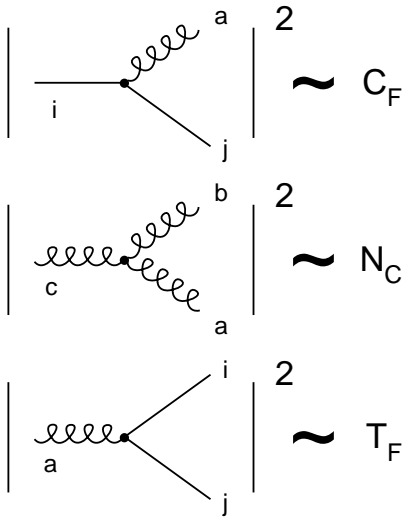
$$\cos \chi_{BZ} \propto (\vec{p}_1 \times \vec{p}_2) \cdot (\vec{p}_3 \times \vec{p}_4) \quad (10)$$

and the Nachtmann-Reiter angle:

$$\cos \theta_{NR}^* \propto (\vec{p}_1 - \vec{p}_2) \cdot (\vec{p}_3 - \vec{p}_4). \quad (11)$$

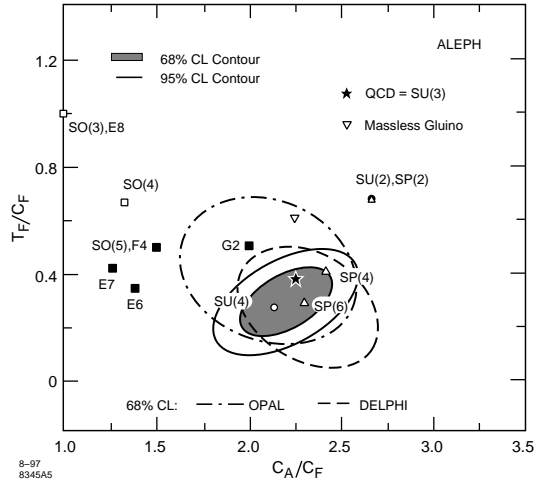
Interestingly the 1-loop corrections discussed in the previous section do not change the shapes of the predicted distributions of these angles [44], so that use of the tree-level calculations, which has been done to date, would appear to be valid.

In studies by the LEP experiments, fits for N_C/C_F and T_F/C_F have been performed to these angular distributions, as well as to the angle α_{34} between jets 3 and 4. Results from the most recent study, by the ALEPH Collaboration, are displayed in Fig. 13 [46], where they are compared with the values for numerous gauge groups. The SU(3) QCD expectation is clearly in good agreement with the data. The expectations from several other gauge models, such as SU(4), SP(4)



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Figure 12: Casimir classification of tree-level QCD couplings.

Figure 13: The T_F/C_F vs. N_C/C_F plane showing recent measurements [46] from LEP experiments, as well as the expectations from numerous gauge groups.

and $SP(6)$, also appear to be compatible with the experimental results. Note, however, that none of these models contains three colour degrees of freedom for quarks, and hence all can be ruled out on that basis. Besides $SU(3)$, only the $U(1)_3$ and $SO(3)$ models contain three quark colours, but both are inconsistent with the measured values of N_C/C_F and T_F/C_F . The results shown in Fig. 13 hence yield the remarkable conclusion that $SU(3)$ is the only known viable gauge model for strong interactions. Also shown in this figure is the theoretical expectation for QCD augmented with a single family of light gluinos [47]; the ALEPH result appears to rule out this expectation at better than 95% confidence level, but this interpretation has been criticised [48].

5 Conclusions

Electron-positron annihilation is an ideal laboratory for strong-interaction measurements. Jets in the final state allow the dynamics of quarks and gluons to be measured precisely. Since the PETRA/PEP era the Z^0 experiments have established the gauge structure of strong interactions via measurement of the Casimir factor ratios N_C/C_F and T_F/C_F , leading us to the conclusion that QCD *is* the correct theory. Differences between quark- and gluon-jets of the same energy have been convincingly demonstrated and, when compared in a consistent fashion, have been found to be in agreement with theoretical expectations. The coupling α_s has been determined to the 5%-level of accuracy from inclusive Z^0 lineshape observables and from hadronic τ decays, as well as from event shape measures and scaling violations in inclusive single-particle fragmentation functions. These $\alpha_s(M_Z^2)$ measurements are internally consistent, and agree with results from lepton-nucleon scattering, hadron-hadron collisions, and lattice gauge theory determined across a wide range of energy scales.

The development of precise silicon vertex detectors has allowed flavour-dependent properties to be studied in both the primary hard process and in jet fragmentation, and the strong coupling has been found to be flavour-independent at the sub-5% level. In addition, high electron-beam polarisation at SLC has allowed interesting new symmetry tests using 3-jet events. Finally, tremendous theoretical effort has resulted in perturbative QCD calculations that are accurate at the 10%-level, and attempts to calculate non-perturbative effects for jet observables are well under way.

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References

- [1] P.N. Burrows 'Precise Tests of QCD in e^+e^- Annihilation', SLAC-PUB-7434; to appear in Proc. XXIV SLAC Summer Institute on Particle Physics (1996).
- [2] W.A. Bardeen, A.J. Buras, D.W. Duke, T. Muta, Phys. Rev. **18** (1978) 3998; D.W. Duke, Rev. Mod. Phys. **52** (1980) 199.
- [3] R.M. Barnett *et al.*, Phys. Rev. **D54** (1996) 77.
- [4] P.N. Burrows, Proc. 3rd International Symposium on Radiative Corrections, August 1-5 1996, Cracow, Poland, Acta Phys. Pol. **B28** (1997) 467.
- [5] M. Shifman, Mod. Phys. Lett. **A10** No. 7 (1995) 605.
- [6] S.G. Gorishny, A. Kataev, S.A. Larin, Phys. Lett. **B259** (1991) 144; L.R. Surguladze, M.A. Samuel, Phys. Rev. Lett. **66** (1991) 560.
- [7] K.H. Chetyrkin, J.H. Kühn, A. Kwiatkowski, Berkeley preprint LBL 36678-Rev (1996); subm. to Phys. Rep.
- [8] A. Blondel, Proc. 28th International Conference on High Energy Physics, Warsaw, Poland, 25-31 July 1996, Eds. Z. Ajduk, A.K. Wroblewski, World Scientific 1997, p. 205.
- [9] The LEP and SLD Collaborations, LEPEWWG/97-01 (April 1997).
- [10] P.B. Renton, Proc. 17th International Symposium on Lepton-Photon Interactions, 10-15 August 1995, Beijing, China, p. 35.
- [11] D. Schaile, Proc. XXVII International Conference on High Energy Physics, July 20-27 1994, Glasgow, Scotland, IoP Publishing, Eds. P.J. Bussey, I.G. Knowles, p. 27.
- [12] F. Le Diberder, A. Pich, Phys. Lett. **B289** (1992) 165.

- [13] E. Braaten, S. Narison, A. Pich, Nucl. Phys. **B373** (1992) 581; F. Le Diberder, A. Pich, Phys. Lett. **B286** (1992) 147.
- [14] M. Neubert, Nucl. Phys. **B463** (1996) 511.
- [15] L. Duflot (ALEPH Collab.), Nucl. Phys. B (Proc. Supp.) **39B** (1995) 322.
- [16] OPAL Collab., R. Akers *et al.*, Z. Phys. **C66** (1995) 543.
- [17] CLEO Collab., T. Coan *et al.*, Phys. Lett. **B356** (1995) 580.
- [18] G. Altarelli, P. Nason, G. Ridolfi, Z. Phys. **C68** (1995) 257.
- [19] SLD Collab., K. Abe *et al.*, Phys. Rev. **D51** (1995) 962.
- [20] R.K. Ellis, D.A. Ross, A.E. Terrano, Phys. Rev. Lett. **45** (1980) 1226; Nucl. Phys. **B178** (1981) 421.
G. Kramer, B. Lampe, Z. Phys. **C39** (1988) 101; Fortschr. Phys. **37** (1989) 161.
Z. Kunszt *et al.*, CERN 89-08 Vol I, (1989) p. 373.
- [21] S. Catani, G. Turnock, B.R. Webber, and L. Trentadue, Phys. Lett. **B263** (1991) 491.
- [22] P.M. Stevenson, Phys. Rev. **D23** (1981) 2916;
G. Grunberg, Phys. Rev. **D29** (1984) 2315;
S.J. Brodsky, G.P. Lepage, P.B. Mackenzie, Phys. Rev. **D28** (1983) 228.
- [23] See *eg.* G.A. Baker Jr., ‘Essentials of Padé Approximants’, Academic Press, 1975.
- [24] P.N. Burrows *et al.*, Phys. Lett. **B382** (1996) 157.
- [25] P.N. Burrows *et al.*, Phys. Lett. **B392** (1997) 223.
- [26] OPAL Collab., G. Alexander *et al.*, OPAL-PN281 (1997).

- [27] M. Schmelling, Proc. 28th International Conference on High Energy Physics, Warsaw, Poland, 25-31 July 1996, Eds. Z. Ajduk, A.K. Wroblewski, World Scientific 1997, p. 91.
- [28] L3 Collab., M. Acciari *et al.*, CERN-PPE/97-074 (1997); subm. to Phys. Lett. B.
- [29] ALEPH Collab., D. Buskulic *et al.*, Phys. Lett. **B357** (1995) 487.
- [30] G. Gurci, W. Furmanski, R. Petronzio, Nucl. Phys. **B175** (1980) 27.
- [31] DELPHI Collab., P. Abreu *et al.*, Phys. Lett. **B398** (1997) 194.
- [32] OPAL Collab., R. Akers *et al.*, Z. Phys. **C60** 397 (1993).
- [33] SLD Collab., K. Abe *et al.*, SLAC-PUB-7573 (July 1997); contributed paper to the XVIII International Symposium on Lepton Photon Interactions, and to the 1997 International Europhysics Conference on High Energy Physics.
- [34] S.J. Brodsky, J.F. Gunion, Phys. Rev. Lett. **37** (1976) 402.
- [35] K. Shizuya, S. Tye, Phys. Rev. Lett. **41** (1978) 787.
M.B. Einhorn, B.G. Weeks, Nucl. Phys. **B146** (1978) 445.
- [36] P.N. Burrows, Nucl. Phys. B (Proc. Suppl.) **23B** (1991) 30.
- [37] OPAL Collab., G. Alexander *et al.*, Phys. Lett. **B388** (1996) 659.
- [38] For a recent review see: J.W. Gary, Proc. XXV International Symposium on Multiparticle Dynamics, Stara Lesna, Slovakia, September 12-16 1995.
- [39] J.W. Gary, Phys. Rev. **D49** (1994) 4503.
- [40] A. Brandenburg, L. Dixon, Y. Shadmi, Phys. Rev. **D53** (1996) 1264.
- [41] SLD Collab., K. Abe *et al.*, Phys. Rev. Lett. **75** (1995) 4173.

- [42] SLD Collab., K. Abe *et al.*, SLAC-PUB-7570 (July 1997); contributed paper to the XVIII International Symposium on Lepton Photon Interactions, and to the 1997 International Europhysics Conference on High Energy Physics.
- [43] TASSO Collab., Phys. Lett. **B214** (1988) 286.
P.N. Burrows, D. Phil. Thesis, Univ. of Oxford, RAL-T-071 (1988).
- [44] L. Dixon, A. Signer, Phys. Rev. Lett. **78** (1997) 811.
L. Dixon, A. Signer, SLAC-PUB-7528 (1997); subm. to Phys. Rev. D.
- [45] O. Nachtmann, A. Reiter, Z. Phys. **C16** (1982) 45.
M. Bengtsson, P.M. Zerwas, Phys. Lett. **B208** (1988) 306.
- [46] ALEPH Collab., R. Barate *et al.*, CERN-PPE/97-002 (1997); subm. to Z. Phys. C.
- [47] See *eg.*, G. Farrar, Phys. Rev. **D51** (1995) 3904.
- [48] G. Farrar, Rutgers Univ. preprint RU-97-22 (1997); hep-ph/9707467.