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Light front Treatment of Nuclei and Deep Inelastic Scattering

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I. INTRODUCTION

A light front treatment of the nuclear wave function is developed and applied, using the mean field approximation, to infinite nuclear matter. The nuclear mesons are shown to carry about a third of the nuclear plus momentum p^+; but their momentum distribution has support only at $p^+ = 0$, and the mesons do not contribute to nuclear deep inelastic scattering. This zero mode effect occurs because the meson fields are independent of space-time position.

II. DISCUSSION

The discovery that the deep inelastic scattering structure function of a bound nucleon differs from that of a free one (the EMC effect[1]) changed the way that physicists viewed the nucleus. With a principal effect that the plus momentum (energy plus third component of the momentum, $p^0 + p^3 \equiv p^+$) carried by the valence quarks is less for a bound nucleon than for a free one, quark and nuclear physics could not be viewed as being independent.

The interpretation of the experiments requires that the role of conventional effects, such as nuclear binding, be assessed and understood[2, 3]. Nuclear binding is supposed to be relevant because the plus momentum of a bound nucleon is reduced by the binding energy, and so is that of its confined quarks. Conservation of momentum implies that if nucleons lose momentum, other constituents such as nuclear pions[4], must gain momentum. This partitioning of the total plus momentum amongst the various constituents is the momentum sum rule. Pions are quark anti-quark pairs so that a specific enhancement of the nuclear antiquark momentum distribution is a testable [5] consequence of this idea. A nuclear Drell Yan experiment [6], was performed, but no influence of nuclear pion enhancement was seen. This led Ref. [7] to question the idea of the pion as a dominant carrier of the nuclear force.

III. NUCLEAR CALCULATION

Here a closer look at the relevant nuclear theory is taken, and the momentum sum rule is studied. This talk is based on Ref.[8].

The structure function depends on the Bjorken variable x_{Bj} which in the parton model is the ratio of the quark plus momentum to that of the target. Thus $x_{Bj} = p^+/k^+$, where k^+ is the plus momentum of a nucleon bound in the nucleus, so a more direct relationship between the necessary nuclear theory and experiment occurs by using a theory in which k^+ is one of the canonical variables. Since k^+ is conjugate to a spatial variable $x^- \equiv t - z$, it is natural to quantize the dynamical variables at the equal light cone time variable of $x^+ \equiv t + z$. To use such a formalism is to use light front quantization, which requires a new derivation of the nuclear wave function, because previous work used the equal time formalism.

Such a derivation is provided here, using a simple model in which the nuclear constituents are nucleons ψ (or ψ'), scalar mesons ϕ[10] and vector mesons V^μ. The Lagrangian $\mathcal{L}$ is given by

$$\begin{aligned}\mathcal{L} = \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2) - \frac{1}{4}V^{\mu\nu}V_{\mu\nu} + \frac{m_v^2}{2}V^\mu V_\mu \\ +\bar{\psi}'\left(\gamma^\mu(i\partial_\mu - g_v\, V_\mu) - M - g_s\phi\right)\psi' \qquad (1)\end{aligned}$$

where the bare masses of the nucleon, scalar and vector mesons are given by M, m_s, m_v, and

$V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. This Lagrangian may be thought of as a low energy effective theory for nuclei under normal conditions.

This hadronic model, when evaluated in mean field approximation, gives[11] at least a qualitatively good description of many (but not all) nuclear properties and reactions. The aim here is to use a simple Lagrangian to study the effects that one might obtain by using a light front formulation. It is useful to simplify this first calculation by studying infinite nuclear matter.

The light front quantization procedure necessary to treat nucleon interactions with scalar and vector mesons was derived by Soper[13] and also by Yan and collaborators[14, 15]. Glazek and Shakin[16] used a Lagrangian containing nucleons and scalar mesons to study infinite nuclear matter. Here both vector and scalar mesons are included, and the nuclear plus momentum distribution is obtained.

Next examine the field equations. The nucleons satisfy

$$\gamma \cdot (i\partial - g_v V)\psi' = (m + g_s\phi)\psi'. \tag{2}$$

The number of independent degrees of freedom for light front field theories is fewer than in the usual theory. One defines projection operators $\Lambda_\pm \equiv \gamma^0\gamma^\pm/2$ and the independent Fermion degree of freedom is $\psi'_+ = \Lambda_+\psi'$. The relation between ψ'_- and ψ'_+ is very complicated unless one may set the plus component of the vector field to zero. This is a matter of a choice of gauge for QED and QCD, but the non-zero mass of the vector meson prevents such a choice here. Instead, one simplifies the equation for ψ'_- by[15] transforming the Fermion field according to $\psi' = e^{-ig_v\Lambda(x)}\psi$ with $\partial^+\Lambda = V^+$ which yields

$$\begin{aligned}(i\partial^- - g_v\bar{V}^-)\psi_+ &= (\boldsymbol{\alpha}_\perp \cdot (\boldsymbol{p}_\perp - g_v\bar{\boldsymbol{V}}_\perp) + \beta(M + g_s\phi))\psi_- \\ i\partial^+\psi_- &= (\boldsymbol{\alpha}_\perp \cdot (\boldsymbol{p}_\perp - g_v\bar{\boldsymbol{V}}_\perp) + \beta(M + g_s\phi))\psi_+\end{aligned} \tag{3}$$

where

$$\partial^+\bar{V}^\mu = \partial^+V^\mu - \partial^\mu V^+ \tag{4}$$

The field equations for the mesons are

$$\partial_\mu V^{\mu\nu} + m_v^2 V^\nu = g_v \bar{\psi}\gamma^\nu\psi$$
$$\partial_\mu\partial^\mu\phi + m_s^2\phi = -g_s\bar{\psi}\psi. \tag{5}$$

We now introduce the mean field approximation[11]. The coupling constants are considered strong and the Fermion density large. Then the meson fields can be approximated as classical- the sources of the meson fields are replaced by their expectation values. In this case, the nucleon mode functions will be plane waves and the nuclear matter ground state can be assumed to be a normal Fermi gas, of Fermi momentum k_F, and of large volume Ω in its rest frame. We consider the case that there is an equal number of protons and neutrons. Then the meson fields are constants given by

$$\phi = -\frac{g_s}{m_s^2}\langle\bar{\psi}\psi\rangle$$
$$V^\mu = \frac{g_v}{m_v^2}\langle\bar{\psi}\gamma^\mu\psi\rangle = \delta^{0,\mu}\frac{g_v\rho_B}{m_v^2}, \tag{6}$$

where $\rho_B = 2k_F^3/3\pi^2$. This result that V^μ is a constant, along with Eqs. (4) and (6), tells us that the only non-vanishing component of $\bar{V}$ is $\bar{V}^- = V^0$. The expectation values refer to the nuclear matter ground state.

With this, the light front Schroedinger equation for the modes of the field operator $\sim e^{ik\cdot x}$ and can be obtained from Eq. (3) as[12]

$$(i\partial^- - g_v\bar{V}^-)\psi_+ = \frac{\boldsymbol{k}_\perp^2 + (M + g_s\phi)^2}{k^+}\psi_+. \tag{7}$$

The light front eigenenergy ($i\partial^- \equiv k^-$) is the sum of a kinetic energy term in which the mass is shifted by the presence of the scalar field, and an constant energy arising from the vector field. Thus the nucleons have a mass $M + g_s\phi$ and move in plane wave states. The nucleon field operator is constructed using the solutions of Eq. (7) as the plane wave basis states. This means that the nuclear matter ground state, defined by operators that create and destroy baryons in eigenstates of Eq. (7), is the correct wave function and that

Equations (6) and (7) represent the solution of the approximate field equations, and the diagonalization of the Hamiltonian.

The computation of the energy and plus momentum distribution proceeds from taking the appropriate expectation values of $T^{\mu\nu}$:

$$P^{\mu} = \frac{1}{2}\int d^2x_{\perp}dx^{-}\langle T^{+\mu}\rangle. \tag{8}$$

We are concerned with the light front energy P^- and momentum P^+. Within the mean field approximation one finds

$$\begin{aligned} T^{+-} &= m_s^2\phi^2 + 2\psi_+^{\dagger}(i\partial^- - g_v\bar{V}^-)\psi_+ \\ T^{++} &= m_v^2V_0^2 + 2\psi_+^{\dagger}i\partial^+\psi_+. \end{aligned} \tag{9}$$

Taking the nuclear matter expectation value of T^{+-} and T^{++} and performing the spatial integral of Eq. (8) leads to the result

$$\frac{P^-}{\Omega} = m_s^2\phi^2 + \frac{4}{(2\pi)^3}\int_F d^2k_{\perp}dk^+\frac{\boldsymbol{k}_{\perp}^2 + (M+g_s\phi)^2}{k^+} \tag{10}$$

$$\frac{P^+}{\Omega} = m_v^2V_0^2 + \frac{4}{(2\pi)^3}\int_F d^2k_{\perp}dk^+k^+. \tag{11}$$

The subscript F denotes that $\mid \vec{k} \mid < k_F$ with k^3 defined by the relation

$$k^+ = \sqrt{(M+g_s\phi)^2 + \vec{k}^2} + k^3. \tag{12}$$

The expression for the energy of the system $E = \frac{1}{2}(P^+ + P^-)$[16], is the same as in the usual treatment[11]. This can be seen by summing equations (10) and (11) and changing integration variables using $\frac{dk^+}{k^+} = \frac{dk^3}{\sqrt{(M+g_s\phi)^2+\vec{k}^2}}$. This equality of energies is a nice check on the present result because a manifestly covariant solution of the present problem, with the usual energy, has been obtained[17]. Moreover, setting $\frac{\partial E}{\partial \phi}$ to zero reproduces the field equation for ϕ, as is also usual. Rotational invariance, here the relation $P^+ = P^-$, follows as the result of minimizing the energy per particle at fixed volume with respect to k_F, or minimizing the energy with respect to the volume[16]. The parameters $g_v^2M^2/m_v^2 = 195.9$ and $g_s^2M^2/m_s^2 = 267.1$ have been chosen [18] so as to give the binding energy per particle of nuclear matter as 15.75 MeV with k_F=1.42 Fm^{-1}. In this case, solving the equation for ϕ gives $M + g_s\phi = 0.56\ M$.

IV. NUCLEAR PLUS MOMENTUM DISTRIBUTIONS

The use of Eq. (11) and these parameters leads immediately to the result that only 65% of the nuclear plus momentum is carried by the nucleons; the remainder is carried by the mesons. This is a much smaller fraction than is found in typical nuclear binding models[2, 3]. The nucleonic momentum distribution which is the input to calculations of the nuclear structure function of primary interest here. This function can be computed from the integrand of Eq.(11). The probability that a nucleon has plus momentum k^+ is determined from the condition that the plus momentum carried by nucleons, P_N^+, is given by $P_N^+/A = \int dk^+\ k^+ f(k^+)$, where $A = \rho_B \Omega$. It is convenient to use the dimensionless variable $y \equiv \frac{k^+}{\bar{M}}$ with $\bar{M} = M - 15.75$ MeV. Then Eq.(11) and simple algebra leads to the equation

$$f(y) = \frac{3}{4}\frac{\bar{M}^3}{k_F^3}\theta(y^+ - y)\theta(y - y^-)\left[\frac{k_f^2}{\bar{M}^2} - (\frac{E_f}{\bar{M}} - y)^2\right], \tag{13}$$

where $y^{\pm} \equiv \frac{E_F \pm k_F}{\bar{M}}$ and $E_F \equiv \sqrt{k_F^2 + (M + g_s\phi)^2}$. Similarly the baryon number distribution $f_B(y)$ (number of baryons per y, normalized to unity) can be determined from the expectation value of $\psi^\dagger\psi$. The result is

$$f_B(y) = \frac{3}{8}\frac{\bar{M}^3}{k_F^3}\theta(y^+ - y)\theta(y - y^-)$$
$$\left[(1 + \frac{E_F^2}{\bar{M}^2 y^2})(\frac{k_f^2}{\bar{M}^2} - (\frac{E_F}{\bar{M}} - y)^2) - \frac{1}{2y^2}(\frac{k_F^4}{\bar{M}^4} - (\frac{E_F}{\bar{M}} - y)^4)\right], \tag{14}$$

which is different than $f(y)$.

The nuclear deep inelastic structure function, F_{2A} can be obtained from the light front distribution function $f(y)$ and the nucleon structure function F_{2N} using the relation[3]

$$\frac{F_{2A}(x)}{A} = \int dy f(y) F_{2N}(x/y), \tag{15}$$

where x is the Bjorken variable computed using the nuclear mass divided by A ($\bar{M}$): $x = Q^2/2\bar{M}\nu$. This formula is the expression of the convolution model in which one means to assess, via $f(y)$, only the influence of nuclear binding. Consider the present effect of having the average value of y equal to 0.65. Frankfurt and Strikman[3] use Eq. (15) to argue that an average of 0.95 is sufficient to explain the 15% depletion effect observed for the Fe nucleus. One may also compare the 0.65 fraction with the result 0.91 computed[19] for nuclear matter, including the effects of correlations, using equal time quantization. The present result then represents a very strong binding effect, even though this infinite nuclear matter result can not be compared directly with the experiments using Fe targets. One might think that the mesons, which cause this binding, would also have huge effects on deep inelastic scattering.

It is necessary to determine the momentum distributions of the mesons. The mesons contribute 0.35 of the total nuclear plus momentum, but we need to know how this is distributed over different individual values. The paramount feature is that ϕ and V^{μ} are the same constants for any and all values of the spatial coordinates $x^-, \boldsymbol{x}_\perp$. This means that the related momentum distribution can only be proportional to a delta function setting both the plus and $\perp$ components of the momentum to zero. This result is attributed to the mean field approximation, in which the meson fields are treated as classical quantitates. Thus the finite plus momentum can be thought of as coming from an infinite number of quanta, each carrying an infinitesimal amount of plus momentum. A plus momentum of 0 can only be accessed experimentally at $x_{Bj} = 0$, which requires an infinite amount of energy. Thus, in the mean field approximation, the scalar and vector mesons can not contribute to deep inelastic scattering. The usual term for a field that is constant over space is a zero mode, and the present Lagrangian provides a simple example. For finite nuclei, the mesons would carry a very small momentum of scale given by the inverse of the nuclear radius, under the mean field approximation. If fluctuations were to be included, the relevant momentum scale would be of the order of the inverse of the average distance between nucleons (about 2 Fm).

V. SUMMARY AND ASSESSMENT

The Lagrangian of Eq. (1) and its evaluation in mean field approximation for nuclear matter have been used to provide a simple but semi-realistic example. It would be premature to compare the present results with data. The specific numerical results of the present work are far less relevant than the emergent central feature that the mesons responsible for nuclear binding need not be accessible in deep inelastic scattering. Another interesting feature is that $f(y)$ and $f_B(y)$ are not the same functions.

More generally, we view the present model as being one of a class of models in which the mean field plays an important role. For such models nuclei would have constituents that contribute to the momentum sum rule but do not contribute to deep inelastic scattering. Thus the predictive and interpretive power of the momentum sum rule is vitiated.

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[9] Our notation is that a four vector A^μ is defined by the plus, minus and perpendicular components as $(A^0 + A^3, A^0 - A^3, \mathbf{A}_\perp)$.

[10] The scalar mesons are meant to represent the two pion exchange potential which causes much of the medium range attraction between nucleons, as well as the effects of a fundamental scalar meson. Thus the pion is an important implicit part of the present Lagrangian.

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