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The QCD Coupling Constant from Tau Decays*

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ABSTRACT

We show that naive perturbation theory up to $0(\alpha_s^3)$ underestimates the theoretical prediction for $R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau + hadrons]}{\Gamma[\tau^- \to \nu_\tau e^- \overline{\nu}_e]}$. We use Padé Summation (PS) and find $\alpha_s(M_\tau) = .307\ (9)$. Using the new 4-loop QCD β-function, we find $\alpha_s(M_Z) = .1164\ (14)$ in agreement with recent values.

The τ lepton is the only known lepton massive enough to decay into hadrons and, hence, its semileptonic decays are important tests of perturbative QCD. Since its discovery in 1975 at the SPEAR e^+e^- storage ring[1], the τ lepton[2] has been a subject of extensive experimental study. In particular the ratio

$$R_\tau \equiv \frac{\Gamma[\tau^- \to \nu_\tau + hadrons]}{\Gamma[\tau^- \to \nu_\tau e^- \overline{\nu}_e]} \tag{1}$$

provides a clean way of determining $\alpha_s(M_\tau)$, the strong coupling constant at the mass of the τ,

$$M_\tau = 1777.1^{+0.4}_{-0.5}\ MeV \tag{2}$$

The experimental average value[3] is

$$R_\tau^{exp} = 3.649(14) \tag{3}$$

Theoretically, R_τ is given by[4]

$$R_\tau = 3(|V_{ud}|^2 + |V_{us}|^2)\ S_{EW}\ [1 + \delta^1_{EW} + \delta^{(0)} + \delta^1] \tag{4}$$

where

$$\begin{aligned} S_{EW} &= 1.0194 \\ and\ \delta^1_{EW} &= .0010 \end{aligned} \tag{5}$$

are the known electroweak corrections, and

$$\delta^1 = -.007(4) \tag{6}$$

is the non-perturbative contribution. We will include the small mass corrections in determining our final error-bars. The perturbative contribution is given by[5]

$$\delta^{(0)} = x + 5.20x^2 + 26.36x^3 + ax^4 \tag{7}$$

where $x = \frac{\alpha_s}{\pi}$ and a has not yet been calculated. However, from our Asymptotic Padé Approximant Prediction (APAP) method we find[6]

$$a = 108 \tag{8}$$

This agrees with the prediction of Kataev and Starshenko[7]

$$a = 105.5 \tag{9}$$

Groote et al and Pich found that

$$\alpha_s(M_\tau) = \begin{cases} .375(7) & ref\ 8 \\ .378(7) & ref\ 8 \\ .354(5) & ref\ 8 \\ .355(25) & ref\ 9 \end{cases} \tag{10}$$

However, if one uses their results it is clear that the perturbation series without the $0(x^4)$ term is converging very slowly. For example for

$$\begin{aligned} \alpha_s(M_\tau) &= .36,\ x = .1146 \\ \delta^{(0)} &= .1146 + .0683 + .0397 + (.0186) \\ &= .2226(.2412) \end{aligned} \tag{11}$$

is clearly not an accurate value for the sum of the series. Even if one includes the ax^4 term (in parenthesis), the perturbation series is still not close to the actual sum of the series. For this reason we will use Padé Summation (PS) to the series.

From eq (7) we construct the [2/2] Padé Approximant (PA). (For details on PA see refs (10 - 13.))

$$[2/2] = \frac{1 - 39.13x + 147.5x^2}{1 - 40.13x + 182.4x^2} \tag{12}$$

We will use the [1/1], [2/1], and [1/2] to estimate our error-bars. By fitting $\delta^{(0)}$ from eq (4),

$$\delta^{(0)} = .99(5), \tag{13}$$

to the [2/2] PA in eq. (12) we obtain

$$\alpha_s(M_\tau) = .307(9) \tag{14}$$

The error-bar is determined by studying how x changes with different PA, mass corrections, and the experimental error in eq. (3). Our result is considerably smaller than the previous results in eq. (10). This is due to the PS which we used to sum the series. In fact, if we restrict ourselves to the perturbation series only up to $0(x^3)$ in eq. (7) we obtain

$$\alpha_s(M_\tau) = \begin{cases} .364 & (wc) \\ .354 & (woc) \end{cases} \tag{15}$$

where (wc) includes all the corrections in eq. (4) while (woc) includes only the naive perturbation series. It is clear that eqs. (15) are consistent with the previous results in eq. (10).

Using our result in eq. (14) we now evolve up to M_Z to determine $\alpha_s(M_Z)$. We use the new 4-loop evolution equations of Chetyrkin et al[14] and the relations between $\Lambda^{(f-1)}$ and $\Lambda^{(f)}$ of Samuel and

Li.[15] These relations differ from those of Marciano[16] by approximately 3%. It is interesting to note that the 4-loop result is needed at low energies to get an accurate value of $\Lambda^{(3)}$, though it is negligible at higher energies. We find from eq. (14) that

$$\begin{aligned} \Lambda^{(3)} &= 315(18)\ MeV \\ \Lambda^{(4)} &= 268(18)\ MeV \\ \textit{and}\ \Lambda^{(5)} &= 190(18)\ MeV \end{aligned} \qquad \textbf{(16)}$$

From $\Lambda^{(5)}$ we find

$$\alpha_s(M_Z) = .1164(14) \qquad \textbf{(17)}$$

It has been claimed that recent measurements of $\alpha_s(M_Z)$ may be grouped into two classes, those made at "low$-Q^2$ ", which tend to cluster at values around 0.112, and those made at "high $-$ Q^2 ", which tend to cluster at values around 0.123. For a review of these measurements see Ref (17). However, examination of the large set of $\alpha_s(M_Z)$ measurements reveals that in fact all are consistent with a "world average central value" of about 0.117, with an uncertainty of $\pm$ .005, and that their grouping into two supposedly discrepant classes is arbitrary and not significant. For example, careful analysis of the SLD Collaboration results[18] of $\alpha_s(M_Z)$ from 15 hadronic event shape observables in e^+e^- annihilation revealed that next-to-leading QCD corrections were not sufficient to properly analyze the data. Using PA to estimate higher-order terms it was found[19] that the scatter in the results was reduced by a factor of more than three. Moreover, the average value of $\alpha_s(M_Z)$ was reduced from

$$\alpha_s(M_Z) = .1226(25)(109) \qquad \textbf{(18)}$$

to

$$\alpha_s(M_Z) = .1147(35) \qquad \textbf{(19)}$$

very close to the result in eq. (17). Moreover values of $\alpha_s(M_Z)$ from Lattice Theory[20] have increased recently and are now consistent with the result in eq. (17). For example ref 20 obtains $\alpha_s(M_Z) = .1174(24)$.

Finally, it is interesting to note the following. In a recent paper by Groote et al[21] the authors obtained a relation between the 2 physical observables R_τ and R, where

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{20}$$

They note that "the data seem to be systematically lower than the prediction by about 7%." The effect, discussed here, which accelerates the convergence of the R_τ naive perturbation theory with PS is in the right direction and of the required magnitude to resolve this discrepancy.

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