BFKL SCATTERING AT LEPII AND A NEXT e^+e^- COLLIDER [*]

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Abstract

We discuss virtual photon scattering in the region dominated by BFKL exchange, and report results for the cross sections at present and future e^+e^- colliders.

Based on talks by D. E. Soper at DIS97 and by F. Hautmann at Photon97

The BFKL equation describes scattering processes in QCD in the limit of large energies and fixed (sufficiently large) momentum transfers. The study that we present in this paper analyzes the prospects for using photon-photon collisions as a probe of QCD dynamics in this region. The quantity we focus on is the total cross section for scattering two photons sufficiently far off shell at large center-of-mass energies, $\gamma^*(Q_A^2) + \gamma^*(Q_B^2) \to$ hadrons, $s \gg Q_A^2, Q_B^2 \gg \Lambda_{QCD}^2$. This process can be observed at high-energy and high-luminosity e^+e^- colliders as well as e^-e^- or $\mu^\pm\mu^-$ colliders, where the photons are produced from the lepton beams by bremsstrahlung. The $\gamma^*\gamma^*$ cross section can be measured in collisions in which both the outgoing leptons are tagged.

The basic motivation for this study is that compared to tests of BFKL dynamics in deeply inelastic lepton-hadron scattering (see, for instance, the review in Ref. [1]) the off-shell photon cross section presents some theoretical advantages, essentially because it does not involve a non-perturbative target. The photons act as color dipoles with small transverse size, so that the QCD interactions can be treated in a fully perturbative framework.

The structure of $\gamma^*\gamma^*$ high-energy scattering is shown schematically in Fig. 1. We work in a frame in which the photons q_A, q_B have zero transverse momenta and are boosted along the positive and negative light-cone directions. In the leading logarithm approximation, the process can be described as the interaction of two $q\overline{q}$ pairs scattering off each other via multiple gluon exchange. The $q\overline{q}$ pairs are in a color-singlet state and interact through their color dipole moments. The gluonic function $\mathcal{F}$ is obtained from the solution to the BFKL equation [2].

The analysis of the transverse-distance scales involved in the scattering illustrates a few distinctive features of this process. The mean transverse size of each $q\overline{q}$ dipole is given, in the first approximation, by the reciprocal of the corresponding photon virtuality:

$$< R_{\perp A} > \sim 1/Q_A \quad , \quad < R_{\perp B} > \sim 1/Q_B \quad . \tag{1}$$

However, fluctuations can bring in much larger transverse sizes. Large-size fluctuations occur as a result of the configurations in which one quark of the pair carries small transverse momentum and a small fraction of the photon longitudinal momentum (the so-called aligned-jet configurations [3]). For example, for the momentum p_A
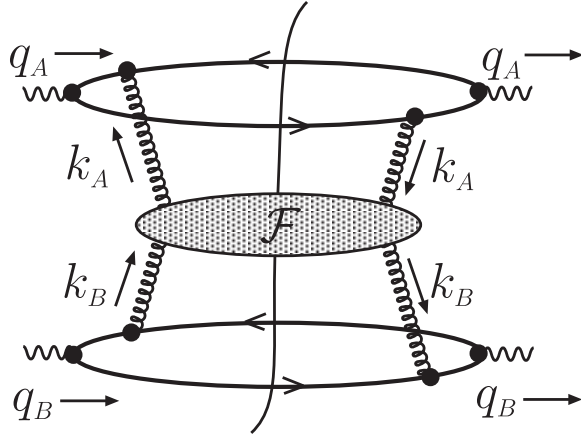
Figure 1: The virtual photon cross section in the high energy limit.

of the quark created by photon A:

$$\mathbf{p}_{\perp A} \ll Q_A \quad , \quad z_A \equiv p_A^+/q_A^+ \ll 1 \quad . \tag{2}$$

The actual size up to which the $q\overline{q}$ pair can fluctuate is controlled by the scale of the system that it scatters off. Therefore, in $\gamma^*\gamma^*$ scattering the fluctuations in the transverse size of each pair are suppressed by the off-shellness of the photon creating the other pair. If *both* photons are sufficiently far off shell, the transverse separation in each $q\overline{q}$ dipole stays small [4]. This can be contrasted with the case of deeply inelastic $e\,p$ scattering (or $e\,\gamma$, where γ is a (quasi-)real photon). In this case, the $q\overline{q}$ pair produced by the virtual photon can fluctuate up to sizes of the order of a hadronic scale, that is, $1/\Lambda_{QCD}$. This results in the deeply inelastic cross section being determined by an interplay of short and long distances.

In principle, the $q\overline{q}$ dipoles in the $\gamma^*\gamma^*$ process could still fluctuate to bigger sizes in correspondence of configurations in which the jet alignment occurs twice, once for each photon. However, such configurations cost an extra overall power of $1/Q^2$ in the cross section (terms proportional to $1/(Q_A^2\,Q_B^2)$ rather than $1/(Q_A\,Q_B)$) [5]. Therefore, they only contribute at the level of sub-leading power corrections to $\sigma(\gamma^*\gamma^*)$.

Even though the $q\overline{q}$ dipoles have small transverse size, sensitivity to large transverse distances may be brought in through the BFKL function $\mathcal{F}$. This indeed is expected to occur when the energy s becomes very large. As s increases, the typical impact parameters dominating the cross section for BFKL exchange grow to be much larger than the size of the colliding objects [6]. One can interpret this as providing an
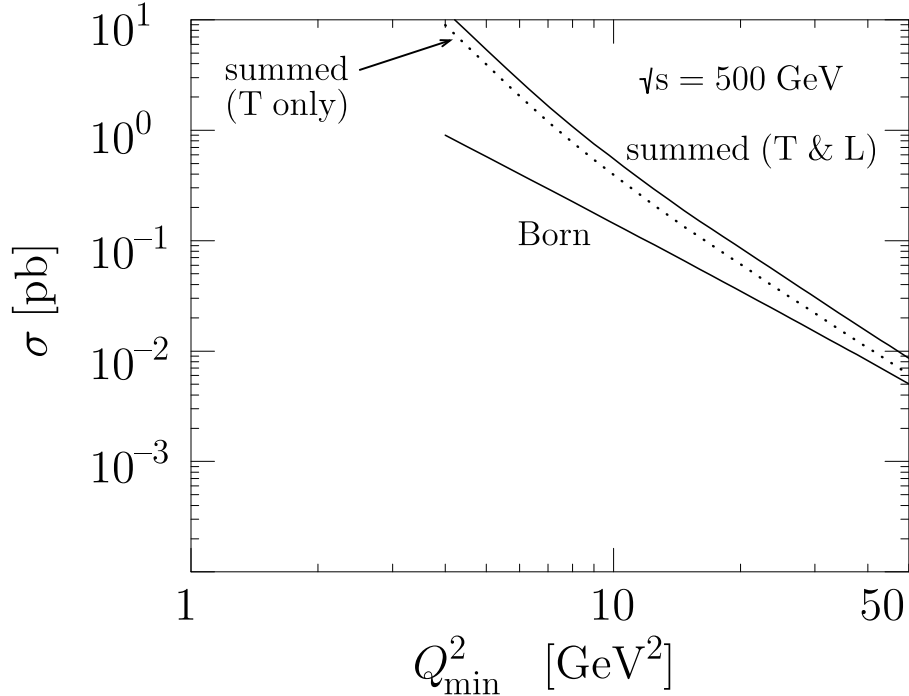
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Figure 2: The $Q^2_{\min}$ dependence of the e^+e^- integrated rate for $\sqrt{s} = 500$ GeV. The choice of the cuts and of the scales in the leading logarithm result is as in Ref. [4]. The dot-dashed and solid lines correspond to the result of using, respectively, the Born and the BFKL-summed expressions for the photon-photon cross section. The dotted curve shows the contribution to the summed result coming from transversely polarized photons.

upper bound on the range of values of $(\alpha_s(Q^2) \ln(s/Q^2))$ in which the simple BFKL approach to virtual photon scattering is expected to give reliable predictions [4].

The calculation of $\sigma(\gamma^*\gamma^*)$ and the form of the result are discussed in detail in Refs. [4, 7]. We recall here the main features:

i) for large virtualities, $\sigma(\gamma^*\gamma^*)$ scales like $1/Q^2$, where $Q^2 \sim \max\{Q^2_A, Q^2_B\}$. This is characteristic of the perturbative QCD prediction. Models based on Regge factorization (which work well in the soft-interaction regime dominating $\gamma\gamma$ scattering near the mass shell) would predict a higher power in $1/Q$.

ii) $\sigma(\gamma^*\gamma^*)$ is affected by logarithmic corrections in the energy s to all orders in
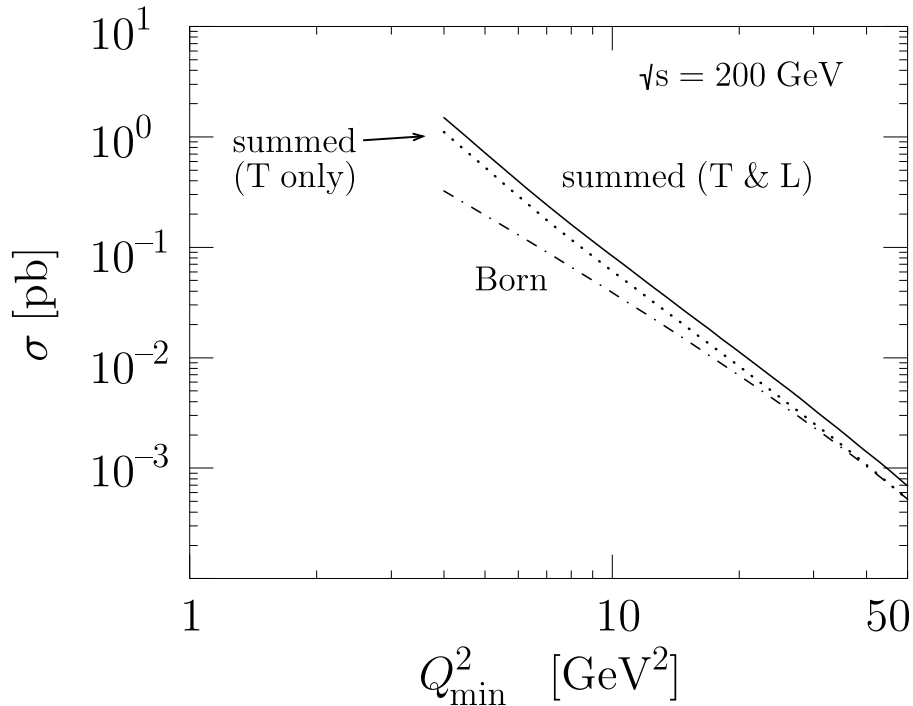
Figure 3: Same as in Fig. 2 for $\sqrt{s} = 200$ GeV.

α_s. As a result of the BFKL summation of these contributions, the cross section rises like a power in s, $\sigma \propto s^\lambda$. The Born approximation to this result (that is, the $\mathcal{O}(\alpha_s^2)$ contribution, corresponding to single gluon exchange in the graph of Fig. 1) gives a constant cross section, $\sigma_{Born} \propto s^0$. This behavior in s can be compared with lower-order calculations which do not include the corrections associated to (single or multiple) gluon exchange. Such calculations would give cross sections that fall off like $1/s$ at large s.

These features are reflected at the level of the e^+e^- scattering process. The e^+e^- cross section is obtained by folding $\sigma(\gamma^*\gamma^*)$ with the flux of photons from each lepton. In Figs. 2 and 3, we integrate this cross section with a lower cut on the photon virtualities (in order that the coupling α_s be small, and that the process be dominated by the perturbative contribution) and a lower cut on the photon-photon c.m.s. energy (in order that the high energy approximation be valid). We plot the result as a function of the lower bound $Q^2_{\min}$, illustrating the expected dependence of

the photon-photon cross section on the photon virtualities. Fig. 2 is for the energy of a future e^+e^- collider. Fig. 3 refers to the LEP collider operating at $\sqrt{s} = 200$ GeV. Details on our choice of cuts may be found in Ref. [4].

From Figs. 2 and 3, for a value of the cut $Q_{\min} = 2$ GeV we find $\sigma \simeq 1.5$ pb at LEP200 energies, and $\sigma \simeq 12$ pb at the energy of a future collider. These cross sections would give rise to about 750 events at LEP200 for a value of the luminosity $L = 500$ pb^{-1}, and about 6×10^5 events at $\sqrt{s} = 500$ GeV for $L = 50$ fb^{-1}. The above value of $Q_{\min}$ would imply detecting leptons scattered through angles down to about 20 mrad at LEP200, and about 8 mrad at a future 500 GeV collider. If instead we take, for instance, $Q_{\min} = 6$ GeV, the minimum angle at a 500 GeV collider is 24 mrad. Then the cross section is about 2×10^{-2} pb, corresponding to about 10^3 events. The dependence on the photon-photon c.m. energy $\sqrt{\hat{s}}$ can be best studied by fixing $Q_{\min}$ and looking at the cross section $d\sigma/(d\ln\hat{s}\,dy)$ (here y is the photon-photon rapidity). In Fig. 4 we plot this cross section at $y = 0$. While at the lowest end of the range in $\sqrt{\hat{s}}$ the curves are strongly dependent on the choice of the cuts, for increasing $\sqrt{\hat{s}}$ the plotted distribution is rather directly related to the behavior of $\sigma(\gamma^*\gamma^*)$ discussed earlier. In particular, as $\sqrt{\hat{s}}$ increases to about 100 GeV we see the Born result flatten out and the summed BFKL result rise, while the contribution from quark exchange is comparatively suppressed. The damping towards the higher end of the range in $\sqrt{\hat{s}}$ affects all curves and is due to the influence of the photon flux factors.

Fig. 4 is for $\sqrt{s} = 500$ GeV. The corresponding curves at LEP200 energies are qualitatively similar. The main difference is that at $\sqrt{s} = 200$ GeV there is less available range for $\sqrt{\hat{s}}$.

We see from the results presented above that at a future e^+e^- collider it should be possible to probe the effects of pomeron exchange in a range of Q^2 where summed perturbation theory applies. One should be able to investigate this region in detail by varying Q_A, Q_B and $\hat{s}$ independently. At LEP200 such studies appear to be more problematic mainly because of limitations in luminosity. Even with a modest luminosity, however, one can access the region of relatively low Q^2 if one can get down to small enough angles. This would allow one to examine experimentally the transition between soft and hard scattering.
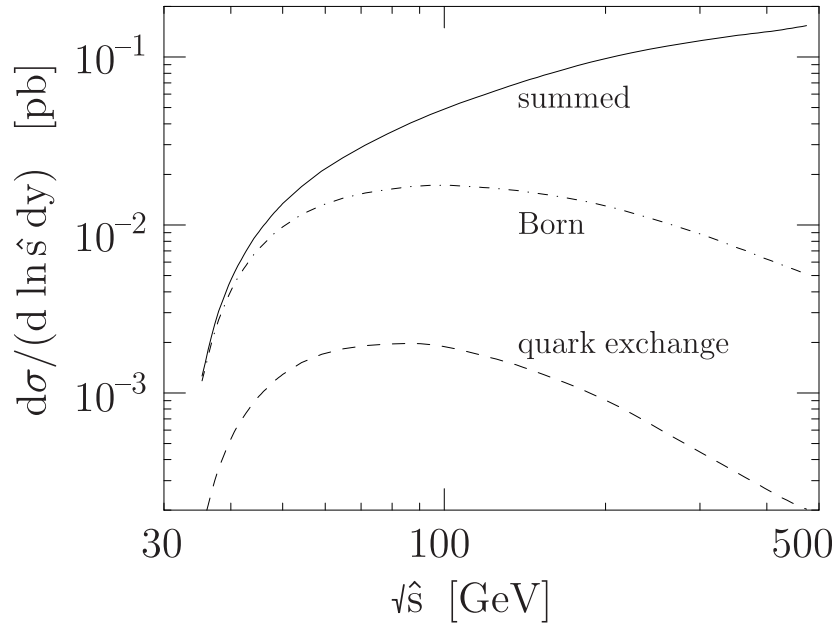
Figure 4: The cross section $d\sigma/(d\ln\hat{s}\,dy)$ at $y = 0$ for $\sqrt{s} = 500$ GeV. We take $Q^2_{\min} = 10$ GeV^2. The solid curve is the summed BFKL result. The dot-dashed curve is the Born result. The dashed curve shows the (purely electromagnetic) contribution arising from the scattering of (transversely polarized) photons via quark exchange.

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