Advanced Techniques for Multiparton Loop Calculations: A Minireview[1]

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Abstract. We present an overview of techniques developed in recent years for the efficient calculation of one-loop multiparton amplitudes, in particular those relying on unitarity and collinear factorization.

INTRODUCTION

Much of the experimental effort in high-energy physics today is directed at searching for new physics beyond the standard model. Successful searches will require a detailed understanding of known physics. At high-energy colliders, particularly hadron-hadron colliders, this requirement implies above all the need for a detailed understanding of perturbative QCD. A detailed theoretical picture of dijet production, for example, is necessary if we are to use it in constraining the gluon distribution. The search for new physics in the single-jet inclusive distribution likewise requires detailed theoretical calculations (in addition to better measurements of the parton distributions than have been available heretofore). Other measurements which rely on our detailed understanding of perturbative QCD include the W mass, W+jet ratios, and top production in semi-leptonic and purely hadronic channels.

In the context of jet physics at high-energy colliders, next-to-leading order calculations represent the first step in providing such a detailed picture. Unlike the case of more inclusive measurements, such as the total hadronic cross section in e^+e^- annihilation, the perturbative expansion for jet observables contains both "ultraviolet" and "infrared" logarithms. The former arise from the truncation of the perturbation series at a fixed order. They manifest themselves in the (unphysical) renormalization-scale dependence of theoretical predictions. This dependence is numerically strong at leading order, because the coupling is still relatively large, and runs relatively quickly. Next-to-leading order effects compensate this sensitivity, and in practice reduce significantly the undesired renormalization-scale dependence. The second type of logarithms arise because of the presence of different scales, a hard scale characterizing the scattering, and softer scales characterizing the size of a jet. They modify the perturbative expansion from one solely in α_s to one in $\alpha_s \ln^2 y_{\rm IR}$ and $\alpha_s \ln y_{\rm IR}$ as well, where $y_{\rm IR}$ is the ratio of the different scales. Only at next-to-leading order can we justify quantitatively the harmless nature of these logarithms, and thereby the applicability of perturbation theory.

ORGANIZING A CALCULATION

At leading order, producing numerical predictions for an n-jet process in lepton-hadron collisions requires knowledge of the parton distribution functions of the proton; of α_s; and of the tree-level matrix element for the $\ell + x \to \ell + n$ parton process. (The n-jet count excludes the remnant.) We can get this matrix element easily from that for $\ell\bar{\ell} \to (n + 1)$ partons. At next-to-leading order, the calculation of the same process also requires the tree-level matrix element for $\ell\bar{\ell} \to (n + 2)$ partons, and the one-loop matrix element for $\ell\bar{\ell} \to (n+1)$ partons. (It furthermore requires a general formalism such as that of refs. [1–4] for cancelling the infrared singularities analytically while allowing a numerical calculation of fully-differential observables.)

It is the calculation of the one-loop matrix elements that requires the bulk of the theoretical effort in producing predictions for a new process. In the traditional Feynman-diagram approach, even amplitudes with four external partons [5] require hard, lengthy calculations; and the situation only gets worse as the number of external legs grows. The difficulties arise from the enormous number of diagrams, the large amount of vertex algebra in each diagram, and the complexity of loop integrals with many powers of the loop momentum in the numerator. A brute-force approach might easily lead to expressions thousands of times larger than an appropriate representation of a result.

The vastly more efficient methods developed in recent years start by taking advantage of earlier developments in both tree-level calculations [6] and string-based methods for loop calculations [7]. These include (a) color de-

composition [8], (b) the spinor helicity method [9], (c) use of supersymmetric decompositions, (d) decomposition into 'primitive' amplitudes, and the use of permutation identities [11] that express subleading-color amplitudes as a sum of permutations over 'primitive' amplitudes. Primitive amplitudes correspond to color-ordered amplitudes with a definite orientation of internal fermion lines. These fundamental building blocks may be computed efficiently using the twin tools of unitarity-based sewing and factorization, developed in refs. [10] and reviewed at length in ref. [12]. These techniques have been used extensively in a series of calculations, including the one-loop matrix elements [11] for $0 \rightarrow q\bar{q}ggg$, the all-multiplicity maximally-helicity violating amplitudes in $N = 4$ and $N = 1$ supersymmetric theories [10], and the one-loop matrix elements for $\ell\bar{\ell} \rightarrow q\bar{q}q'\bar{q}'$ [13] and $\ell\bar{\ell} \rightarrow q\bar{q}gg$ [14].

CALCULATION OF PRIMITIVE AMPLITUDES

A one-loop amplitude, in general, contains absorptive pieces. The corresponding dispersive terms can be determined via dispersion relations from their cuts, which are just given by products of tree amplitudes. (The approach can be extended beyond one loop; for first steps in this direction see another contribution to this session [15].) In practice, one does not need to use dispersion relations explicitly, but only implicitly, to determine the integral functions that appear in any given amplitude, along with their coefficients. The restricted number of integral functions that can appear implies that some cut-free pieces are also determined by this approach of sewing tree amplitudes. Indeed, in supersymmetric theories, the entire amplitude is given by this technique.

The key point in the unitarity-based method is that we sew tree *amplitudes*, not tree diagrams. A calculation thus takes advantage of all the cancellations and reductions in numbers of terms that have already occurred in the process of computing the tree amplitudes.

A calculation using the unitarity-based method proceeds as follows. We want to compute the coefficients c_i of each of the box, triangle or bubble integrals that might appear in the amplitude. (The set of possible integrals can be determined by a 'gedanken' reduction using Passarino-Veltman [16] and van Neerven-Vermaseren [17], or equivalent [18] techniques.) We consider in turn cuts in all possible channels. For a given channel, we form the cut in that channel, summing over all intermediate states; this gives rise to a phase-space integral of the form

$$\int d^D \text{LIPS}(-\ell_1, \ell_2) \, A_L^{\text{tree}}(-\ell_1, \ldots, \ell_2) A_R^{\text{tree}}(-\ell_2, \ldots, \ell_1) \,, \tag{1}$$

where $\ell_{1,2}$ are the four-dimensional on-shell momenta crossing the cut, and where the $A_{L,R}^{\text{tree}}$ are the color-ordered tree subamplitudes on the two sides

of the cut. Using the Cutkosky rules, we can rewrite this expression as the absorptive or imaginary part of a loop amplitude,

$$\left[\int \frac{d^D \ell_1}{(2\pi)^D} A_L^{\text{tree}} \frac{1}{\ell_2^2 + i\varepsilon} A_R^{\text{tree}} \frac{1}{\ell_1^2 + i\varepsilon} \right]_{\text{cut}} . \tag{2}$$

In this expression, we may use the on-shell conditions $\ell_1^2 = 0 = \ell_2^2$ freely in all factors except the inserted propagators. Using a power-counting theorem [10,12] in those cases where it applies (or simply up to a polynomial ambiguity to be fixed as described below, in cases where it does not apply), we can recover the real parts by dropping the subscript "cut". We then perform the usual reductions on the resulting loop integral, and extract the coefficient of any integral function containing a cut in the given channel. (Functions which don't contain a cut in the given channel should be dropped.) Finally, we reassemble the final answer by considering all channels.

The box integrals have cuts in more than one channel. Considering both channels provides us with a cross check — the coefficients as computed in both channels must agree — or alternatively we could reduce the amount of work we must do by considering only one channel. The latter choice is particularly appropriate when computing amplitudes in an $N = 4$ supersymmetric gauge theory; in this theory, all amplitudes can be written in terms of scalar boxes. (We assume the use of a supersymmetrically-consistent version of dimensional regularization, such as dimensional reduction, throughout.)

In non-supersymmetric theories, there are remaining cut-free or polynomial terms which are not determined by sewing tree amplitudes in $D = 4$. It is possible to determine them if we use the cuts at $\mathcal{O}(\epsilon)$ rather than just to $\mathcal{O}(\epsilon^0)$. The reason is that amplitudes in a massless gauge theory have an over-all power of $(-s)^{-\epsilon}$, where s is one of the momentum invariants of the external legs. The polynomial terms therefore do contain cuts at $\mathcal{O}(\epsilon)$, and can be deduced by sewing tree amplitudes, where the momenta crossing the cuts are taken to be on-shell in $(4 - 2\epsilon)$ dimensions rather than four dimensions. For scalars (and fermions [19]), this is equivalent to computing massive rather than massless amplitudes, followed by an appropriately weighted integration over the "mass" (really the (-2ϵ)-dimensional component of the momentum).

However, for practical purposes, it is better to use collinear factorization to determine the polynomial terms. This technique relies on the universal factorization of one-loop amplitudes in the collinear limit [20,10], which for leading-color amplitudes reads,

$$A_{n;1}^{1-\text{loop}}(\cdots, a, b) \xrightarrow{a \| b}$$
$$\sum_{\lambda_P = \pm} \left[C_{-\lambda_P}^{\text{tree}}(a^{\lambda_a}, b^{\lambda_b}) A_{n-1;1}^{1-\text{loop}}(\cdots; P^{\lambda_P}) + C_{-\lambda_P}^{1-\text{loop}}(a^{\lambda_a}, b^{\lambda_b}) A_{n-1}^{\text{tree}}(\cdots; P^{\lambda_P}) \right] \tag{3}$$

where the splitting amplitudes C are universal functions depending only on the collinear momenta and their helicities λ, and on the momentum fraction

z ($k_P = k_a + k_b$, $k_a = zk_P$). In general, the terms deduced by sewing trees will not yield the correct limit; one must add polynomial pieces. One caveat in applying this technique is that we have no proof that this determines the missing polynomial pieces uniquely. (It is likely true for more than five external legs; for five external legs, there is an ambiguity arising from the existence of a term which contains no cuts but is collinear-finite.) The results obtained using this technique thus need to be checked (for example, numerically) against results from another method.

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