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The Role of the Vacuum Insertion Approximation in Calculating CP Asymmetries in B Decays

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CP asymmetries in B decays into final CP eigenstates are in many cases theoretically clean. In particular, they do not depend on the values of hadronic parameters. The sign of the asymmetries, however, does depend on the sign of the B_B parameter. Furthermore, the information from ϵ_K that all angles of the unitarity triangles lie in the range $\{0, \pi\}$ depends on the sign of the B_K parameter. Consequently, in the (unlikely) case that the vacuum insertion approximation is such a poor approximation that either B_B or B_K is negative, the sign of CP asymmetries in neutral B decays will be opposite to the standard predictions. Various subtleties concerning the role of $K - \bar{K}$ mixing in the case of final states with a single K_S or K_L , such as the $B \rightarrow \psi K_S$ decay, are clarified.

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1. Introduction and Formalism

CP asymmetries in B decays into final CP eigenstates [1-3] will provide stringent tests of the Kobayashi-Maskawa mechanism of CP violation. For decay processes that depend on a single CKM phase, such as the $B \rightarrow \psi K_S$ mode, the Standard Model prediction is theoretically very clean (for reviews, see *e.g.* [4-6]). In particular, while the magnitude of neutral meson mixing amplitudes, namely Δm_B and Δm_K , suffers from large hadronic uncertainties in the matrix elements (parameterized, respectively, by $B_B f_B^2$ and B_K), the CP asymmetries are independent of the value of these parameters. It is a little known fact, however, that the *sign* of the asymmetries does depend on the sign of B_B and, in an indirect way [7], also on the sign of B_K . In this work we explain how this dependence arises and describe the consequences in (the unlikely) case that the vacuum insertion approximation is surprisingly poor so that it gives the wrong sign of the matrix elements.

Before we start a detailed and technical analysis of the sign dependence of the otherwise clean CP asymmetries, we give the general argument for the existence of this dependence. In the decays of neutral B mesons to CP eigenstates, the CP violating asymmetry arises solely from an interference between an amplitude which involves $B - \bar{B}$ mixing, and one which does not. The relative phase of these two interfering amplitudes includes the sign of the hadronic matrix element for $B - \bar{B}$ mixing. Since this matrix element is determined by the CP conserving strong interactions, its sign is the same in the decay of a $B_{\text{phys}}^0(t)$ and in that of a $\bar{B}_{\text{phys}}^0(t)$. A reversal of this sign would obviously reverse the sign of the contribution of the interference term to both the decay rate for $B_{\text{phys}}^0(t)$ and the decay rate for $\bar{B}_{\text{phys}}^0(t)$. Thus, a reversal of the sign of the hadronic matrix element would cause a reversal of the CP violating asymmetry between these two decay rates.

As there are many subtle points in this discussion, we repeat here the analysis of CP violation in B and K decays with particular attention to signs. We focus on the neutral B meson system, but the analysis in this section applies equally well to the neutral K system. Our phase convention is defined by

$$\text{CP}|B^0\rangle = \omega_B|\bar{B}^0\rangle, \quad \text{CP}|\bar{B}^0\rangle = \omega_B^*|B^0\rangle, \quad (|\omega_B| = 1). \quad (1.1)$$

Physical observables do not depend on the phase factor ω_B . We define q and p to be the

components of the neutral B interaction eigenstates in the mass eigenstates,

$$|B_{1,2}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle. \quad (1.2)$$

We further define

$$M_{12} - \frac{i}{2}\Gamma_{12} \equiv \langle B^0 | \mathcal{H}_{\text{eff}}^{\Delta b=2} | \bar{B}^0 \rangle, \quad (1.3)$$

where M and Γ are hermitian matrices, so that

$$M_{12}^* = M_{21}, \quad \Gamma_{12}^* = \Gamma_{21}. \quad (1.4)$$

The mass and width difference between the physical states are given by

$$\Delta m \equiv M_2 - M_1, \quad \Delta\Gamma \equiv \Gamma_2 - \Gamma_1. \quad (1.5)$$

Solving the eigenvalue equations gives

$$\begin{aligned} (\Delta m)^2 - \frac{1}{4}(\Delta\Gamma)^2 &= (4|M_{12}|^2 - |\Gamma_{12}|^2), \\ \Delta m \Delta\Gamma &= 4\text{Re}(M_{12}\Gamma_{12}^*), \end{aligned} \quad (1.6)$$

$$\frac{q}{p} = -\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - \frac{i}{2}\Delta\Gamma}. \quad (1.7)$$

The quantity (q/p) plays an important role in the calculation of CP asymmetries in neutral B decays and will introduce, as we shall see, some dependence on hadronic physics.

2. The Vacuum Insertion Approximation

The effective Hamiltonian that is relevant to M_{12} is of the form

$$\mathcal{H}_{\text{eff}}^{\Delta b=2} \propto e^{+2i\phi_B} [\bar{d}\gamma^\mu(1 - \gamma_5)b]^2 + e^{-2i\phi_B} [\bar{b}\gamma^\mu(1 - \gamma_5)d]^2 \quad (2.1)$$

where $2\phi_B$ is a CP violating (weak) phase. (We use the Standard Model $V - A$ amplitude, but the results can be generalized to any Dirac structure.) For example, within the Standard Model

$$\phi_B = \arg(V_{tb}V_{td}^*). \quad (2.2)$$

(We implicitly assume here that long distance contributions to $B-\bar{B}$ mixing are negligible.)

The M_{12} matrix element is often calculated in the vacuum insertion approximation (VIA):

$$M_{12}^{\text{VIA}} = \langle B^0 | \mathcal{O}^{\Delta b=1} | 0 \rangle \langle 0 | \mathcal{O}^{\Delta b=1} | \bar{B}^0 \rangle, \quad (2.3)$$

where

$$\mathcal{O}^{\Delta b=1} \propto e^{+i\phi_B} [\bar{d}\gamma^\mu(1-\gamma_5)b] + e^{-i\phi_B} [\bar{b}\gamma^\mu(1-\gamma_5)d]. \quad (2.4)$$

Under CP transformations,

$$\bar{\psi}_i \gamma^\mu (1-\gamma_5) \psi_j \rightarrow -\bar{\psi}_j \gamma_\mu (1-\gamma_5) \psi_i, \quad (2.5)$$

thus we learn that

$$\langle 0 | \mathcal{O}^{\Delta b=1} | \bar{B}^0 \rangle = -\omega_B^* e^{2i\phi_B} \langle 0 | \mathcal{O}^{\Delta b=1} | B^0 \rangle. \quad (2.6)$$

From the hermiticity of $\mathcal{O}^{\Delta b=1}$ we know that $\langle B^0 | \mathcal{O}^{\Delta b=1} | 0 \rangle = \langle 0 | \mathcal{O}^{\Delta b=1} | B^0 \rangle^*$. This fact, in combination with (2.3) and (2.6), gives

$$M_{12}^{\text{VIA}} = -\omega_B^* e^{2i\phi_B} |M_{12}^{\text{VIA}}|. \quad (2.7)$$

The ratio between the true value of M_{12} and its value in the VIA is conventionally parameterized by a factor B_B :

$$M_{12} = -\omega_B^* e^{2i\phi_B} B_B |M_{12}^{\text{VIA}}|. \quad (2.8)$$

As the strong interactions conserve CP, the B_B parameter is real. Yet its sign could a-priori be positive or negative.

3. The CP Asymmetries in $B \rightarrow D^+ D^-$ and $B \rightarrow \psi K_S$

To see how the various phases and signs affect calculations of CP violation, we consider CP asymmetries in neutral B decays into final CP eigenstates:

$$a_{f_{CP}} = \frac{\Gamma(B_{\text{phys}}^0(t) \rightarrow f_{CP}) - \Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f_{CP})}{\Gamma(B_{\text{phys}}^0(t) \rightarrow f_{CP}) + \Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f_{CP})}. \quad (3.1)$$

We now introduce the various ingredients that enter the calculation of such asymmetries.

For the neutral B system, we define

$$\Delta m_B > 0 \quad (\implies |B_1\rangle \equiv |B_L\rangle, \quad |B_2\rangle \equiv |B_H\rangle), \quad (3.2)$$

($L(H)$ stand for light (heavy)). Taking into account that $\Delta m_B \gg |\Delta\Gamma_B|$, eqs. (1.6) and (1.7) simplify into

$$\Delta m = 2|M_{12}|, \quad \Delta\Gamma = 2\text{Re}(M_{12}\Gamma_{12}^*)/|M_{12}|, \quad (3.3)$$

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|}. \quad (3.4)$$

Note that q/p (and therefore also $a_{f_{CP}}$) is independent of $\Delta\Gamma$. In particular, the relative sign between Δm and $\Delta\Gamma$ does not play a role here. Putting (2.8) in (3.4) we finally get

$$\frac{q}{p} = \omega_B e^{-2i\phi_B} \text{sign}(B_B). \quad (3.5)$$

Additional phase dependence of CP asymmetries comes from decay amplitudes. We define A_f and \bar{A}_f according to

$$A_f = \langle f | \mathcal{H}_d | B^0 \rangle, \quad \bar{A}_f = \langle f | \mathcal{H}_d | \bar{B}^0 \rangle. \quad (3.6)$$

The decay Hamiltonian is of the form

$$\mathcal{H}_d \propto e^{+i\phi_f} [\bar{q}\gamma^\mu(1-\gamma_5)d][\bar{b}\gamma_\mu(1-\gamma_5)q] + e^{-i\phi_f} [\bar{q}\gamma^\mu(1-\gamma_5)b][\bar{d}\gamma_\mu(1-\gamma_5)q], \quad (3.7)$$

where ϕ_f is the appropriate weak phase. (For simplicity we use a $V-A$ decay amplitude, but the results hold for any Dirac structure.) From (2.5) we learn that under a CP transformation the two terms in (3.7) are interchanged except for the $e^{+i\phi_f}$ and $e^{-i\phi_f}$ phase factors. Then

$$\bar{A}_f = \omega_f \omega_B^* e^{-2i\phi_f} A_f, \quad (3.8)$$

where $\text{CP}|f\rangle = \omega_f|\bar{f}\rangle$. For a final CP eigenstate, $f = f_{CP}$, the phase factor ω_f is replaced by $\eta_{f_{CP}} = \pm 1$, the CP eigenvalue of the final state. Then

$$\frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}} = \eta_f \omega_B^* e^{-2i\phi_f}. \quad (3.9)$$

An important role in CP violation is played by a complex quantity λ_f , defined by

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}. \quad (3.10)$$

For B decays into final CP eigenstates, we find from (3.5) and (3.9):

$$\lambda_{f_{CP}} = \eta_{f_{CP}} e^{-2i(\phi_B + \phi_f)} \text{sign}(B_B), \quad (3.11)$$

which is independent of phase conventions. The asymmetry $a_{f_{CP}}$ of eq. (3.1) takes a particularly simple form when the decay amplitude is dominated by a single weak phase

$$a_{f_{CP}} = -\text{Im}\lambda_{f_{CP}} \sin(\Delta m_B t). \quad (3.12)$$

From eq. (3.11) we find then that

$$\text{Im}\lambda_{f_{CP}} = -\eta_{f_{CP}} \text{sign}(B_B) \sin[2(\phi_B + \phi_f)]. \quad (3.13)$$

To take an example, we now calculate the CP asymmetry in $B \rightarrow D^+ D^-$. Within the Standard Model and neglecting penguin diagrams, the decay phase defined in (3.7) is given by

$$\phi_{D^+ D^-} = \arg(V_{cd} V_{cb}^*). \quad (3.14)$$

(Unlike (2.2), which is sensitive to new physics, for tree level processes such as $b \rightarrow c\bar{c}d$, the Standard Model tree level diagram is likely to dominate even in the presence of new physics. Therefore (3.14) is likely to hold almost model independently.) Using (2.2) and (3.14), and taking into account that $\eta_{D^+ D^-} = +1$, we find for λ defined in (3.11):

$$\lambda_{D^+ D^-} = \text{sign}(B_B) \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left(\frac{V_{cd}^* V_{cb}}{V_{cd} V_{cb}^*} \right), \quad (3.15)$$

$$\text{Im}\lambda_{D^+ D^-} = -\sin(2\beta) \text{sign}(B_B), \quad (3.16)$$

where

$$\beta \equiv \arg \left[-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right]. \quad (3.17)$$

Eq. (3.16) is often displayed in the literature without its dependence on B_B . The reason is that it is widely believed that the vacuum insertion approximation gives a reasonable

approximation to the true values of the relevant matrix elements. (Lattice calculations strongly support this notion [8].) In particular, it is believed that it gives the correct sign of the matrix elements. One should not forget, however, that the dependence on the hadronic physics does exist.

The situation is somewhat more complicated in decays with a single K_S (or K_L) in the final state. There is some confusion in the literature concerning such decays which we would like to clarify. The three main points concerning this mode are the following:

- a. In $B \rightarrow \psi K_S$, the kaon will be experimentally identified by its decay to two pions within roughly one K_S lifetime.
- b. The smallness of ϵ_K implies that the contribution from $K_L \rightarrow \pi\pi$ within roughly one K_S lifetime is negligible.
- c. The smallness of ϵ_K also implies that K_S is almost purely a CP-even state.

This situation allows a straightforward derivation of the asymmetry. In particular, it implies that the relative phase between the direct $K^0 \rightarrow \pi\pi$ amplitude and the $K^0 \rightarrow \bar{K}^0 \rightarrow \pi\pi$ amplitude is very small and practically does not affect the CP asymmetry. Using the notation $\psi(2\pi)_K$ to describe the final state, the amplitude ratio is given by

$$\frac{\bar{A}_{\psi(2\pi)_K}}{A_{\psi(2\pi)_K}} = \eta_{\psi(2\pi)_K} \omega_B^* e^{-2i\phi_{\psi(2\pi)_K}}, \quad (3.18)$$

where $\eta_{\psi(2\pi)_K} = -1$. The relevant phase is found simply from the decay chain $B^0 \rightarrow \psi K^0 \rightarrow \psi(2\pi)_K$. Within the Standard Model, but practically model-independently, it is given by

$$\phi_{\psi(2\pi)_K} = \arg(V_{cs} V_{cb}^* V_{us}^* V_{ud}). \quad (3.19)$$

Using (3.18) and (3.19), we get the Standard Model value for $\lambda_{\psi(2\pi)_K}$:

$$\lambda_{\psi(2\pi)_K} = -\text{sign}(B_B) \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left(\frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*} \right) \left(\frac{V_{ud}^* V_{us}}{V_{ud} V_{us}^*} \right). \quad (3.20)$$

Taking into account that unitarity of the three-generation CKM matrix implies that, to a very high accuracy, $\left(\frac{V_{ud}^* V_{us}}{V_{ud} V_{us}^*} \right) = \left(\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \right)$, the Standard Model prediction for the CP asymmetry in $B \rightarrow \psi(2\pi)_K$ is

$$\text{Im}\lambda_{\psi(2\pi)_K} = \sin(2\beta)\text{sign}(B_B). \quad (3.21)$$

Notice that, to get (3.21), it is not essential whether the typical kaon mixing time is shorter or longer than the decay time. The only important information about $K - \bar{K}$ mixing is that, to an excellent approximation, its amplitude is aligned with that of the $K \rightarrow \pi\pi$ decay amplitude.

Another point of interest is the fact that one can learn about new physics in $K - \bar{K}$ mixing from a comparison of $a_{D^+D^-}$ and $a_{\psi K_S}$ [9]. (We assume here that the tree contribution is dominant among the Standard Model contributions to $B \rightarrow D^+D^-$.) This may seem puzzling in view of our discussion above, where we argued that (3.20) is independent of the physics that is responsible for $K - \bar{K}$ mixing. Indeed, allowing new physics in $B - \bar{B}$ mixing and in $K - \bar{K}$ mixing but not in the relevant decay processes, $b \rightarrow c\bar{c}d$, $b \rightarrow c\bar{c}s$ and $s \rightarrow u\bar{u}d$, we have

$$\lambda_{D^+D^-} = \left(\frac{q}{p}\right)_B \omega_B^* \left(\frac{V_{cd}^* V_{cb}}{V_{cd} V_{cb}^*}\right), \quad (3.22)$$

and

$$\lambda_{\psi K_S} = - \left(\frac{q}{p}\right)_B \omega_B^* \left(\frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*}\right) \left(\frac{V_{ud}^* V_{us}}{V_{ud} V_{us}^*}\right). \quad (3.23)$$

Then, if experiments find

$$a_{D^+D^-} \neq -a_{\psi K_S}, \quad (3.24)$$

this will necessarily require a violation of the Standard Model relation

$$\left(\frac{V_{cs}^* V_{cd}}{V_{us}^* V_{ud}}\right) \approx -1. \quad (3.25)$$

However, (3.25) holds if either of the following two conditions is valid:

- a. The three generation CKM matrix is unitary;
- b. $K - \bar{K}$ mixing is dominated by the Standard Model box diagrams with intermediate charm and up quarks.

Therefore, (3.24) will signal that (a) the quark sector is larger than just the three standard generations, and (b) there is a new physics contribution to $K - \bar{K}$ mixing.

4. The Role of $K - \bar{K}$ Mixing

In contrast to $B - \bar{B}$ mixing, long distance contributions are potentially significant in $K - \bar{K}$ mixing. As we do not know how to calculate these contributions reliably, we will

just parameterize them by

$$\tilde{B}_K \equiv B_K \frac{[M_{12}(\text{LD}) + M_{12}(\text{SD})]}{M_{12}(\text{SD})}, \quad (4.1)$$

where LD (SD) stand for long (short) distance. Here, B_K is the K -system short distance mixing parameter, analogous to B_B (see (2.8)):

$$M_{12}(\text{SD}) = -\omega_K^* e^{2i\phi_K} B_K |M_{12}^{\text{VIA}}(\text{SD})|, \quad (4.2)$$

where ω_K is defined through

$$\text{CP}|K^0\rangle = \omega_K |\bar{K}^0\rangle, \quad \text{CP}|\bar{K}^0\rangle = \omega_K^* |K^0\rangle, \quad (|\omega_K| = 1). \quad (4.3)$$

The points that we would like to emphasize, concerning these parameters, are the following:

- (i) As we saw in the last section, neither $\text{sign}(\tilde{B}_K)$ nor $\text{sign}(B_K)$ affect the sign of the CP asymmetry in $B \rightarrow \psi K_S$, which depends only on the fact that K_S is (approximately) CP even.
- (ii) $\text{Sign}(\tilde{B}_K)$ does affect the sign of $\Delta m_K \equiv m(K_L) - m(K_S)$, and from the positivity, experimentally, of Δm_K , we know that $\text{sign}(\tilde{B}_K) = +1$.
- (iii) $\text{Sign}(B_K)$, which is not known from experiment and need not agree with $\text{sign}(\tilde{B}_K)$, does play an indirect, but essential, role in predicting the signs of CP asymmetries in neutral B decays. For, a reversal of $\text{sign}(B_K)$ would reverse the signs of such quantities as $\sin 2\beta$.

We now explain points (ii) and (iii) in some detail. First, we show that the experimental fact that the heavier kaon mass eigenstate is, to an excellent approximation, CP odd (or, equivalently, does not decay to final two pions), namely that (ignoring CP violation)

$$\lambda_{K \rightarrow \pi\pi} \equiv \left(\frac{q}{p}\right)_K \frac{\bar{A}_{\pi\pi}^K}{A_{\pi\pi}^K} = 1, \quad (4.4)$$

fixes the sign of \tilde{B}_K to be positive. Here, q_K and p_K are defined by

$$|K_{S,L}\rangle = p_K |K^0\rangle \pm q_K |\bar{K}^0\rangle, \quad (4.5)$$

where $L(S)$ stand for long (short), and we have chosen $\Delta\Gamma_K < 0$. It is experimentally known that the long-lived kaon is heavier [10], namely $\Delta m_K > 0$. Neglecting the CP

violating effects, which are of $\mathcal{O}(10^{-3})$, and going through the same analysis as in the B system, we find

$$\left(\frac{q}{p}\right)_K = \omega_K e^{-2i\phi_K} \text{sign}(\tilde{B}_K). \quad (4.6)$$

For the amplitude ratio, we have

$$\frac{\bar{A}_{\pi\pi}^K}{A_{\pi\pi}^K} = \eta_{\pi\pi} \omega_K^* e^{-2i\phi_{\pi\pi}}. \quad (4.7)$$

where $\eta_{\pi\pi} = +1$. We get

$$\lambda_{K \rightarrow \pi\pi} = e^{-2i(\phi_K + \phi_{\pi\pi})} \text{sign}(\tilde{B}_K). \quad (4.8)$$

Within the Standard Model, $(M_{12})_K$ is described by box diagrams with intermediate charm and up quarks, leading to

$$\phi_K = \arg(V_{cs}V_{cd}^*). \quad (4.9)$$

The $s \rightarrow u\bar{u}d$ decay is dominated by the W -mediated tree diagram (this holds model independently), leading to

$$\phi_{\pi\pi} = \arg(V_{us}^*V_{ud}). \quad (4.10)$$

With three quark generations, $\arg(V_{cs}V_{cd}^*) = \arg(V_{us}V_{ud}^*)[\text{mod } \pi]$ to within a few milliradians. (Were this not the case, we would not know ϕ_K since the long-distance part involves $V_{us}V_{ud}^*$ while the dominant box diagram in the short-distance part depends on $V_{cs}V_{cd}^*$.)

Then,

$$\lambda_{K \rightarrow \pi\pi} = \text{sign}(\tilde{B}_K) \implies \text{sign}(\tilde{B}_K) = +1. \quad (4.11)$$

Next, we would like to ask whether we can tell the sign of $\sin 2\beta$ from the existing measurements of CP violation in K decays? Note that all angles of the unitarity triangle are either in the range $\{0, \pi\}$ or in the range $\{\pi, 2\pi\}$. Then, if we know $\text{sign}(\sin \phi)$, where ϕ is any of the three angles of the unitarity triangle, then we know $\text{sign}(\sin \beta)$. Furthermore, as $|V_{ub}/V_{cb}| \leq 0.10$ (it suffices here that $|V_{ub}/V_{cb}| \leq \sin \theta_C = 0.22$, namely that β is either in the range $\{0, \pi/2\}$ or $\{3\pi/2, 2\pi\}$), we learn that $\text{sign}(\sin \beta) = \text{sign}(\sin 2\beta)$. The question is then whether the measurement of ϵ_K tells us unambiguously $\text{sign}(\sin \phi)$.

To find the answer, we have to analyze precisely those $\mathcal{O}(10^{-3})$ effects that we neglected in (4.4) and write instead

$$\lambda_{K \rightarrow \pi\pi} = 1 - 2\varepsilon_K. \quad (4.12)$$

(This expression holds to zeroth order in A_2/A_0 , where A_I is the decay amplitude into two pions in isospin I state. To first order in A_2/A_0 , it is $\lambda_0 = (q/p)_K(\bar{A}_0/A_0)$ which appears on the left hand side of (4.12). However, this distinction is irrelevant to our discussion here.) Naively, using

$$\lambda_{K \rightarrow \pi\pi} = \text{sign}(\tilde{B}_K) \left(\frac{V_{cs}^* V_{cd}}{V_{cs} V_{cd}^*} \right) \left(\frac{V_{ud}^* V_{us}}{V_{ud} V_{us}^*} \right), \quad (4.13)$$

we would conclude that we get a clean determination of one small phase. However, as ε_K is of $\mathcal{O}(10^{-3})$, we need to include other effects of this order that we neglected in M_{12}^K , particularly the small phase difference between M_{12} and Γ_{12} and the contributions proportional to $V_{ts} V_{td}^*$ and $(V_{ts} V_{td}^*)^2$. After a lengthy but well-known and straightforward calculation [6], the resulting constraint is

$$\text{sign}(B_K) \sin \gamma > 0, \quad (4.14)$$

where

$$\gamma \equiv \arg \left[-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right]. \quad (4.15)$$

Note that it is indeed B_K which appears in (4.14) and not the \tilde{B}_K parameter defined in (4.1). (The long distance contributions to M_{12} are in phase with Γ_{12} and therefore do not contribute.) Consequently, we cannot say that the sign of B_K is experimentally determined. Only if the LD contribution is smaller than the SD one, or if it is large but has the same sign as the SD one, then $\text{sign}(\tilde{B}_K) = \text{sign}(B_K)$. However, if we are not willing to state that $|M_{12}^K(\text{LD})| < |M_{12}^K(\text{SD})|$, then the Standard Model result that $\sin \gamma > 0$ depends on the validity of the VIA at least to the extent that $B_K > 0$ [7]. (Lattice calculations [11,8], the $1/N$ approach [12-13], QCD sum rules [14-15], and various other methods [16-20] support $B_K > 0$.)

5. Conclusions

To summarize our main points:

1. The Standard Model predictions for the values of CP asymmetries in B^0 decays into final CP eigenstates are independent of the values of hadronic parameters. However, the sign of all asymmetries depend on the sign of B_B , that is the ratio between the short distance contributions to $B - \bar{B}$ mixing and their value in the vacuum insertion approximation.

2. In decays into final states with a single neutral kaon, where the kaon is identified by its decay to two pions, there is no dependence on the phase of $K - \bar{K}$ mixing. Perhaps a better way of making this statement is to say that the relevant phase is known experimentally.

3. Still, the Standard Model predictions for the sign of the asymmetries depends on information from ε_K which does depend on the sign of B_K (the analog of B_B for the K system).

4. The sign of B_K is not known experimentally. The experimental fact that the heavier neutral kaon is, to an excellent approximation, CP odd, fixes the sign of another parameter, \tilde{B}_K , which (unlike B_K) depends also on the long distance contributions to $K - \bar{K}$ mixing. If long distance contributions are larger than the short distance ones, the sign of B_K could, in principle, differ from the sign of \tilde{B}_K .

We emphasize that, while we gave the two explicit examples of $B \rightarrow D^+D^-$ and $B \rightarrow \psi K_S$, the same analysis holds for any B decays into final CP eigenstates that are dominated by a single weak phase.

Very likely, the vacuum insertion approximation is a reasonable approximation for the matrix elements of the $\Delta b = 2$ and $\Delta s = 2$ four-quark operators. However, one has to bear in mind that the Standard Model predictions are not entirely independent of this approximation:

- (i) If $B_B < 0$ and $B_K > 0$, all the asymmetries will have an opposite sign to the standard prediction;
- (ii) If $B_K < 0$ (which requires that the long distance contributions to Δm_K are larger in magnitude and opposite in sign to the short distance ones) and $B_B > 0$ then, again,

- all the asymmetries will have an opposite sign to the standard prediction;
- (iii) If $B_B < 0$ and $B_K < 0$, all the asymmetries will have the predicted sign because the two sign errors cancel.

If, as expected, experiments find $\text{Im}\lambda_{D^+D^-} < 0$ and $\text{Im}\lambda_{\psi K_S} > 0$, it will give an experimental support (though not a completely rigorous evidence) that the vacuum insertion approximation is a reasonable method to estimate the matrix elements of the relevant four quark operators.

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