SLAC–PUB–7607
July 1997

Effect of Feedback and Noise on Fast Ion Instability*

A. W. Chao and G. V. Stupakov
Stanford Linear Accelerator Center, Stanford University, Stanford CA 94309

Abstract

One can use a feedback system to suppress the fast ion instability. However, the feedback noise (and also other sources of noise in the machine) continously excites the transient oscillations in the electron beam that are amplified through the electron interaction with the ions. We calculate the equlibrium level of these oscillations under the influence of the feedback and show how they grow exponentially from the head to the tail of the bunch train in a linear theory. Nonlinear saturation effects are assume negligible.

Presented at International Workshop on Multi-Bunch Instabilities in Future Electron and Positron Accelerators
Tsukuba, Japan
July 15 - 18, 1997

*Work supported by Department of Energy contract DE–AC03–76SF00515.

Effect of Feedback and Noise on Fast Ion Instability

Abstract

One can use a feedback system to suppress the fast ion instability. However, the feedback noise (and also other sources of noise in the machine) continously excites the transient oscillations in the electron beam that are amplified through the electron interaction with the ions. We calculate the equlibrium level of these oscillations under the influence of the feedback and show how they grow exponentially from the head to the tail of the bunch train in a linear theory. Nonlinear saturation effects are assume negligible.

1 Introduction

A fast beam-ion instability which is caused by the interaction of an electron bunch train with the residual gas ions [1, 2] can be of potential danger in future high-current, low-emittance accelerators. The instability mechanism assumes that the ions are not trapped from turn-to-turn, and is the same in both linacs and storage rings. The ions generated by the head of the bunch train oscillate in the transverse direction and resonantly interact with the betatron oscillations of the subsequent bunches, causing the growth of the initial perturbation of the beam. First experimental observation of the fast ion instability on the Advanced Light Source at the LBL has been recently reported in [3].

The original model of the instability developed in Refs. [1, 2] assumes no damping in the system. The analysis applies when the beam or the beam train is injected with a displacement which has a snap-shot pattern $\sim e^{-i(\omega_\beta+\omega_I)s/c}$. This initial displacement excites the bunch tail oscillation through ions. In the absence of saturation effects, the amplitude of the oscillations grows as $\exp(z\sqrt{s/s_0})$ where

$$s_0 = \frac{2\omega_\beta}{\kappa\omega_I}, \tag{1}$$

ω_β is the betatron frequency, ω_I is the frequency of the ion oscillations, and κ is the coefficient responsible for the beam-ion interaction,

$$\kappa \equiv \frac{4\dot{\lambda}_{ion} r_e}{3\gamma c\sigma_y\left(\sigma_x+\sigma_y\right)}\ , \tag{2}$$

where γ denotes the relativistic factor for the beam, r_e is the classical electron radius, $\sigma_{x,y}$ is the horizontal and vertical rms-beam size respectively, and $\dot{\lambda}_{ion}$ is the number of ions per meter generated by the beam per unit time. The characteristic instability growth time is $\tau_{\text{instability}} = s_0/cl^2$, with l the length of the bunch train.

When there is an external damping – radiation damping, for example – the behavior of this instability is quite different. What happens then is the entire growth, including the beam head and tail, would be damped by the external damping. This is true even if the damping rate τ_d^{-1} is much weaker than the instability growth rate, i.e. even if $\tau_d^{-1} \ll \tau_{\text{instability}}^{-1}$.†

To see this, consider the case of a beam train. The first bunch does not see any ions and executes a free betatron oscillation. The ions it produces excite the second bunch. However, with an external damping, even a very weak one, the oscillation of the first bunch slowly decays in $t \approx \tau_d$. After the first bunch oscillation damps out, the second bunch is no longer driven, and it starts to damp. It will take another τ_d to damp after the first bunch has been damped; Thus its oscillation decays in $t \approx 2\tau_d$. With the first and the second bunches stop oscillating, the third bunch, regardless of the fact that it has been driven to a large amplitude during this time, begins to damp. Eventually, it is damped out, etc. With an external damping, therefore, the fast ion instability is only a transient effect. A small injection error would very quickly grow to a large oscillation, especially towards the tail of the

†This statement is not to be confused with our conclusion later, Eq.(20).

beam train. However, with time, the oscillation of the entire train decays and, if not excited again, will stay quiet.

We see here one special property of the fast ion instability. To damp out a conventional instability, one would need an external damping rate larger than the instability growth rate. But this is not true here. The fast ion instability is in this sense not a true instability; it is intrinsically a transient effect which is particularly sluggish.

2 Feedback System

One might envision therefore that the best way to deal with the fast ion instability is to ignore it, and let radiation damping take care of it. Unfortunately, the beam is constantly excited by various noise effects in an accelerator: power supply ripples, collective instabilities, etc.

A feedback system of course also damps the fast ion instability. However, feedback systems also carry noise, which constantly excite the fast ion instability, and this feedback noise may dominate over all other noise sources. So we now have a situation where the feedback system provides simultaneously the excitation and the damping. The beam responds to it very similarly to a single electron responding to quantum excitation and radiation damping. The net result is that each bunch in a beam train will reach a certain rms oscillation amplitude which is determined by an equilibrium between the feedback damping and the feedback noise. Existing theories [1, 2] mostly describe the transient behavior. What we are interested in (for example, to compare with some experimental observations) however, is the statistical equilibrium when a feedback is turned on. In the following, we analyze this equilibrium. We treat a linear theory of this problem, while Ref.[4] discusses the subject including nonlinear effects.

Let the feedback damping time be τ_d, and its noise be characterized by a random force $f(s,z)$ acting on the beam. The equation for the amplitude of the electron oscillations can be obtained using the approach of Refs. [1, 2]

$$\frac{\partial \tilde{y}}{\partial s} + \frac{1}{c\tau_d}\tilde{y} = \frac{1}{2s_0}\int_0^z dz' z' \tilde{y}(s,z') + f(s,z). \tag{3}$$

Without the second term on the left hand side and with $f = 0$ we have the case studied in Refs. [1, 2]. The new terms take into account the damping caused by the feedback system with the damping time τ_d and the noise modeled by the force $f(s,z)$.

The solution of Eq. (3) is

$$\begin{aligned}\tilde{y}(s,z) &= \int_{-\infty}^{s} ds' f(s',z) e^{\frac{s'-s}{c\tau_d}} \\ &- \int_0^z dz' \int_{-\infty}^{s} ds' f(s',z') e^{\frac{s'-s}{c\tau_d}} \frac{\partial}{\partial z'} I_0\left(\sqrt{\frac{(z^2-z'^2)(s-s')}{s_0}}\right)\end{aligned} \tag{4}$$

where I_0 is the modified Bessel function of the zeroth order. The first term in Eq.(4) is the direct response of the beam to the noise kicks. The second term is the response due to coupling to the ions.

Existing analysis [1, 2] is for the case $f(s,z) \propto \delta(s)$, yielding

$$\tilde{y}(s,z) \propto e^{-s/c\tau_d} I_0(\sqrt{z^2 s/s_0}), \tag{5}$$

which is the known result.

In case we have a constant source of random noise acting on the beam and $f(s,z)$ is a random function, a more adequate description of the beam motion would be in terms of the average square of the amplitude of oscillations. To calculate $\langle \tilde{y}^2(s,z) \rangle$ we assume that the force f is a δ-correlated random noise

$$\langle f(s,z) f(s',z') \rangle = F\delta(s-s')\delta(z-z') \tag{6}$$

which is appropriate for a wide-band feedback system. The parameter F can be related to the average square of amplitude of the betatron oscillations under the influence of the noise without ions. In this case the amplitude $\tilde{y}$ is given by the first term in Eq. (4)

$$\tilde{y}(s,z) = \int_{-\infty}^{s} ds' f(s',z) e^{\frac{s'-s}{c\tau_d}}, \tag{7}$$

and using Eq. (6) we have

$$\langle \tilde{y}(s,z)\tilde{y}(s,z')\rangle = \frac{c\tau_d}{2} F\delta(z-z'). \tag{8}$$

So far we have assumed a long continuous beam. In case the beam consists of a train of discrete bunches, the quantity

$$\frac{1}{l}\int_0^l dz \langle \tilde{y}(s,z)\tilde{y}(s,z')\rangle = \frac{c\tau_d}{2l} F \tag{9}$$

should be equated with the following sum

$$\frac{1}{N_b}\sum_{k=1}^{N_b} \langle \tilde{y}_i \tilde{y}_k\rangle = \frac{\langle \tilde{y}_0^2\rangle}{N_b}, \tag{10}$$

where N_b is the number of bunches in the beam, l is the length of the bunch train, and we have assumed that, in the absence of ions, the amplitudes of different bunches are uncorrelated, $\langle \tilde{y}_i \tilde{y}_k\rangle = \langle \tilde{y}_0^2\rangle \delta_{i,k}$. This gives an expression for F,

$$F = \frac{2l\langle \tilde{y}_0^2\rangle}{N_b c\tau_d}. \tag{11}$$

Now, returning to the case with the ions, we will assume that the second term in Eq. (4) dominates, and neglect the first term. Using Eq. (6), we have

$$\langle \tilde{y}^2(z) \rangle = F \int_0^z dz' \int_0^\infty ds e^{-\frac{2s}{c\tau_d}} \left[\frac{\partial}{\partial z'} I_0 \left(\sqrt{\frac{s(z^2 - z'^2)}{s_0}} \right) \right]^2 \tag{12}$$

Note that, being the statistical average, this $\langle \tilde{y}^2(z) \rangle$ is independent of s. Note also that one obtains correlation between $\tilde{y}(z_1)$ and $\tilde{y}(z_2)$ for two different bunches as

$$\langle \tilde{y}(z_1) \tilde{y}(z_2) \rangle = \langle \tilde{y}^2(z_1) \rangle, \quad \text{if} \quad z_2 > z_1 \tag{13}$$

Let us now consider the bunches in the asymptotic regime with

$$\bar{\eta} \equiv z \sqrt{\frac{c\tau_d}{s_0}} \gg 1 \tag{14}$$

Using the asymptotic representation $I_0(x) \approx (e^x / \sqrt{2\pi x})$, Eq. (12) becomes

$$\begin{aligned} \langle \tilde{y}^2(z) \rangle &\approx \frac{F}{2\pi} \int_0^z dz' \int_0^\infty ds \frac{e^{-2s/c\tau_d}}{\left(\frac{s}{s_0}(z^2 - z'^2)\right)^{1/2}} \left(\frac{\partial}{\partial z'} e^{\sqrt{s(z^2 - z'^2)/s_0}} \right)^2 \\ &\approx \frac{F}{2\pi} \int_0^z dz' \int_0^\infty ds\, z'^2 \sqrt{\frac{s}{s_0}} \frac{e^{-2s/c\tau_d + 2\sqrt{s(z^2 - z'^2)/s_0}}}{(z^2 - z'^2)^{3/2}}. \end{aligned} \tag{15}$$

Introducing new variables $\eta = \sqrt{s(z^2 - z'^2)/s_0}$ and $\eta' = z' \sqrt{c\tau_d / s_0}$ we have

$$\begin{aligned} \langle \tilde{y}^2(z) \rangle &\approx \frac{F (c\tau_d)^{3/2}}{\pi s_0^{1/2}} \int_0^{\bar{\eta}} \eta'^2 \, d\eta' \int_0^\infty \frac{\eta^2 \, d\eta}{(\bar{\eta}^2 - \eta'^2)^3} \exp\left(-2 \frac{\eta^2}{\bar{\eta}^2 - \eta'^2} + 2\eta \right) \\ &\approx \frac{F (c\tau_d)^{3/2}}{4\sqrt{2\pi} s_0^{1/2}} \int_0^{\bar{\eta}} d\eta' \frac{\eta'^2}{(\bar{\eta}^2 - \eta'^2)^{1/2}} \exp\left(\frac{\bar{\eta}^2 - \eta'^2}{2} \right) \end{aligned}$$

$$\approx \quad F\frac{(c\tau_d)^{3/2}}{8s_0^{1/2}}\frac{e^{\bar{\eta}^2/2}}{\bar{\eta}}. \tag{16}$$

Due to the factor $e^{\bar{\eta}^2/2}$, the dependence of $\langle\tilde{y}^2(z)\rangle$ on $\bar{\eta}$ for $\bar{\eta} \gg 1$ is very strong.

We next consider the limit opposite to Eq.(14), i.e., when $\bar{\eta} \ll 1$. Using Eq.(12), we obtain

$$\langle\tilde{y}^2(z)\rangle \approx F\frac{z^3c^3\tau_d^3}{48s_0^2} = F\frac{(c\tau_d)^{3/2}}{48s_0^{1/2}}\bar{\eta}^3 \tag{17}$$

Equations (16) and (17) are our main results. One may relate the average square results Eqs.(16) and (17) to the average square of the first bunch by using Eq.(11).

Figure 1 shows the behavior of the normalized mean square amplitude

$$g \equiv \frac{\langle\tilde{y}^2(z)\rangle s_0^{1/2}}{F(c\tau_d)^{3/2}} = \frac{\langle\tilde{y}^2(z)\rangle}{\langle\tilde{y}_0^2\rangle}\frac{N_b}{2l}\sqrt{\frac{s_0}{c\tau_d}} \tag{18}$$

versus $\bar{\eta}$. The two curves correspond to the $\bar{\eta} \gg 1$ and $\bar{\eta} \ll 1$ behaviors according to Eqs.(16) and (17) respectively. The solid portions of the curves represent their respective region of applicability.

In order to avoid an enhancement of beam emittance due to fast ion instability, one should avoid the exponential regime when $\bar{\eta} \gg 1$. If one

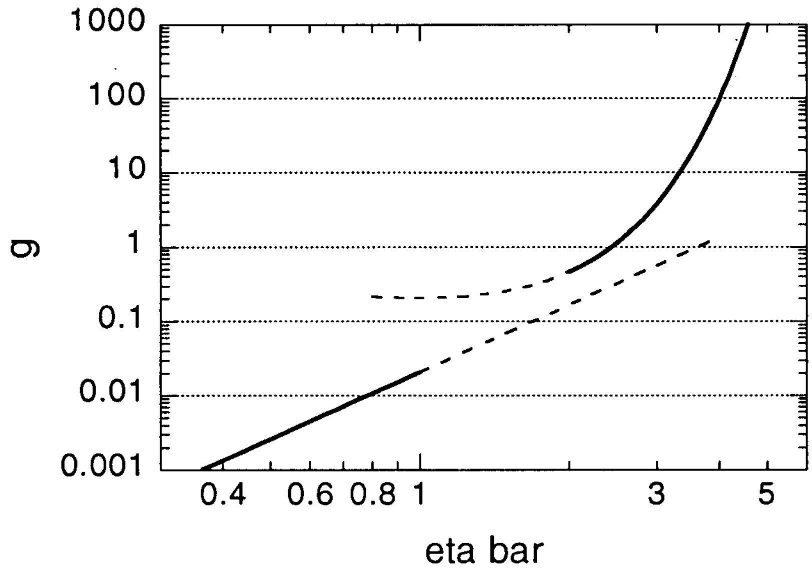

Figure 1: Normalized asymptotic mean square amplitude g as a function of $\bar{\eta}$.

adopts that as the operating condition, then one is led to require

$$l < \sqrt{\frac{s_0}{c\tau_d}} \tag{19}$$

or equivalently

$$\tau_{\text{instability}} > \tau_d \tag{20}$$

where $\tau_{\text{instability}} = s_0/cl^2$. Indeed, when Eq.(19) [or (20)] is satisfied, it follows that the last bunch in the bunch train (with $z = l$) has

$$\frac{\langle \tilde{y}^2(l) \rangle}{\langle \tilde{y}_0^2 \rangle} \approx \frac{1}{24N_b} \left(\frac{l^2 c\tau_d}{s_0} \right)^2 \ll 1 \tag{21}$$

and the effective emittance growth is negligible. On the other hand, due to the extremely rapid dependence of $\langle \tilde{y}(z) \rangle$ on z when $\bar{\eta} \gg 1$, the tolerable value of $\bar{\eta}$ is not far from $\bar{\eta} = 1$.

We would like to thank Sam Heifets for illuminating discussions.

References

[1] T. O. Raubenheimer and F. Zimmermann. *Phys. Rev. E*, **52**, 5487, (1995).

[2] G. V. Stupakov, T. O. Raubenheimer, and F. Zimmermann. *Phys. Rev. E*, **52**, 5499, (1995).

[3] J. Byrd et al. *Phys. Rev. Lett.*, **79**, 79 (1997).

[4] S. Heifets, this workshop.