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CP Violation in K and B Decays *

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Abstract

We review basic aspects of the phenomenology of CP violation in the decays of K and B mesons. In particular we discuss the commonly used classification of CP violation – CP violation in the mass matrix, in the interference of mixing with decay, and in the decay amplitude itself – and the related notions of direct and indirect CP violation. These concepts are illustrated with explicit examples. We also emphasize the highlights of this field including the clean observables $B(K_L \to \pi^0 \nu\bar{\nu})$ and $\mathcal{A}_{CP}(B \to J/\Psi K_S)$. The latter quantity serves to demonstrate the general features of large, mixing induced CP violation in B decays.

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1 Introduction

Until today CP violation has only been observed in a few decay modes of the long-lived neutral kaon, where it appears as a very small ($\mathcal{O}(10^{-3})$) effect. Despite continuing efforts since the first observation of this phenomenon in 1964 and respectable progress in both experiment and theory, our understanding of CP violation has so far remained rather limited. Upcoming new experiments with K and B mesons are likely to improve this situation substantially. The great effort being invested into these studies is motivated by the fundamental implications that CP violation has for our understanding of nature: CP violation defines an absolute, physical distinction between matter and antimatter. It is also one of the necessary conditions for the dynamical generation of the observed baryon asymmetry in the universe. In addition CP violation provides a testing ground for Standard Model flavor dynamics – the physics of quark masses and mixing.
The source of CP violation in the Standard Model (SM) is the Cabibbo-Kobayashi-Maskawa (CKM) matrix V entering the charged-current weak interaction Lagrangian

$$\mathcal{L}_{CC} = \frac{g_W}{2\sqrt{2}} V_{ij} \bar{u}_i \gamma^\mu (1-\gamma_5) d_j W^+_\mu + h.c. \tag{1}$$

where $(u_1, u_2, u_3) \equiv (u, c, t)$, $(d_1, d_2, d_3) \equiv (d, s, b)$ are the mass eigenstates of the six quark flavors and a summation over $i, j = 1, 2, 3$ is understood. The unitary CKM matrix ($V^\dagger V = 1$) arises from diagonalizing the quark mass matrix and relating the original weak eigenstates of quark flavor to the physical mass eigenstates. The off-diagonal elements of V describe the strength of weak, charged current transitions between different generations of quarks.
In general, a $n \times n$ unitary matrix has n^2 free (real) parameters. Not all of them are physical quantities in the present case since one has the freedom of redefining the $2n$ fields u_i and d_j ($i, j = 1, \ldots, n$) by arbitrary phases α_i and β_j, respectively. From (1) one sees that only the differences $\alpha_i - \beta_j$ can affect V in this redefinition. There are $2n - 1$ independent $\alpha_i - \beta_j$. The number of independent, physical parameters that characterize V is therefore $n^2 - (2n-1) = (n-1)^2$. Out of these $(n-1)^2$, $n(n-1)/2$, the number of parameters of a real, orthogonal $n \times n$ matrix, represent rotation angles. The remaining $(n-1)(n-2)/2$ are complex phases. Obviously, then, for one or two generations of quarks the matrix V can be chosen to be real. For the realistic case of three generations, however, a physical complex phase is in general present in V [1]. As a consequence, if this phase $\delta \neq 0, \pi$, the weak interaction Lagrangian (1) is not invariant under CP. (A further requirement for this to be true is that all three up-type quark masses must be different from each other and the same must hold for the down-type quarks. Otherwise an arbitrary unitary rotation may be performed on the degenerate quark fields and the complex phase be removed. Also, none of the rotation angles must be 0 or $\pi/2$.) In the sections following this Introduction we will discuss how this violation of CP symmetry at the level of the fundamental Lagrangian manifests itself in observable CP asymmetries occuring in the weak decays of K and B mesons.
The CKM matrix has the following explicit form

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\varrho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{2}$$

where the second expression is a convenient parametrization in terms of λ, A, ϱ and η due to Wolfenstein. It is organized as a series expansion in powers of $\lambda = 0.22$ (the sine of the Cabibbo angle) to exhibit the hierarchy among the transitions between generations. Ordering transitions $i \to j$ according to decreasing strength, this hierarchy reads $i \to i > 1 \to 2 > 2 \to 3 > 1 \to 3$, as is manifest in (2). The explicit parametrization shown in (2) is valid through order $\mathcal{O}(\lambda^3)$, an approxiamtion that is sufficient for most practical applications. Higher order terms can be taken into account if necessary [2].

The unitarity structure of the CKM matrix is conventionally displayed in the so-called unitarity triangle (Fig. 1). This triangle is a graphical representation of the unitarity relation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ (normalized by $-V_{cd}V_{cb}^*$) in the complex plane of Wolfenstein parameters (ϱ, η). The angles α, β and γ of the unitarity triangle are phase convention independent and can be determined in CP violation experiments. The area of the unitarity triangle, which is proportional to η, is a measure of CP nonconservation in the Standard Model.

The framework for a theoretical treatment of weak decays in general, and CP violating processes in particular, is provided by low energy effective Hamiltonians, which have the generic form

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(m_t, M_W/\mu, \alpha_s) Q_i \tag{3}$$

Here G_F is the Fermi constant, V_{CKM} the appropriate combination of CKM elements, C_i are Wilson coefficients, which include also strong interaction effects, and the Q_i are local four-fermion operators. (3) provides a systematic approximation that applies to processes where the relevant energy scale is much smaller than the W-boson or the top quark mass, such as for instance K and B meson decays. An example for a typical operator is $Q_2 = (\bar{s}u)_{V-A}(\bar{u}d)_{V-A}$, which appears in the analysis of nonleptonic kaon decays. In essence the operators Q_i are nothing else than (effective) interaction vertices and the coefficients C_i the corresponding coupling constants. The Hamiltonians (3) can be derived from the fundamental Standard Model Lagrangian using operator product expansion and renormalization group techniques. They may be viewed as the modern generalization of the original Fermi-theory of weak interactions. To calculate decay amplitudes, matrix elements of the operators have to be evaluated between

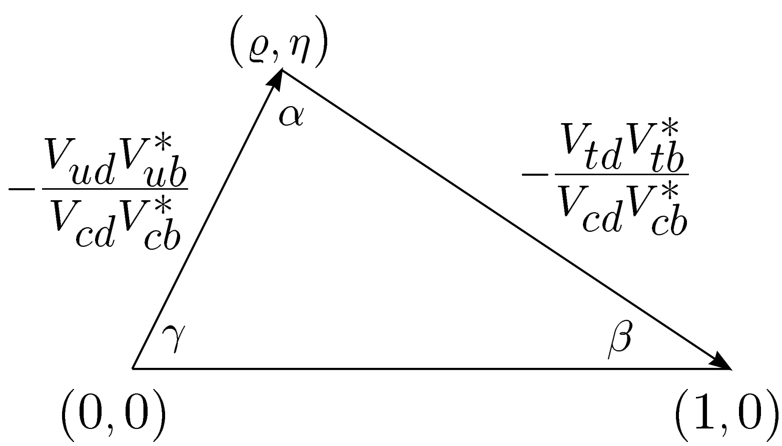

Figure 1: The normalized unitarity triangle in the (ϱ, η) plane

the initial and final states under consideration. This is a problem that involves nonperturbative QCD dynamics – in general a difficult task not yet satisfactorily solved in many cases. The coefficients C_i on the other hand are calculable in perturbation theory, as they incorporate the short distance contributions to the decay amplitude. The factorization of short distance and long distance contributions (Wilson coefficients and operator matrix elements, respectively), inherent in the effective Hamiltonian approach, is a key feature of this framework. Although we will not further elaborate on these issues here, the effective Hamiltonian picture should be kept in mind as the theoretical basis for weak decay phenomenology. A review of the current status of this subject as well as an introduction to the basic concepts may be found in [2]. For a general introduction to CP violation see [3].
The outline of this talk is as follows. After this Introduction we briefly recall the physics of particle-antiparticle mixing, which is crucial for the discussion of CP violation in neutral K and B meson decays. We then describe a classification of CP violating phenomena in K and B decays. To illustrate the concepts we will here use kaon processes as specific examples. Subsequently we discuss the rare decay mode $K_L \to \pi^0 \nu\bar{\nu}$ and some of the basic issues of CP violating asymmetries in B decays. A short summary concludes our presentation.

2 Particle-Antiparticle Mixing

Neutral K and B mesons can mix with their antiparticles through second order weak interactions. They form two-state systems ($K^0 - \bar{K}^0$, $B_d - \bar{B}_d$, $B_s - \bar{B}_s$) that are described by Hamiltonian matrices $\hat{H}$ of the form

$$\hat{H} = \begin{pmatrix} M_{11} & M_{12} \\ M^*_{12} & M_{11} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma^*_{12} & \Gamma_{11} \end{pmatrix} \tag{4}$$

where CPT invariance has been assumed. The absorptive part Γ_{ij} of $\hat{H}$ accounts for the weak decay of the neutral meson $F = K,\ B_d,\ B_s$. In Fig. 2 we show typical diagrams that give rise to the off-diagonal elements of $\hat{H}$ for the example of the kaon system. Diagonalizing the Hamiltonian $\hat{H}$ yields the physical eigenstates $F_{H,L}$. They are linear combinations of the strong interaction eigenstates F and $\bar{F}$ and can be written as

$$F_H = \mathcal{N}_{\bar{\varepsilon}} \left[(1+\bar{\varepsilon})F + (1-\bar{\varepsilon})\bar{F} \right] \equiv pF + q\bar{F} \tag{5}$$

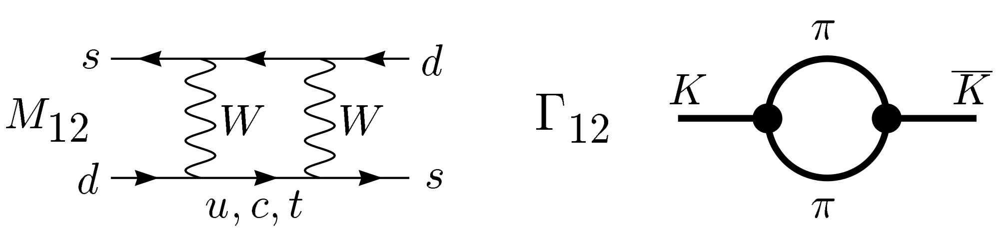

Figure 2: Diagrams contributing to M_{12} and Γ_{12} in the neutral kaon system.

Table 1: Important properties of neutral K and B meson systems. Here $\Gamma \equiv (\Gamma_H + \Gamma_L)/2$. The kaon entries and ΔM for B_d are experimental results, the remaining numbers theoretical expectations.

	K^0	B_d	B_s
$\Delta\Gamma/\Gamma$	$2.0,\ \Gamma_L = 579 \cdot \Gamma_H$	~ 0	$\sim 0.16 \pm 0.10$
$\Delta M/\Gamma$	0.95	0.73 ± 0.05	$\sim 25 \pm 15$

$$F_L = \mathcal{N}_{\bar{\varepsilon}} \left[(1+\bar{\varepsilon})F - (1-\bar{\varepsilon})\bar{F} \right] \equiv pF - q\bar{F} \tag{6}$$

with the normalization factor $\mathcal{N}_{\bar{\varepsilon}} = 1/\sqrt{2(1+|\bar{\varepsilon}|^2)}$. Here $\bar{\varepsilon}$ is determined by

$$\frac{1-\bar{\varepsilon}}{1+\bar{\varepsilon}} \equiv \frac{q}{p} = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{\left(\Delta M + \frac{i}{2}\Delta\Gamma\right)/2} \tag{7}$$

where ΔM and $\Delta\Gamma$ are the differences of the eigenvalues $M_{H,L} - i\Gamma_{H,L}/2$ corresponding to the eigenstates $F_{H,L}$

$$\Delta M \equiv M_H - M_L > 0 \qquad\qquad \Delta\Gamma \equiv \Gamma_L - \Gamma_H \tag{8}$$

The labels H and L denote, respectively, the heavier and the lighter eigenstate so that ΔM is positive by definition. We employ here the CP phase convention $CP \cdot F = -\bar{F}$. Using the SM results for M_{12}, Γ_{12} and standard phase conventions for the CKM matrix (see (2)), one finds in the limit of CP conservation ($\eta = 0$) that $\bar{\varepsilon} = 0$. With (5), (6) it follows that F_H is CP odd and F_L is CP even in this limit, which is close to realistic since CP violation is a small effect. As we shall see explicitly later on, the real part of $\bar{\varepsilon}$ is a physical observable, while the imaginary part is not. In particular $(1-\bar{\varepsilon})/(1+\bar{\varepsilon})$ is a phase convention dependent, unphysical quantity.

Important characteristics of the three cases $F = K^0$, B_d, B_s are collected in Table 1. A crucial feature of the kaon system is the very large difference in decay rates between the two eigenstates, the lighter eigenstate decaying much more rapidly than the heavier one. For the kaon system the states F_L and F_H are therefore commonly denoted as short-lived (K_S) and long-lived (K_L) eigenstates, respectively. The same hierarchy in decay rates is expected for the B_s mesons, although far less pronounced as $\Gamma_H/\Gamma_L = \mathcal{O}(1)$. In the case of B_d $\Delta\Gamma/\Gamma$ is essentially negligible. The labeling of eigenstates as heavy/light is therefore more common for B mesons. The basic reason for this pattern is the small number of decay channels for the neutral kaons. Decay into the predominant CP even two-pion final states $\pi^+\pi^-$, $\pi^0\pi^0$ is only available for K_S, but not (to first approximation) for the (almost) CP odd state K_L. The latter can decay into three pions, which however is kinematically strongly suppressed, leading to a much longer K_L lifetime. This somewhat accidental feature is absent for B mesons, which have many more decay modes due to their larger mass. We may summarize this discussion by noting that in general the following correspondence holds for the eigenstates of the neutral K and B systems. One has **H**eavy=**L**ong-lived≈CP odd, and **L**ight=**S**hort-lived≈CP even, where the CP assignments are only approximate due to CP violation.

3 Classification of CP Violation

The CP noninvariance of the fundamental weak interaction Lagrangian leads to a violation of CP symmetry at the phenomenological level, in particular in decays of K and B mesons. For instance, processes forbidden by CP symmetry may occur or transitions related to each other by CP conjugation may have a different rate. The phenomenology of CP violating decays is very rich, already for kaons and even more so for B mesons. In this situation it is certainly helpful to have a classification of the various possible mechanisms at hand. One that is commonly used in the literature on this subject employs the following terminology.

a) *CP violation in the mixing matrix.* This type of effect is based on CP violation in the two-state mixing Hamiltonian $\hat{H}$ (4) itself and is measured by the observable quantity $\text{Im}(\Gamma_{12}/M_{12})$. It is related to a change in flavor by two units, $\Delta S(\Delta B) = 2$.

b) *CP violation in the decay amplitude.* This class of phenomena is characterized by CP violation originating directly in the amplitude for a given decay. It is entirely independent of particle-antiparticle mixing and can therefore occur for charged mesons ($K^{\pm}$, $B^{\pm}$) as well. Here the transitions have $\Delta S(\Delta B) = 1$.

c) *CP violation in the interference of mixing and decay.* In this case the interference of two amplitudes, necessary in general to induce observable CP violation, takes place between the mixing amplitude and the decay amplitude in decays of neutral K and B mesons. This very important class is sometimes also refered to as *mixing-induced* CP violation, a terminology not to be confused with a).

Complementary to this classification is the widely used notion of *direct* versus *indirect* CP violation. It is motivated historically by the hypothesis of a new superweak interaction [4, 5], that was proposed as early as 1964 by Wolfenstein to account for the CP violation observed in $K_L \to \pi^+\pi^-$ decay. This new CP violating interaction would lead to a local four-quark vertex that changes flavor quantum number (strangeness or beauty) by two units. Its only effect would be a CP violating contribution to M_{12}, so that all observed CP violation could be attributed to particle-antiparticle mixing alone. Today, after the advent of the three generation SM, the CKM mechanism of CP violation appears more natural. In principal the superweak scenario represents a logical possibility, leading to a different pattern of observable CP violation effects. In fact, all experimental measurements available to date are still consistent with the superweak hypothesis.

Now, any CP violating effect that can be entirely assigned to CP violation in M_{12} (as for the superweak case) is termed *indirect CP violation*. Conversely, any effect that can not be described in this way and explicitly requires CP violating phases in the decay amplitude itself is called *direct CP violation*. It follows that class a) represents indirect, class b) direct CP violation. Class c) contains aspects of both. In this latter case the magnitude of CP violation observed in any one decay mode (within the neutral kaon system, say) could by itself be ascribed to mixing, thus corresponding to an indirect effect. On the other hand, a difference in the degree of CP violation between two different modes would reveal a direct effect.

The classification a) – c) is especially common in the context of B physics but it applies to kaon physics as well. To emphasize this point and to provide concrete examples for the above general concepts, we will next illustrate these classes by important applications in kaon decays. We will also use this opportunity to discuss several aspects of kaon CP violation in more detail. After all K physics is the area from which our entire present experimental knowledge of CP violation derives. For a general review of CP violation in kaon decays see [6].

a) – Lepton Charge Asymmetry

The lepton charge asymmetry in semileptonic K_L decay is an example for CP violation in the mixing matrix. It is probably the most obvious manifestation of CP nonconservation in kaon decays. The observable considered here reads ($l = e$ or μ)

$$\begin{aligned}\Delta &= \frac{\Gamma(K_L \to \pi^- l^+ \nu) - \Gamma(K_L \to \pi^+ l^- \bar{\nu})}{\Gamma(K_L \to \pi^- l^+ \nu) + \Gamma(K_L \to \pi^+ l^- \bar{\nu})} = \frac{|1+\bar{\varepsilon}|^2 - |1-\bar{\varepsilon}|^2}{|1+\bar{\varepsilon}|^2 + |1-\bar{\varepsilon}|^2} \\ &\approx 2\text{Re}\,\bar{\varepsilon} \approx \frac{1}{4}\text{Im}\frac{\Gamma_{12}}{M_{12}} \end{aligned} \tag{9}$$

If CP was a good symmetry of nature, K_L would be a CP eigenstate and the two processes compared in (9) were related by a CP transformation. The rate difference Δ should vanish. Experimentally one finds however [7]

$$\Delta_{exp} = (3.27 \pm 0.12) \cdot 10^{-3} \tag{10}$$

a clear signal of CP violation. The second equality in (9) follows from (5), as applied to K_L, noting that the positive lepton l^+ can only originate from $K \sim (\bar{s}d)$, l^- only from $\bar{K} \sim (\bar{d}s)$. This is true to leading order in SM weak interactions and holds to sufficient accuracy for our purpose. The charge of the lepton essentially serves to tag the strangeness of the K, thus picking out either only the K or only the $\bar{K}$ component. Any phase in the semileptonic amplitudes is irrelevant and the CP violation effect is purely in the mixing matrix itself. In fact, as indicated in (9), Δ is determined by $\text{Im}(\Gamma_{12}/M_{12})$, the physical measure of CP violation in the mixing matrix.

From (10) we see that $\Delta > 0$. This empirical fact can be used to define positive electric charge in an absolute, physical sense. Positive charge is the charge of the lepton more copiously produced in semileptonic K_L decay. This definition is unambiguous and would even hold in an antimatter world. Also, using some parity violation experiment, this result implies in addition an absolute definition of left and right. These are quite remarkable facts. They clearly provide part of the motivation to try to learn more about the origin of CP violation.

b) – CP Violation in the Decay Amplitude

Observable CP violation may also occur through interference effects in the decay amplitudes themselves (pure direct CP violation). This case is conceptually perhaps the simplest mechanism for CP violation and the basic features are here particularly transparent. Consider a situation where two different components contribute to the amplitude of a K meson decaying into a final state f

$$A \equiv A(K \to f) = A_1 e^{i\delta_1} e^{i\phi_1} + A_2 e^{i\delta_2} e^{i\phi_2} \tag{11}$$

Here A_i ($i = 1, 2$) are real amplitudes and δ_i are complex phases from CP conserving interactions. The δ_i are usually strong interaction rescattering phases. Finally the ϕ_i are weak phases, that is phases coming from the CKM matrix in the SM. The corresponding amplitude for the CP conjugated process $\bar{K} \to \bar{f}$ then reads (the explicit minus signs are due to our convention $CP \cdot K = -\bar{K}$, $(CP \cdot f = \bar{f})$)

$$\bar{A} \equiv A(\bar{K} \to \bar{f}) = -A_1 e^{i\delta_1} e^{-i\phi_1} - A_2 e^{i\delta_2} e^{-i\phi_2} \tag{12}$$

Since now all quarks are replaced by antiquarks (and vice versa) compared to (11), the weak phases change sign. The CP invariant strong phases remain the same. From (11) and (12) one finds immediately

$$|A|^2 - |\bar{A}|^2 \sim A_1 A_2 \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2) \tag{13}$$

The conditions for a nonvanishing difference between the decay rates of $K \to f$ and the CP conjugate $\bar{K} \to \bar{f}$, that is direct CP violation, can be read off from (13). There need to be two interfering amplitudes A_1, A_2 and these amplitudes must simultaneously have both different weak (ϕ_i) and different strong phases (δ_i). Although the strong interaction phases can of course not generate CP violation by themselves, they are still a necessary requirement for the weak phase differences to show up as observable CP asymmetries. It is obvious from (11) and (12) that in the absence of strong phases A and $\bar{A}$ would have the same absolute value despite their different weak phases, since then $A = -\bar{A}^*$.
A specific example is given by the decays $K(\bar{K}) \to \pi^+\pi^-$ (here $f = \pi^+\pi^- = \bar{f}$). The amplitudes can be written as

$$\begin{aligned} A_{+-} &= \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \frac{1}{\sqrt{3}} A_2 e^{i\delta_2} \\ \bar{A}_{+-} &= -\sqrt{\frac{2}{3}} A_0^* e^{i\delta_0} - \frac{1}{\sqrt{3}} A_2^* e^{i\delta_2} \end{aligned} \tag{14}$$

where $A_{0,2} = \langle \pi\pi(I = 0, 2)|\mathcal{H}_W|K\rangle$ are the transition amplitudes of K to the isospin-0 and isospin-2 components of the $\pi^+\pi^-$ final state. They still include the weak phases, but the strong phases have been factored out and written explicitly in (14). Taking the modulus squared of the amplitudes we get

$$\begin{aligned} \frac{\Gamma(K \to \pi^+\pi^-) - \Gamma(\bar{K} \to \pi^+\pi^-)}{\Gamma(K \to \pi^+\pi^-) + \Gamma(\bar{K} \to \pi^+\pi^-)} &= \sqrt{2} \sin(\delta_0 - \delta_2) \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right) \\ &= 2 \text{ Re } \varepsilon' \end{aligned} \tag{15}$$

The quantity so defined is just twice the real part of the famous parameter ε', the measure of direct CP violation in $K \to \pi\pi$ decays. The real parts of $A_{0,2}$ can be extracted from experiment. The imaginary parts have to be calculated using the effective Hamiltonian formalism briefly sketched in the Introduction. Ultimately the amplitudes derive from quark level diagrams. The most important contributions, the gluon penguin and the electroweak penguin, are depicted in Fig. 3. The importance of the electroweak penguin graph might be surprising at first sight; after all it is a contribution suppressed by small electroweak couplings compared to the strong interaction effect represented by the gluon penguin diagram. However, there are several circumstances that actually conspire so as to enhance the impact of the electroweak

sector substantially. First of all, the electroweak diagrams contribute to ImA_2, in contrast to the gluon penguins, which correspond to pure $\Delta I = 1/2$ operators (the gluon coupling conserves isospin) and can only lead to an isospin-0 final state, starting from a kaon with isospin 1/2. Furthermore, the suppression $\sim \alpha/\alpha_s$ from coupling constants is largely compensated by the fact that Re$A_0 \gg$ ReA_2, reflecting the empirical $\Delta I = 1/2$ rule in nonleptonic kaon decays. In addition the electroweak contribution grows strongly with the top quark mass [8, 9] and turns out to be quite substantial for the actual value $\bar{m}_t(m_t) = 167\ GeV$ ($\overline{MS}$-mass). Entering with sign opposite to the (positive) gluon penguin contribution, the electroweak penguin contribution tends to cancel the latter. This feature makes a precise theoretical prediction of ε', which anyhow suffers from large hadronic uncertainties, even more difficult. The typical order of magnitude of ε' can however be understood from (15). The size of ImA_i/ReA_i is essentially determined by the small CKM parameters that carry the complex phase and which are related to the top quark in the loop diagrams from Fig. 3. Roughly speaking ImA_i/Re$A_i \sim$ Im$V^*_{ts}V_{td} \sim 10^{-4}$. Empirically we have, from the $\Delta I = 1/2$ rule, ReA_2/Re$A_0 \sim 10^{-2}$. This leads to a natural size of ε' of $\sim 10^{-6}$, or possibly even smaller due to the cancellations mentioned before.
We should stress that the quantity in (15) is not the observable actually used to determine ε' experimentally. We have discussed it here because it is of conceptual interest as the simplest manifestation of ε'. The realistic analysis requires a more general consideration of $K_L, K_S \to \pi\pi$ decays to which we will turn in the following paragraph.

c) – Mixing Induced CP Violation in $K \to \pi\pi$: ε, ε'

In this section we will illustrate the concept of mixing-induced CP violation with the example of $K \to \pi\pi$ decays. These are important processes, since CP violation has first been seen in $K_L \to \pi^+\pi^-$ and as of today our most precise experimental knowledge about this phenomenon still comes from the study of $K \to \pi\pi$ transitions. There are two distinct final states and in a strong interaction eigenbasis the transitions are $K^0, \bar{K}^0 \to \pi\pi(I = 0), \pi\pi(I = 2)$, with definite isospin for $\pi\pi$. Alternatively, using the physical eigenbasis for both initial and final states, one has $K_L, K_S \to \pi^+\pi^-, \pi^0\pi^0$.
Consider next the amplitude for K_L going into the CP even state $\pi\pi(I = 0)$, which can

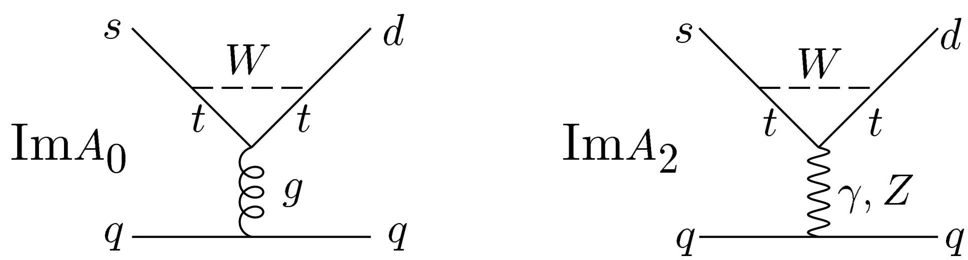

Figure 3: Gluon penguin and electroweak penguin diagram contributions to the parameter ε'.

proceed via K ($\sim (1+\bar{\varepsilon})A_0$) or via $\bar{K}$ ($\sim (1-\bar{\varepsilon})A_0^*$). Hence (to first order in small quantities)

$$A(K_L \to \pi\pi(I=0)) \sim (1+\bar{\varepsilon})A_0 e^{i\delta_0} - (1-\bar{\varepsilon})A_0^* e^{i\delta_0} \sim \bar{\varepsilon} + i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} = \varepsilon \tag{16}$$

This defines the parameter ε, characterizing mixing-induced CP violation. Note that ε involves a component from mixing ($\bar{\varepsilon}$) as well as from the decay amplitude ($\mathrm{Im}A_0/\mathrm{Re}A_0$). Neither of those is physical separately, but ε is. Note also that the physical quantity $\mathrm{Re}\bar{\varepsilon}$ discussed above satisfies $\mathrm{Re}\bar{\varepsilon} = \mathrm{Re}\varepsilon$. More generally one can form the following two CP violating observables

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \qquad \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \tag{17}$$

These amplitude ratios involve the physical initial and final states and are directly measurable in experiment. They are related to ε and ε' through

$$\eta_{+-} = \varepsilon + \varepsilon' \qquad \eta_{00} = \varepsilon - 2\varepsilon' \tag{18}$$

The phase of ε is given by $\varepsilon = |\varepsilon| \exp(i\pi/4)$. The relative phase between ε' and ε can be determined theoretically. It is close to zero so that to very good approximation $\varepsilon'/\varepsilon = \mathrm{Re}\varepsilon'/\varepsilon$. Both η_{+-} and η_{00} measure mixing-induced CP violation (interference between mixing and decay). Each of them considered separately could be attributed to CP violation in $K - \bar{K}$ mixing and would therefore represent indirect CP violation. On the other hand, a nonvanishing difference $\eta_{+-} - \eta_{00} = 3\varepsilon' \neq 0$ is a signal of direct CP violation. Experimentally one has [7]

$$|\varepsilon| = (2.282 \pm 0.019) \cdot 10^{-3} \tag{19}$$

Theoretically ε is related to the first diagram shown in Fig. 2. Comparison of the theoretical expression [2] with the experimental result yields an important constraint on the CKM phase δ (this is the phase of the CKM matrix in standard parametrization [7]; it coincides with the phase γ of the unitarity triangle). The quantity ε' can be measured as the ratio $\mathrm{Re}\varepsilon'/\varepsilon \approx \varepsilon'/\varepsilon$ using the double ratio of rates

$$\left|\frac{\eta_{+-}}{\eta_{00}}\right|^2 \doteq 1 + 6\ \mathrm{Re}\frac{\varepsilon'}{\varepsilon} \tag{20}$$

Currently the following measurements are available

$$\mathrm{Re}\frac{\varepsilon'}{\varepsilon} = \left\{ \begin{array}{ll} (23 \pm 7) \cdot 10^{-4} & \text{CERN NA31} \\ (7.4 \pm 5.9) \cdot 10^{-4} & \text{FNAL E731} \end{array} \right. \tag{21}$$

These results are somewhat inconclusive and it remains presently still open whether or not a direct CP violation effect exists in $K \to \pi\pi$ decays. As mentioned before, the theoretical predictions suffer from large hadronic uncertainties. A representative range from a recent analysis of Buras et al. [10] is

$$2 \cdot 10^{-4} \le \varepsilon'/\varepsilon \le 19 \cdot 10^{-4} \tag{22}$$

Similar results have been obtained by other groups [11, 12, 13]. Currently running or future experiments at CERN, FNAL and Frascati aim at an improved sensitivity of $\Delta\varepsilon'/\varepsilon \approx 10^{-4}$. If ε'/ε is not too small, the new round of measurements has a good chance to finally resolve the question of direct CP violation in $K \to \pi\pi$ experimentally.

4 The Rare Decay $K_L \to \pi^0 \nu\bar{\nu}$

One of the most promising opportunities for future studies of flavor physics and CP violation is the rare decay $K_L \to \pi^0 \nu\bar{\nu}$. This process combines strongly motivated phenomenological interest (sensitivity to high energy scales, top quark mass and CKM couplings) with a situation where all theoretical uncertainties are exceedingly well under control. With these features $K_L \to \pi^0 \nu\bar{\nu}$ is unparalleled in the phenomenology of weak decays.
$K_L \to \pi^0 \nu\bar{\nu}$ is a flavor-changing neutral current process, induced at one-loop order in the SM. It proceeds entirely through short distance weak interactions because the neutrinos can couple only to heavy gauge bosons (W, Z). The transition can be effectively described by a local $(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$ interaction (and $h.c.$), whose coupling strength is calculable from the SM. This interaction is semileptonic and the required hadronic matrix element $\langle \pi^0 | (\bar{s}d)_V | K^0 \rangle$ can be extracted from the well measured decay $K^+ \to \pi^0 e^+ \nu$ using isospin symmetry. The knowledge of short distance QCD effects at next-to-leading order ($\mathcal{O}(\alpha_s)$) [14], essentially eliminates the dominant theoretical uncertainty in this decay mode from scale dependence. The process is theoretically under control to an accuracy of better than $\pm 3\%$.
In the limit of conserved CP, the relevant hadronic matrix element would be $\langle \pi^0 | (\bar{s}d)_V + (\bar{d}s)_V | K_L \rangle$. Because of the CP properties of K_L, π^0 and the transition current this matrix element is zero in this limit. In the SM $K_L \to \pi^0 \nu\bar{\nu}$ therefore measures a violation of CP symmetry. It belongs to the class of mixing-induced CP violation. Considering the amplitude ratio $\eta_{\pi^0\nu\bar{\nu}} = A(K_L \to \pi^0 \nu\bar{\nu})/A(K_S \to \pi^0 \nu\bar{\nu})$, which is analogous to η_{+-} for $K \to \pi^+\pi^-$ (17), one finds $\eta_{\pi^0\nu\bar{\nu}} = \mathcal{O}(1)$ in the SM, essentially because $K \to \pi^0 \nu\bar{\nu}$ is a rare decay. Thus we have $\eta_{\pi^0\nu\bar{\nu}} \gg \eta_{+-} = \mathcal{O}(10^{-3})$, which means that $K_L \to \pi^0 \nu\bar{\nu}$ is a signal of very large direct CP violation within the SM. The branching ratio $B(K_L \to \pi^0 \nu\bar{\nu})$ is proportional to $(\mathrm{Im} V_{ts}^* V_{td})^2$, which makes it an ideal measure of $\mathrm{Im} V_{ts}^* V_{td}$ or the parameter η.
The current SM prediction for the branching ratio is $B(K_L \to \pi^0 \nu\bar{\nu}) = (2.8 \pm 1.7) \cdot 10^{-11}$ [15], where the sizable range reflects our presently still quite limited knowledge of CKM parameters, but not intrinsic theoretical uncertainties, which are negligible. Using the experimental limit on $K^+ \to \pi^+ \nu\bar{\nu}$, a model independent upper bound can be set at $B(K_L \to \pi^0 \nu\bar{\nu}) < 1.1 \cdot 10^{-8}$ [16]. Current experimental searches, not optimized for this process, have yielded a (published [7]) upper bound of $5.8 \cdot 10^{-5}$ (Fermilab E799). Dedicated experiments will aim at an actual measurement of $K_L \to \pi^0 \nu\bar{\nu}$ in the future. A proposal already exists at Brookhaven (BNL E926) and there are further plans at Fermilab and KEK.

5 CP Violation in B Decays

Decays of B mesons offer a wide range of possibilities to expand our knowledge of CP violation and to test further what we have learned from the kaon system. Among those are truly superb opportunities with esssentially no theoretical uncertainty and predicted large CP asymmetries. The prototype observable is the time-dependent CP asymmetry in $B_d(\bar{B}_d) \to J/\Psi K_S$, which is without doubt the highlight of this field. We will first focus on this case in the following because of its importance and because it exhibits the characteristic features of a large class of CP violating observables in B physics. We will briefly mention further possibilities later on.

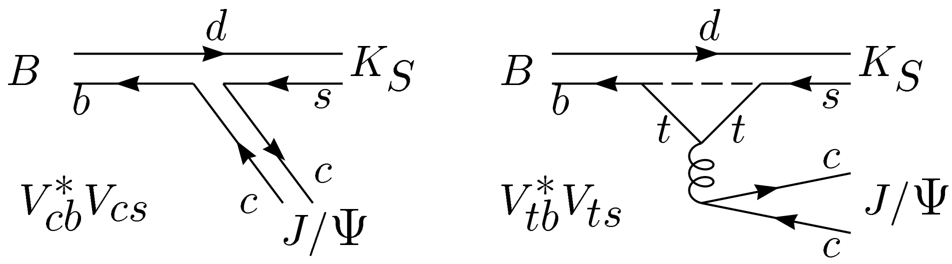

Figure 4: Representative diagrams contributing to $B_d \to J/\psi K_S$.

5.1 $B_d \to J/\Psi K_S$

The CP asymmetry in $B_d \to J/\Psi K_S$ belongs to the class of mixing-induced CP violation, that is CP violation in the interference of mixing and decay. In the kaon system an essentially pure beam of a definite eigenstate, the K_L, can easily be produced due to the vast difference in lifetimes between K_L and K_S, which is ideal for CP violation studies. Since the lifetime difference between eigenstates is negligibly small for the $B_d - \bar{B}_d$ system, the same method can not be applied in this case. Instead explicit flavor tagging (determination of the flavor of one of the B mesons (produced in pairs), for instance by means of the lepton charge in the semileptonic decay of the other) is required and one has to consider the time dependence of $B-\bar{B}$ mixing.† Solving the time dependent Schrödinger equation with the mixing Hamiltonian $\hat{H}$ (4), and neglecting $\Delta\Gamma$, one has

$$B(t) = e^{-iMt-\frac{1}{2}\Gamma t}\left[\cos\frac{\Delta M t}{2}\ B - \frac{q}{p} i \sin\frac{\Delta M t}{2}\ \bar{B}\right] \tag{23}$$

$$\bar{B}(t) = e^{-iMt-\frac{1}{2}\Gamma t}\left[\cos\frac{\Delta M t}{2}\ \bar{B} - \frac{p}{q} i \sin\frac{\Delta M t}{2}\ B\right] \tag{24}$$

$B(t)$ and $\bar{B}(t)$ are the time evolved states that started out as flavor eigenstates B and $\bar{B}$, respectively, at time $t = 0$.

The CP asymmetry in $B_d \to J/\Psi K_S$ is the prime example of the important class of asymmetries in neutral B mesons decaying into a CP eigenstate, in this case $f = J/\Psi K_S$, which is CP odd. There are two basic contributions to the decay amplitude, distinguished by the combination of CKM parameters $V^*_{cb}V_{cs}$ or $V^*_{tb}V_{ts}$. Representative diagrams are shown in Fig. 4. The third possible factor $V^*_{ub}V_{us}$ can be expressed in terms of the above two by CKM unitarity, $V^*_{ub}V_{us} = -V^*_{cb}V_{cs} - V^*_{tb}V_{ts}$. Choosing the latter two as independent parameters is useful in the present case, since $V^*_{ub}V_{us}$ is Cabibbo suppressed.

A crucial feature of the $B_d \to J/\Psi K_S$ mode is that the relative weak phase between $V^*_{cb}V_{cs}$ and $V^*_{tb}V_{ts}$ is negligibly small. Consequently the $B_d \to J/\Psi K_S$ amplitude can to excellent approximation be represented as $A(B_d \to J/\Psi K_S) = V^*_{cb}V_{cs} \cdot A_{red}$ as far as the weak phase structure is concerned. The quantity A_{red} involves nontrivial hadronic dynamics, but it will

†This latter strategy can in principle also be used for neutral kaons and is in fact the method realized in the CPLEAR experiment at CERN (see M. Mikuz, these proceedings).

drop out when forming the ratio that defines the asymmetry (see (25) below). This fact lies at the bottom of the theoretically clean nature of the $B_d \to J/\Psi K_S$ asymmetry.
Using this property of the amplitude we can now see how the mixing-induced asymmetry comes about. As illustrated in Fig. 5, an initial B state can decay to the CP self-conjugate final state f via two different paths: directly ($B \to f$), or through mixing ($B \to \bar{B} \to f$), since the same final state can be reached by both B and $\bar{B}$. The mixing phase phase ($B \to \bar{B}$) is determined by the box diagram, similar to the first graph in Fig. 2, and reads $(V^*_{tb}V_{td})^2/|V^*_{tb}V_{td}|^2 \equiv V^*_{tb}V_{td}/(V_{tb}V^*_{td})$. The two different decay paths therefore have a relative phase of $V^*_{tb}V_{td}V_{cb}V^*_{cs}/(V_{tb}V^*_{td}V^*_{cb}V_{cs}) = \exp(-2i\beta)$. The CP conjugate situation (starting out with $\bar{B}$) has the opposite phase. Putting everything together (using (23), (24)) one finds the time-dependent asymmetry

$$\begin{aligned} \mathcal{A}_{CP}(B_d \to J/\Psi K_S) &\equiv \frac{\Gamma(B(t) \to \Psi K_S) - \Gamma(\bar{B}(t) \to \Psi K_S)}{\Gamma(B(t) \to \Psi K_S) + \Gamma(\bar{B}(t) \to \Psi K_S)} \\ &= -\sin 2\beta \cdot \sin \Delta M t \end{aligned} \tag{25}$$

A few points about this result are worth emphasizing, the first two of which summarize the basic reasons why $\mathcal{A}_{CP}(B_d \to J/\Psi K_S)$ plays such an important role in flavor physics.

- As mentioned before, the part of the amplitude containing the dependence on the uncalculable hadronic dynamics has canceled out in the asymmetry. The asymmetry depends only on the CKM quantity $\sin 2\beta$. This result holds to within a theoretical uncertainty of less than 1%.

- The effect is quite large in the SM, where one expects approximately $\sin 2\beta \approx 0.6 \pm 0.2$. This information comes from the observed CP violation in the kaon system, which implies that the CP phase η must not be too small. It follows (see Fig. 1) that also $\sin 2\beta$ has to be sizable.

- Two a priori unrelated features of the fundamental SM parameters are very helpful to make a measurement of (25) feasible. First, the B_d lifetime (about $1.5ps$) is relatively large due to the smallness of $V_{cb} \approx 0.04$, which is crucial for being able to resolve the time dependence. Furthermore, ΔM is sizable due to the large top quark mass such

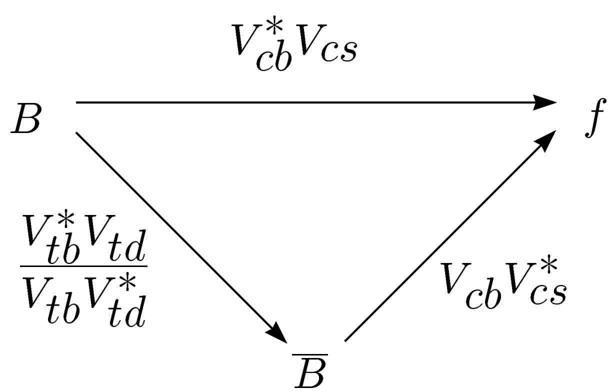

Figure 5: Possible decay paths for an initial B meson to decay into the final state $f = J/\Psi K_S$ that is common to B and $\bar{B}$.

that ΔM turns out to be of almost the same size as Γ (see Table 1), which is almost perfect for optimizing the effect of mixing.

5.2 Other Possibilities

A case very similar to $B_d(\bar{B}_d) \to J/\Psi K_S$ is the CP asymmetry for $B_d(\bar{B}_d) \to \pi^+\pi^-$. Here the dominant contribution to the amplitude has CKM factor $V^*_{ub}V_{ud}$ ($V_{ub}V^*_{ud}$) and consequently the relative phase between the mixed decay $B \to \bar{B} \to f$ and the direct decay $B \to f$ is given by $V^*_{tb}V_{td}V_{ub}V^*_{ud}/(V_{tb}V^*_{td}V^*_{ub}V_{ud}) = \exp(-2i(\beta+\gamma)) = \exp(2i\alpha)$. Accordingly the CP asymmetry is a measure of $\sin 2\alpha$. The situation is, however, somewhat complicated by the second, non-negligible contribution to the decay amplitude from penguin graphs. This contribution comes with CKM factor $V^*_{tb}V_{td}$, which, unlike the case of $B \to J/\Psi K_S$, has a different phase than the leading contribution ($\sim V^*_{ub}V_{ud}$). Consequently, the amplitude no longer has the simple structure of the $B \to J/\Psi K_S$ amplitude with its single weak phase where all hadronic uncertainties cancel, and some poorly calculable hadronic dynamics will invariably enter the CP asymmetry $\mathcal{A}_{CP}(B_d \to \pi^+\pi^-)$ ('penguin pollution'). Strategies have been devised to eliminate this uncertainty, for instance using additional information from related modes as $B_d(\bar{B}_d) \to \pi^0\pi^0$ and $B^\pm \to \pi^\pm\pi^0$ together with isospin symmetry [17]. Assuming this has been achieved, $B \to \pi\pi$ determines $\sin 2\alpha$, an example of mixing-induced CP violation just as the case of $B \to J/\Psi K_S$ and $\sin 2\beta$. As explained before, each of these cases considered separately represents indirect CP violation. However any deviation from the equality $\sin 2\beta = -\sin 2\alpha$ would reveal a direct CP violation effect [18] (the minus sign appears here due to the opposite CP parities of $J/\Psi K_S$ (CP odd) and $\pi^+\pi^-$ (CP even)).
A good example of direct CP violation is provided by the decays $B^\pm \to D^0_{(CP+)}K^\pm$. No flavor-tagging or time-dependent measurements are required here and the asymmetries can be used to extract the angle γ in a clean way [19, 20].
Also B_s mesons offer opportunities for interesting CP violation studies, although they are more challenging experimentally because of the very large oscillation frequency $\Delta M/\Gamma > 10$. For instance, $B_s \to J/\Psi\phi$ is the B_s analog of $B_d \to J/\Psi K_S$ decay. The asymmetry is Cabibbo suppressed in this case but would allow, in principle, a clean determination of η. A measurement of γ is possible with $B_s \to D^+_s K^-$ [21].
There are many more strategies and scenarios discussed in the literature. In our brief account we have focused on those cases that can yield insight into the mechanisms of CP violation with exceptionally small theoretical uncertainties. For general reviews see e.g. [22, 23].

6 Conclusions

The violation of CP symmetry has so far been observed in just five decay modes of the long-lived neutral kaon, namely $K_L \to \pi^+\pi^-$, $\pi^0\pi^0$, $\pi e\nu$, $\pi\mu\nu$, $\pi^+\pi^-\gamma$. All asymmetries can be described by a single complex parameter ε. The question of direct CP violation in $K \to \pi\pi$, measured by ε'/ε, is still open and currently further pursued by ongoing projects. Although our knowledge of this phenomenon is rather limited, the established pattern of CP violation with kaons, $\varepsilon \sim 10^{-3}$ and $\varepsilon' \lesssim 10^{-6}$, is well accounted for by the three generation Standard Model. The smallness of ε and ε' is related to the size of $\mathrm{Im}V^*_{ts}V_{td} = A^2\lambda^5\eta \sim 10^{-4}$.

This quantity is small due to suppressed intergenerational quark mixing ($\sim \lambda^5$), but not due to smallness of the CP violating phase ($\sim \eta$), which in fact is quite substantial (typically $\eta \approx 0.3 - 0.4$). As a consequence, large asymmetries are predicted in the B meson sector, which has many decay channels and a very rich phenomenology. The highlight of this latter area is $\mathcal{A}_{CP}(B_d \to J/\Psi K_S) \sim \sin 2\beta \approx 0.6 \pm 0.2$, exhibiting a large effect with essentially no theoretical uncertainties and good experimental feasibility.
Theoretical progress during recent years that is of relevance for this type of physics includes heavy quark effective theory, the calculation of higher order QCD effects and improvements in lattice QCD computations. In many cases a serious remaining problem is the nonperturbative strong dynamics governing weak decay matrix elements (ε'/ε, $B \to \pi\pi$). Exceptions are clean observables with practically negligible theoretical error. The prime examples of this class are $\mathcal{A}_{CP}(B_d \to J/\Psi K_S)$ ($\sim \sin 2\beta$) and $B(K_L \to \pi^0 \nu\bar{\nu})$ ($\sim \eta^2$). Also important for a further understanding of CP violation is the study of flavor-changing neutral current rare decays with small theoretical ambiguities such as $K^+ \to \pi^+ \nu\bar{\nu}$, $B \to X_s \gamma$, $B \to X_s e^+ e^-$, $B \to l^+ l^-$, $B \to X_s \nu\bar{\nu}$ or $\Delta M_{B_s}/\Delta M_{B_d}$. Since the number of clean processes is very limited, and much complementary information is needed, all of them should be pursued as far as possible.
CP violation is intimately connected with flavor dynamics, the least understood sector of our current Standard Model. It is therefore also closely related to the question of electroweak symmetry breaking, presently one of the most urgent open problems in fundamental physics. The coming years hold great promise for decisive progress in our knowledge about CP violation and for obtaining a clearer picture of what may still lie behind this remarkable phenomenon.

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