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The Top Cross Section in the Standard Model *

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Abstract

We recalculate the $t\bar{t}$ total cross section, $\sigma_{t\bar{t}}$, using Padé summation. Our result, $\sigma_{t\bar{t}} = 6.9 \pm 0.3\, pb$ is in agreement with the latest results from CDF and DØ at Fermilab.

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The sixth (and final?) quark, the top quark t, was discovered by CDF and DØ at Fermilab in 1995 [1]. Its large mass is remarkable when compared with the masses of the other quarks. The latest value from DØ and CDF is [2]

$$m_t = 176.6 \pm 5.5 \; GeV/c^2 \; . \tag{1}$$

From the beginning it was noticed that the $t\bar{t}$ total cross section, $\sigma_{t\bar{t}}$, was larger than the Standard Model prediction.

The latest experimental values [3] are

$$\sigma_{t\bar{t}}^{\rm expt} = \begin{cases} 5.5 \pm 1.8 \; pb & \text{DØ} \\ 7.5 \; {}^{+1.9}_{-1.6} \; pb & \text{CDF} \end{cases} \tag{2}$$

Combining these two values we obtain

$$\sigma_{t\bar{t}}^{\rm expt} = 6.6 \pm 1.3 \; pb \; . \tag{3}$$

This is uncomfortably higher than the theoretical predictions:

$$\sigma_{t\bar{t}}^{\rm th} = \begin{cases} 5.52 \; {}^{+0.07}_{-0.45} \; pb & \text{(Berger and Contopanagos [4])} \\ 4.75 \; {}^{+0.73}_{-0.62} \; pb & \text{(Cantani } et\ al.\text{ [5])} \\ 4.79 \pm 0.67 \; pb & \text{(Laenen } et\ al.\text{ [6])} \end{cases} \tag{4}$$

In this letter we make use of Padé Approximants (PA) to obtain a more precise value for $\sigma_{t\bar{t}}^{th}$. In the last several years, we have demonstrated that PA can be very useful in QCD [7, 8, 9, 10], both in predicting unknown higher-order coefficients (Asymptotic Padé Approximant Predictions–APAP) as well as in summing the perturbative series (Padé Summation–PS).

Given a perturbative series for some physical observable A that has been calculated to some finite order n,

$$A \sim A_n = A_0 + A_1 x + A_2 x^2 + \cdots + A_n x^n \tag{5}$$

where $x = \alpha_s/\pi$, the corresponding PA's are ratios of polynomials

$$[N/M] = \frac{a_0 + a_1 x + \cdots + a_N x^N}{1 + b_1 x + \cdots + b_M x^M} \tag{6}$$

with $N{+}M = n$, chosen such that they reproduce the known coefficients $A_0,\ A_1 \ldots,\ A_n$ when expanded back in a Taylor series. It is clear that the PA $[N/M]$ includes some higher-order effects: the hope is that it resums physically relevant higher-order contributions so that $[N/M]$ will be a better approximation to the observable A than the original truncated power series A_n.

In this letter we use Padé Summation (PA) to resum the $t\bar{t}$ total cross section to obtain a better, more precise theoretical prediction for $\sigma^{th}_{t\bar{t}}$.

From Ref. [4], using $\mu = m_t$, we find that the $q\bar{q}$ contribution is given by

$$\sigma^{(0)}_{q\bar{q}} = 3.69 \; pb \tag{7}$$

and

$$\sigma^{(0+1)}_{q\bar{q}} = 4.43 \; pb \; . \tag{8}$$

PA gives

$$\sigma_{q\bar{q}} = \frac{3.69 \, pb}{1 - 6.1516 \, x} = 4.62 \pm 0.15 \, pb \tag{9}$$

where

$$x = \frac{\alpha_s}{\pi} = 0.03260 \; . \tag{10}$$

Similarly for the gluon contribution we get

$$\sigma_{gg} = \frac{0.34 \, pb}{1 - 26.1638 \, x} = 2.31 \pm 0.25 \, pb \; . \tag{11}$$

Thus we obtain for our theoretical prediction

$$\sigma^{th}_{t\bar{t}} = \sigma_{q\bar{q}} + \sigma_{gg} = 6.9 \pm 0.3 \, pb \; . \tag{12}$$

Comparing this with the experimental result in Eq. (3),

$$\sigma^{\rm expt}_{t\bar{t}} = 6.6 \pm 1.3 \, pb \tag{13}$$

we find excellent agreement. The difference is

$$\Delta = \sigma^{th}_{t\bar{t}} - \sigma^{\rm expt}_{t\bar{t}} = 0.3 \pm 1.3 \, pb \; . \tag{14}$$

The lesson to be learned here is that leading-order (LO) and next-to-leading-order (NLO) QCD is not sufficient to properly make use of what are now very precise experimental measurements. Similar results were found previously in the SLD determinations [11] of $\alpha_s(M_Z)$ at SLAC and the measurement [12] $\sigma_{W+1 \; jet}/\sigma_{W+0 \; jets}$ at DØ at Fermilab. In the first case, the scatter was reduced by a factor of two.

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