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Effects of Focusing on Radiation Damping and Quantum Excitation in Electron Storage Rings*

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Abstract

In this Letter we calculate the effects of a linearly varying focusing field on radiation damping and quantum excitation in an electron storage ring using a quantum mechanical perturbation approach. This model correctly predicts the limits of pure bending and pure focusing. We find that quantum excitation can be exponentially suppressed by the focusing field when the radiation formation length is comparable to the transverse oscillation wavelength. This new result on quantum excitation may have interesting applications in the generation of ultra-low emittance beams.
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In an electron storage ring, synchrotron radiation created by bending magnets gives rise to the radiation damping of the beam emittances in all three degrees of freedom [1]. It is well known [1, 2, 3] that the damping effects are counteracted by quantum excitation due to random photon emissions, which leads to natural emittances when the damping and the excitation rates balance. Electron storage rings routinely obtain such natural emittances, and the art of lattice design in modern synchrotron radiation sources or damping rings is to minimize these emittances under various constraints.

On the other hand, it has been shown [4] that in a straight, continuous focusing channel, the transverse damping rate is independent of the particle energy, and that no quantum excitation is induced. In fact, the final normalized transverse emittance in an ideal focusing system is limited only by the uncertainty principle and is equal to one half of the Compton wavelength of the electron, which is much smaller than the natural transverse emittance achieved in a normal damping ring.

Therefore, the radiation reaction in a focusing system is very different from that in a bending magnet. Although the transverse focusing quadruples are present in a storage ring to confine the beam, and they can modify the individual radiation damping rates by coupling with the bending fields in a combined-function system [3], their contributions to the overall radiation effects are usually negligible compared to the bending dipoles. The length associated with a typical photon emission (the radiation formation length) is on the order of ρ/γ [2, 3], where ρ is the bending radius and γ is the electron energy in units of its rest energy mc^2. The standard treatment of quantum excitation can be quasi-classical because the radiation formation length is much shorter than the transverse oscillation wavelength. Thus, one can model the radiation to be instantaneous with a continuous spectrum of frequencies and treat the quantum nature of radiation as fluctuations about the average rate [1, 3]. In Ref. [2], radiation damping and quantum excitation were analyzed by rigorous quantum mechanical approach for a weak focusing synchrotron. The results agree with those of Ref. [1, 3] and confirm the quasi-classical picture of quantum excitation.

However, as the strength of the transverse focusing increases or as the bending field gradually decreases, the radiation formation length and the transverse oscillation wavelength may become comparable. The radiation in this case can not be regarded as instantaneous. Thus, it is desirable to have a general treatment of radiation effects in a storage ring with arbitrarily strong bending and focusing present. In this Letter, we extend the quantum mechanical analysis developed in Ref. [4] to include the bending case and show that quantum excitation can be suppressed by a strong focusing environment. Both the pure bending and the pure focusing are two limiting cases of the general result. Finally, we point out that there exists an interesting regime that might be useful for ultra-low emittance generation.

We consider here a simple model of storage rings with a continuous, linear focusing field around a circular electron orbit provided by a uniform magnetic field. The model for the focusing field used below is electrostatic in origin such as that created by a dilute cloud of positive ions. The more realistic magnetic focusing field will be discussed later. Suppose that a reference electron with momentum p_0 has a circular trajectory with radius ρ, the three components of the vector potential $\boldsymbol{A}$ for the uniform bending field in the familiar curvilinear coordinates system (x, s, y) (Figure 1) are [5]

$$\begin{aligned} A_x &= A_y = 0, \\ A_s &\equiv (\boldsymbol{A}\cdot\hat{\boldsymbol{s}})\left(1+\frac{x}{\rho}\right) = -\frac{cp_0}{e}\left(\frac{x}{\rho}+\frac{x^2}{2\rho^2}\right). \end{aligned} \tag{1}$$

Let the focusing force $(-Kx)$ be in the transverse x direction and neglect the electron motion in the other transverse y direction, the total energy of the electron can be decomposed as

$$\begin{aligned} E &= \sqrt{m^2c^4 + p_x^2c^2 + \frac{(p_s - eA_sc)^2\,c^2}{(1+x\rho)^2}} + \frac{1}{2}Kx^2 \\ &\approx E_s + \frac{p_x^2c^2}{2E_s} + \frac{1}{2}K_e\,(x - x_\epsilon)^2 - \frac{1}{2}K_e x_\epsilon^2. \end{aligned} \tag{2}$$

Here the longitudinal energy, $E_s = \sqrt{m^2c^4 + p_s^2c^2}$, the effective focusing strength, $K_e = K + p_0^2c^2/E_s\rho^2$, the equilibrium orbit displacement, $x_\epsilon = (p_s - p_0)c/(K_e\rho)$ and the betatron oscillation frequency, $\omega_\beta = \sqrt{K_ec^2/E_s} \equiv c/\beta$ are all functions of p_s. Thus, the transverse motion is a harmonic oscillation that is coupled with the longitudinal momentum through the equilibrium orbit displacement and the oscillation frequency. Following the treatment of Ref. [4], the eigenenergies of the electron in this system are

$$E_{(n,p_s)} = E_s + \hbar\omega_\beta\left(n+\frac{1}{2}\right) - \frac{1}{2}K_e x_\epsilon^2, \tag{3}$$

and the eigenstates are

$$\begin{aligned} \psi_{(n,p_s)}(\boldsymbol{r}) &= \left(\frac{1}{2\pi\rho}\right)^{1/2} \exp\left(i\frac{p_s}{\hbar}s\right)\chi_{(n,p_s)}(x), \\ \chi_{(n,p_s)}(x) &= \left(\frac{C_n}{x_0}\right)^{1/2} \exp\left[-\frac{(x-x_\epsilon)^2}{2x_0^2}\right] H_n\left(\frac{x-x_\epsilon}{x_0}\right), \end{aligned} \tag{4}$$

where $n = 0, 1, 2, ...$ is the transverse quantum level, $C_n = (2^n n!\sqrt{\pi})^{-1}$ is a normalization constant, $x_0 = \sqrt{\hbar c^2/E_s\omega_s}$ is the ground state $(n = 0)$ oscillation amplitude, and H_n is the n^{th} order Hermite polynomial. Both the eigenenergies and eigenstates are functions of n and p_s, the two quantum numbers that correspond to constants of motion in the absence of radiation.

The change of the transverse quantum level n due to spontaneous radiation can be calculated with first-order, time-dependent perturbation theory. The transition rate W_{fi} from an initial state $i(n, p_s)$ into a final state $f(n', p_s')$ with the emission of a photon ($\boldsymbol{k}_\gamma = k_\gamma\hat{\boldsymbol{k}}_\gamma, \omega_\gamma = k_\gamma c$) is given by [4]

$$W_{fi} = \int \frac{d\boldsymbol{k}_\gamma}{(2\pi)^3}\frac{2\pi ce^2}{\hbar k_\gamma}\sum_{\lambda=1}^{2}\left|\langle f|e^{-i\boldsymbol{k}_\gamma\cdot\boldsymbol{r}}(\boldsymbol{v}\cdot\hat{\boldsymbol{e}}_\lambda)|i\rangle\right|^2 \times 2\pi\delta(\omega_\gamma - \omega_{fi}), \tag{5}$$

where $\omega_{fi} = (E_i - E_f)/\hbar$, $\boldsymbol{v} \simeq (\boldsymbol{p} - e\boldsymbol{A}/c)c^2/E$ is the electron velocity operator, and $\hat{\boldsymbol{e}}_{1,2}$ are two orthogonal unit polarization vectors ($\hat{\boldsymbol{e}}_{1,2} \cdot \hat{\boldsymbol{k}}_\gamma = 0$).

Since we are interested in the total radiation effects, we can integrate Eq. (5) over the momentum space of the photons. First, let us expand the δ function

$$\delta(\omega_\gamma - \omega_{fi}) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dt e^{-i(\omega_\gamma - \omega_{fi})t}, \tag{6}$$

and write Eq. (5) as

$$\begin{aligned} W_{fi} &= \frac{e^2 c}{\pi\hbar}\int_{-\infty}^{+\infty} dt e^{i\omega_{fi}t} \int\int \frac{k_\gamma dk_\gamma d\Omega_\gamma}{4\pi} e^{-ik_\gamma ct} \\ &\quad \times \sum_{\lambda=1}^{2} \int d\boldsymbol{r}_1 \psi_f^*(\boldsymbol{r}_1) e^{-i\boldsymbol{k}_\gamma\cdot\boldsymbol{r}_1}(\boldsymbol{v}_1\cdot\hat{\boldsymbol{e}}_\lambda)\psi_i(\boldsymbol{r}_1) \int d\boldsymbol{r}_2 \psi_f(\boldsymbol{r}_2) e^{i\boldsymbol{k}_\gamma\cdot\boldsymbol{r}_2}(\boldsymbol{v}_2^\dagger\cdot\hat{\boldsymbol{e}}_\lambda)\psi_i^*(\boldsymbol{r}_1), \end{aligned} \tag{7}$$

where $\boldsymbol{v}_{1,2}$ is used to distinguish between the velocity operators that operate on coordinate $\boldsymbol{r}_1 = (x_1, s_1)$ and $\boldsymbol{r}_2 = (x_2, s_2)$. By applying the polarization sum rule

$$\sum_{\lambda=1}^{2}(\boldsymbol{v}_1\cdot\hat{\boldsymbol{e}}_\lambda)(\boldsymbol{v}_2^\dagger\cdot\hat{\boldsymbol{e}}_\lambda) = \boldsymbol{v}_1\cdot\boldsymbol{v}_2^\dagger - (\boldsymbol{v}_1\cdot\hat{\boldsymbol{k}}_\gamma)(\boldsymbol{v}_2^\dagger\cdot\hat{\boldsymbol{k}}_\gamma), \tag{8}$$

and introducing the Green's function [2]

$$G(t,r) = -\int\int \frac{k_\gamma dk_\gamma d\Omega_\gamma}{4\pi} e^{-ik_\gamma ct + i\boldsymbol{k}_\gamma\cdot\boldsymbol{r}} = \lim_{\epsilon\to+0}\frac{1}{c^2(t-i\epsilon)^2 - r^2}, \tag{9}$$

we obtain

$$W_{fi} = \frac{e^2 c}{\pi\hbar}\int_{-\infty}^{+\infty} dt e^{i\omega_{fi}t}\int\int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \psi_f^*(\boldsymbol{r}_1)\psi_f(\boldsymbol{r}_2) G(t, |\boldsymbol{r}_1 - \boldsymbol{r}_2|)\left(1 - \frac{\boldsymbol{v}_1\cdot\boldsymbol{v}_2^\dagger}{c^2}\right)\psi_i(\boldsymbol{r}_1)\psi_i^*(\boldsymbol{r}_2). \tag{10}$$

We make the change of variables $\phi = (s_1 - s_2)/\rho$ and $\phi' = (s_1 + s_2)/\rho$, and insert Eq. (4) into Eq. (10) to write

$$\begin{aligned} W_{fi} = \quad & \frac{e^2 c}{\pi\hbar}\int_{-\infty}^{+\infty} dt e^{i\omega_{fi}t}\int_{-2\pi}^{2\pi}\frac{d\phi}{2\pi}\exp\left[i\frac{(p_s - p_s')\rho\phi}{\hbar}\right] \\ & \times \int\int dx_1 dx_2 \chi_f(x_1)\chi_f(x_2) G V \chi_i(x_1)\chi_i(x_2), \end{aligned} \tag{11}$$

where

$$G(t, x_1, x_2, \phi) = \left[c^2(t-i\epsilon)^2 - (\rho+x_1)^2 - (\rho+x_2)^2 + 2(\rho+x_1)(\rho+x_2)\cos\phi\right]^{-1}, \tag{12}$$

$$\begin{aligned} V(x_1, x_2, \phi) &= 1 - \frac{\boldsymbol{v}_1\cdot\boldsymbol{v}_2^\dagger}{c^2} = 1 - \frac{(\boldsymbol{p}_1 - e\boldsymbol{A}_1/c)\cdot(\boldsymbol{p}_2 - e\boldsymbol{A}_2/c)^\dagger}{E^2/c^2} \\ &\simeq 1 - \frac{(p_{x1}p_{x2} + p_s^2)c^2}{E^2}\cos\phi - \frac{(p_{x1} + p_{x2})p_s c^2}{E^2}\sin\phi + O\left(\frac{x_{1,2}^2}{\rho^2}\right). \end{aligned} \tag{13}$$

In the last equation, $p_{x1} = -i\hbar\partial/\partial x_1$ and $p_{x2} = -i\hbar\partial/\partial x_2$ are the transverse momentum operators, and $p_s \simeq p_0$ is the eigenvalue of both operators p_{s1} and p_{s2}.

From Eq. (3), we can make the approximation $\omega_{fi} \simeq v(p_s - p'_s)/\hbar + \omega_\beta(n - n')$ with $v = p_s c^2/E$. Expanding the final transverse wavefunction in terms of the initial equilibrium orbit displacement, i.e.,

$$\chi_{(n',p'_s)}(x) \simeq \left[1 + \frac{(p_s - p'_s)c}{K_e\rho}\frac{\partial}{\partial x}\right]\chi_{(n',p_s)}(x), \tag{14}$$

and introducing the notation $(p_s - p'_s)\rho/\hbar = l$, we finally arrive at

$$\begin{aligned} W_{fi} = \quad & \frac{e^2 c}{\pi\hbar}\int_{-\infty}^{+\infty} dt e^{i(n-n')\omega_\beta t}\int_{-2\pi}^{2\pi}\frac{d\phi}{2\pi}\exp\left[il\left(\phi - \frac{vt}{\rho}\right)\right] \\ & \times\int\int dx_1 dx_2 \chi_{n'}(x_1)\chi_{n'}(x_2)F_l\chi_n(x_1)\chi_n(x_2), \end{aligned} \tag{15}$$

where we have dropped the subscript p_s from all the transverse wavefunctions and defined a transverse operator

$$F_l(t, x_1, x_2, \phi) \equiv \left(1 + il\frac{p_{x1}c}{K_e\rho^2}\right)\left(1 + il\frac{p_{x2}c}{K_e\rho^2}\right)G(t, x_1, x_2, \phi)V(x_1, x_2, \phi). \tag{16}$$

From Ref. [4], the rate of change of the transverse quantum number is given by

$$\frac{dn}{dt} = \sum_{n'}\sum_{p'_s}(n' - n)W_{fi} = \sum_{n'}\sum_{l}(n' - n)W_{fi}. \tag{17}$$

The sum over l can be first carried out using the set of relations

$$\begin{aligned} \sum_l \exp\left[il\left(\phi - \frac{vt}{\rho}\right)\right] &= 2\pi\delta\left(\phi - \frac{vt}{\rho}\right), \\ \sum_l il\exp\left[il\left(\phi - \frac{vt}{\rho}\right)\right] &= 2\pi\delta'\left(\phi - \frac{vt}{\rho}\right), \end{aligned} \tag{18}$$

where the prime means derivative with respect to ϕ. Integration by parts over ϕ yields

$$\frac{dn}{dt} = \frac{e^2 c}{\pi\hbar}\sum_{n'}(n' - n)\int_{-\infty}^{+\infty} dt e^{i(n-n')\omega_\beta t}\int\int dx_1 dx_2 \chi_{n'}(x_1)\chi_{n'}(x_2)F\chi_n(x_1)\chi_n(x_2), \tag{19}$$

where

$$F(t, x_1, x_2) = \left(1 - \frac{p_{x1}c}{K_e\rho^2}\frac{\partial}{\partial\phi}\right)\left(1 - \frac{p_{x2}c}{K_e\rho^2}\frac{\partial}{\partial\phi}\right)GV, \tag{20}$$

and the derivative with respect to ϕ is to be evaluated at $\phi = vt/\rho$ due to the delta functions in Eq. (18).

The Green's function in Eq. (12) plays the role of determining the major contribution of the time integral. Let us define a dimensionless time variable $\tau = ct/\rho$ and expand $\cos\phi$ in the denominator of Eq. (12) to obtain

$$G \simeq \left[I(\tau,\phi) - \left(\frac{x_1 - x_2}{\rho}\right)^2 - \frac{x_1 + x_2}{\rho}\phi^2 \right]^{-1} \rho^{-2}, \tag{21}$$

where $I(\tau,\phi) = (\tau - i\epsilon)^2 - \phi^2 + \phi^4/12 \simeq \tau^2(\gamma^{-2} + \tau^2/12)$ since $\phi = vt/\rho$. The time integral is significant only when $\tau \sim \phi \sim 1/\gamma$, or $ct \sim \rho/\gamma$ (the radiation formation length). Thus, we can also expand Eq. (13) for small ϕ to obtain

$$V \simeq \frac{1}{\gamma^2} + \frac{\phi^2}{2} + \frac{p_{x1}p_{x2}c^2}{E^2} - \frac{(p_{x1} + p_{x2})c}{E}\phi + O\left(\frac{x_{1,2}^2}{\rho^2}\right). \tag{22}$$

We can further expand the Green's function

$$G \simeq \frac{1}{\rho^2}\left[\frac{1}{I} + \frac{(x_1 - x_2)^2}{\rho^2}\frac{1}{I^2} + \frac{(x_1 + x_2)}{\rho}\frac{\phi^2}{I^2} + \frac{(x_1 + x_2)^2}{\rho^2}\frac{\phi^4}{I^3} + \ldots\right]. \tag{23}$$

To evaluate the double integrals of the transverse coordinates in Eq. (19), let us write p_x and x in Eq. (20), Eq. (22), and Eq. (23) in terms of the raising and lowering operators (a and $a^\dagger$) of the harmonic oscillator

$$\begin{aligned} p_x &= -i\sqrt{\frac{E_s\omega_\beta\hbar}{2c^2}}(a - a^\dagger), \\ x - x_\epsilon &= \sqrt{\frac{c^2\hbar}{2E_s\omega_\beta}}(a + a^\dagger). \end{aligned} \tag{24}$$

Applying these operators to the transverse wavefunctions leads to three types of selection rules. Those generated by constant terms have the selection rule $n' = n$, and thus have no contribution to the summation over n' due to the multiplying factor $(n' - n)$ in Eq. (19). Those generated by terms proportional to $p_{x1}p_{x2}$, x_1x_2, $p_{x1}x_2$ and $p_{x2}x_1$ have the selection rule $n' = n \pm 1$, and are the lowest order terms in $\hbar$. Those generated by $x_1^2x_2^2$, $x_1^2x_2p_{x2}$ and $x_1p_{x1}x_2^2$ have the selection rule $n' = n \pm 2$, but they are higher order terms in $\hbar$ and will be ignored. Thus, the summation over n' can be greatly reduced by the selection rule $n' = n \pm 1$. Finally, the time integral can be performed using the residue technique in the complex τ plane. After some lengthy but straightforward algebraic manipulations, we obtain

$$\frac{dn}{dt} = -\frac{2}{3}\frac{e^2\gamma^3}{\rho^2 mc}\left(\xi^2 - 1\right)n + \frac{e^2\gamma^3}{\rho^2 mc}\frac{\exp(-2\sqrt{3}\xi)}{144\xi^3}F(\xi), \tag{25}$$

where

$$F(\xi) = 55\sqrt{3} + 330\xi + 262\sqrt{3}\xi^2 + 300\xi^3 + 48\sqrt{3}\xi^4, \tag{26}$$

and $\xi = (\rho/\gamma)/\beta$ is the ratio of the radiation formation length over the reduced betatron wavelength.

From Ref. [4], the normalized transverse emittance ε_N of the beam is related to the average of the transverse quantum level n by $\varepsilon_N = \lambda\!\!\!^{-}_c \langle n + \frac{1}{2} \rangle$. Thus, we have

$$\frac{d\varepsilon_N}{dt} = -\Gamma_b \left[\left(\xi^2 - 1 \right) \left(\varepsilon_N - \frac{\lambda\!\!\!^{-}_c}{2} \right) - \lambda\!\!\!^{-}_c \frac{\exp(-2\sqrt{3}\xi)}{96\xi^3} F(\xi) \right], \tag{27}$$

where $\Gamma_b = 2e^2\gamma^3/(3\rho^2 mc)$ is the damping constant due to the bending field. Equation (27) describes the general result of radiation (anti-)damping (the first term) and quantum excitation (the second term) in this combined-function system. The relative amount of radiation damping and quantum excitation, as well as the natural emittance (if any), can be determined by a single dimensionless parameter ξ, which is a measure of the radiation formation length in units of the reduced betatron wavelength.

In normal synchrotrons and storage rings, the radiation formation length is much shorter then the reduced betatron wavelength, i.e., $\rho/\gamma \ll \beta$ or $\xi \ll 1$, Eq. (27) becomes

$$\begin{aligned} \frac{d\varepsilon_N}{dt} &= \Gamma_b \left\{ \left(\varepsilon_N - \frac{\lambda\!\!\!^{-}_c}{2} \right) + \lambda\!\!\!^{-}_c \frac{55\sqrt{3}}{96\xi^3} \right\} \\ &= \Gamma_b \left\{ \left(\varepsilon_N - \frac{\lambda\!\!\!^{-}_c}{2} \right) + \lambda\!\!\!^{-}_c \frac{55\sqrt{3}\gamma^3}{96\nu^3} \right\}, \end{aligned} \tag{28}$$

where $\nu = \rho/\beta$ is the betatron tune for the simple system. The first term of Eq. (28) is anti-damping instead of damping because the combined-function system studied here has a negative horizontal damping partition number ($J_x = -1$) [3]. However, the second term of Eq. (28) gives the same quantum excitation rate as using the quasi-classical model in a smooth storage ring [3].

In the opposite limit where $\rho \to \infty$ (a straight focusing channel), we have $\rho/\gamma \gg \beta$ or $\xi \gg 1$, Eq. (27) then reduces to

$$\frac{d\varepsilon_N}{dt} = -\Gamma_b \xi^2 \left(\varepsilon_N - \frac{\lambda\!\!\!^{-}_c}{2} \right) = -\Gamma_c \left(\varepsilon_N - \frac{\lambda\!\!\!^{-}_c}{2} \right), \tag{29}$$

where $\Gamma_c = \Gamma_b \xi^2 = 2r_e K/3mc$ is the damping rate due to the focusing field, and $r_e = e^2/mc^2$ is the classical electron radius. As expected, no quantum excitation is induced, and the fundamental emittance $\lambda\!\!\!^{-}_c/2$ can be reached in the ideal focusing channel [4].

In the intermediate regime where the radiation formation length is on the order of the reduced betatron wavelength ($\rho/\gamma \sim \beta$), the rate of quantum excitation is exponentially suppressed according to Eq. (27) and starts to depart from the result based on the quasi-classical model (Figure 2). A physical interpretation can be given as follows: The transverse energy levels of the electron are well separated as a result of the strong focusing force. Radiative transition to higher transverse levels becomes impossible for the electron with almost all photon emissions, and hence the quantum excitation is suppressed by the focusing environment.

Finally, we note that all of the above results can be extended to alternating-gradient and separated-function systems when longitudinal variations of both bending and focusing fields are short compared with the radiation formation length. Thus, the electrons in such lattices will damp instead of anti-damp, and the natural emittance may be substantially reduced because of the enhancement of radiation damping and the suppression of quantum excitation due to the focusing environment. In Ref. [6], we have provided some preliminary lattice considerations on a focusing-dominated damping ring based on these effects. Ultra-low emittance electron beams produced in such a damping ring may have interesting applications in novel accelerators or light sources requiring very low emittance.

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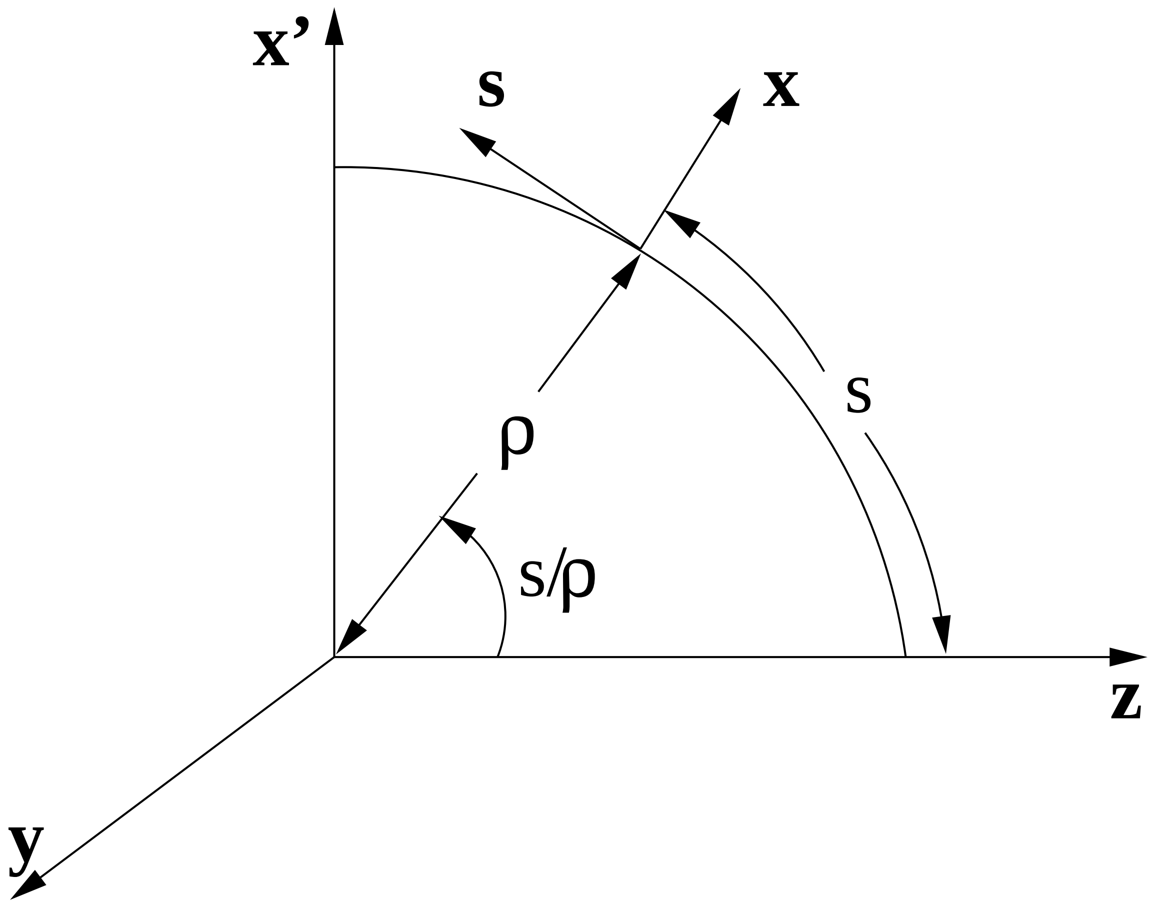

Figure 1: The curvilinear coordinate system (x, s, y) and the Cartesian coordinate system (x', y, z).

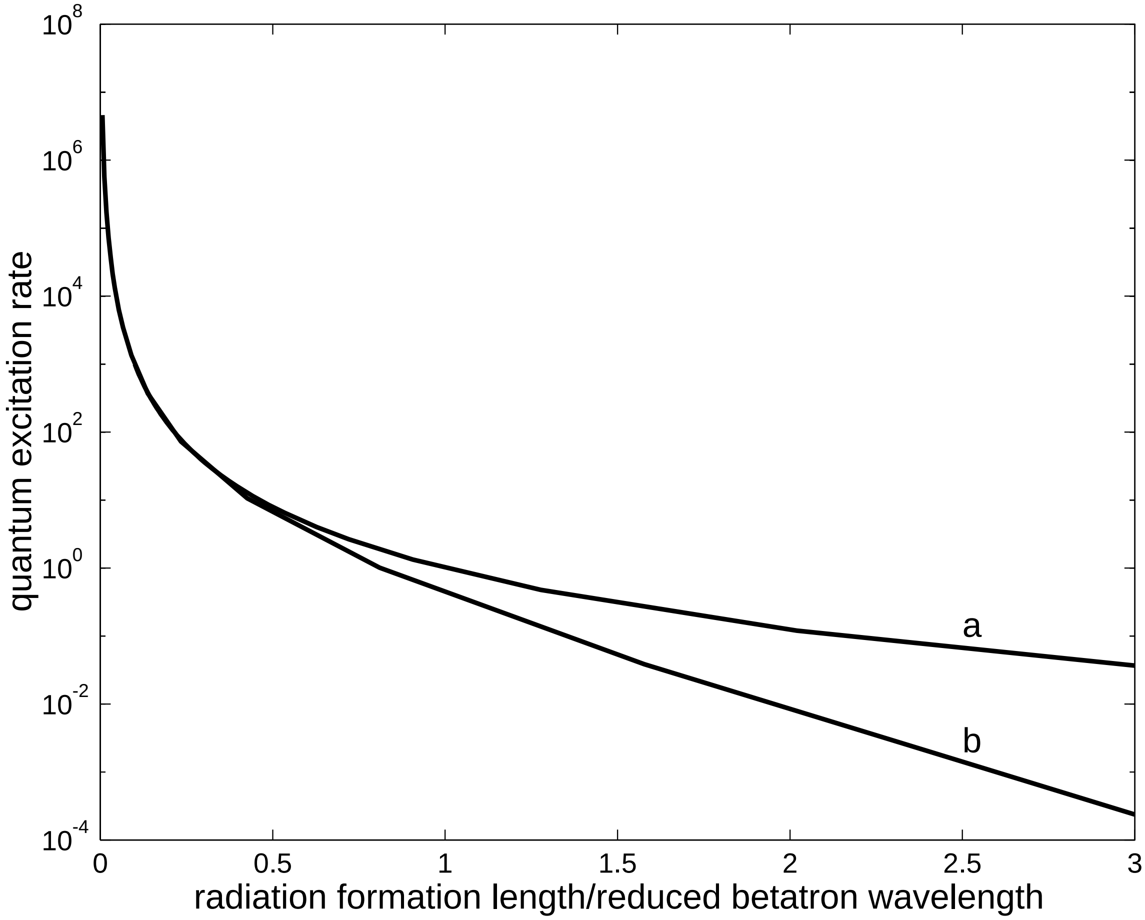

Figure 2: Quantum excitation rate in units of $\Gamma_b \lambda\!\!\!^{-}_c$, predicted by (a) the quasi-classical model, i.e., the second term of Eq. (28), and (b) the quantum mechanical perturbation approach, i.e., the second term of Eq. (27).