

## Chiral Limit of Nuclear Physics

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### Abstract

We study nuclear physics in the chiral limit ( $m_u, m_d = 0$ ) in which the pion mass vanishes. We find that the deuteron mass is changed little, but that P-wave nucleon-nucleon scattering volumes are infinite. This motivates an investigation of the possibilities that there could be a two-nucleon  ${}^3P_0$  bound state, and that the nuclear matter ground state is likely to be a condensed state of nucleons paired to those quantum numbers. However, the short distance repulsion in the nucleon-nucleon potential is not affected by the chiral limit and prevents such new chiral possibilities. Thus the chiral limit physics of nuclei is very similar to that of nature. Using the chiral limit to simplify QCD sum rule calculations of nuclear matter properties seems to be a reasonable approximation.

The derivative coupling of pions to nucleons, which results from spontaneously broken chiral symmetry, plays a major role in the structure of nuclei. This coupling, suppressed by the small momenta typical of nuclear physics, prevents the nuclear ground state, a system of strongly interacting hadrons, from being a pion soup. The net result is that the pion dominated internucleon interactions play a significant, but not dominant, role in the structure of the nuclei. The effects of the two pion exchange potential are believed to account for some of the needed mid-range attraction, but are not strong enough to induce a phase transition from the normal Fermi liquid that is believed to describe the ground state of heavy nuclei.

During recent years there has been a renewed interest in applying chiral Lagrangian models and QCD sum rules towards the description of microscopic nuclear structure[1, 2]. An important element of these analyses is the implicit assumption that the chiral limit  $m_q \rightarrow 0$ , in which  $m_\pi \rightarrow 0$  as well, is smooth enough and does not change qualitatively properties of nuclei. This is reflected in an assertion that the major nuclear characteristics can be expressed through various vacuum condensates which can be determined in the  $m_q \rightarrow 0$  limit. In particular, it was suggested that in this limit one can use the  $\omega, \sigma$ -model where pion degrees of freedom are essentially irrelevant[3].

This assumption raises a question: What is the nature of nuclei in the  $m_q \rightarrow 0$  limit? One can see immediately that the one pion exchange potential acquires a long range tensor interaction between nucleons  $\propto r^{-3}$ . This new interaction raises the possibility that in the chiral limit the structure of the nuclear ground state may be very different from that of actual nuclei. In the absence of quark masses the only scale in QCD is  $\Lambda_{QCD} \approx 200$  MeV, so that one might naively expect that the binding energy of the deuteron and the binding energy per nucleon in nuclear matter might be of that order of magnitude.

The purpose of the present note is to analyze nuclear properties in the chiral limit. This is based on the chiral limit of the nucleon-nucleon  $NN$  interaction. To take this limit we need to analyze the different scales. Our view is that the  $NN$  interaction can be understood in terms of two mass scales: the  $m_\pi$  scale which governs the long range physics and the  $m_\rho$  scale which governs short range physics. We explicitly set  $m_\pi$  to zero in the long range part of the nucleon-nucleon potential, but assume that the

short distance physics is unaffected by taking the chiral limit. Thus the short range  $NN$  interaction is taken to be the same as for realistic  $NN$  interactions. Consider, for example, the Bonn one boson exchange model[4], in which the short range part of the potential is essentially due to the exchange of  $\omega$ , and  $\rho$  mesons. Our assumption is that the masses and the interaction strengths of the vector mesons are not changed in the chiral limit.

The effects of the chiral limit on the medium range attraction are more subtle. It is typical (as is the case in Ref.[4]) to account for this physics by using the exchange of a  $\sigma$  meson of mass 550 MeV. Here we shall assume that the exchange of a  $\sigma$  meson is not influenced by taking the chiral limit, even though this could also be a phenomenological method of including the contribution of the exchange of uncorrelated pions to the two pion exchange potential. In this case, one would expect that taking the chiral limit would influence the medium range attraction. We do not investigate such effects here and our conclusions are based on setting  $m_\pi$  to zero in the one pion exchange potential (OPEP). Note also that contribution of correlated two pion exchange due to excitation of a  $\Delta$  isobar in the intermediate state is also not affected significantly by the change of the pion mass since the range of this interaction is mainly determined by the mass difference of the nucleon and  $\Delta$  which practically does not depend on  $m_q$ .

We proceed by arguing that the the value of the pion-nucleon coupling constant  $g$  should be little affected by taking in the chiral limit. Indeed, in this limit the Goldberger-Trieman relation ( $g/2m = g_A/2f_\pi$ ) would hold exactly and the coupling constant would change only if the nucleon mass  $m$  and the pion decay constant  $f_\pi$  were to depend on the pion mass. The quantity  $f_\pi$  depends on the wave function of the  $q\bar{q}$  component at the origin, which is expected to be essentially independent of the value of  $m_\pi$ . Similarly, the nucleon mass  $m$  is not expected to be significantly different in the chiral limit, see Ref. [5]. The value of  $m_\pi^2$  enters, via chiral loop graphs, as a term of the form  $m_q \ln(m_q)$ , which vanishes in the chiral limit. We thus conclude that the nucleon mass  $m$  and  $f_\pi$  are not significantly changed by considering the chiral limit, and that  $g$  should remain close to its measured value.

Thus in the chiral limit  $m_\pi \rightarrow 0$  the OPEP becomes:

$$V(\vec{r}) = -\frac{g^2}{4\pi} \frac{1}{12m^2} \vec{\tau}_1 \cdot \vec{\tau}_2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 \frac{1}{12\pi} \frac{\Lambda^3}{2} \exp(-\Lambda r) + \frac{g^2}{4m^2} \vec{\tau}_1 \cdot \vec{\tau}_2 S_{12} \frac{1}{4\pi} \left[ \frac{1}{r^3} - \left( \frac{1}{r^3} + \frac{\Lambda}{r^2} + \frac{\Lambda^2}{2r} + \frac{\Lambda^3}{6} \right) \exp(-\Lambda r) \right] \quad (1)$$

where the parameter  $\Lambda$ [4] governs the falloff of the monopole  $\pi$ -nucleon form factor. For values of  $r$  such that  $r \gg 1/\Lambda$  one is left with a tensor term that falls off rather slowly as  $1/r^3$ .

Does the chiral limit affect the deuteron? From Eq. (1) we see that the long range attraction present in the central potential is absent in the chiral limit, and that the attractive tensor force is enhanced. We compute the change in the deuteron binding energy by using first order perturbation theory and find a reduction of about 1 MeV. Specifically, if the deuteron wave function of the Paris NN potential[6] is used, about 2 MeV of attraction (from the central part of the OPEP) is lost, and 0.9 MeV is gained (from the tensor part of the OPEP), which is a loss of 1.1 MeV of binding. For the Bonn potential, the numbers are 2 MeV and  $-1.3$  MeV, or a loss of about 0.7 MeV. This small change seems inconsequential.

We next turn to the  $P$ -waves. The scattering volume is a measure of the  $P$ -wave scattering at low energies. As discussed by Ericson and Weise [7] this volume is qualitatively reproduced by computing the scattering amplitude from the OPEP in first order Born approximation.

In the chiral limit (and ignoring the effects of the form factor) the spin-triplet  $P$ -wave Born amplitudes are given by

$$\left[ \frac{e^{i\delta} \sin\delta}{p} \right]_{^3P_J} = -\frac{g^2}{16\pi M} \langle L=1, S=1, J | S_{12} | L=1, S=1, J \rangle \int r^2 dr \frac{j_1^2(pr)}{r^3}, \quad (2)$$

in which we keep only the long ranged term of Eq. (1) and where  $p$  is the relative momentum,  $\langle L, S, J | S_{12} | L, S, J \rangle = \{-(2L+2)/(2L-1), 2, -2L/(2L+3)\}$ , for  $J = \{L-1, L, L+1\}$ , or  $\{-4, 2, -4/5\}$  for  $L=1$ . The radial integral is dimensionless and it equals  $1/4$ . Thus for  $p \rightarrow 0$ , the phase shift behaves as  $\delta(^3P_J) \propto p$  and the scattering volume  $a(^3P_J)$ , defined by the relation  $a(^3P_J) = \lim_{p \rightarrow 0} \delta(^3P_J)/p^3$  is infinite in the chiral limit.

This behaviour of the phase shift and the infinity of  $a(^3P_J)$  is not an artifact of either retaining only the longest ranged term of Eq. (1) or using the first order Born

approximation. Any other shorter range component of the  $NN$  interaction potential leads to a contribution to the phase shift proportional to  $p^3$ . One can also consider very small momenta  $p$ , for which the phase shift is very small and thus the first order Born approximation holds.

Even though we have discussed only  $P$ -waves, one can show that for all non-zero angular momentum waves the phase shifts have a qualitatively similar behaviour, i.e.  $\delta(^3L_J) \propto p$ . This means that there is significant scattering at long range for all partial waves. Another way of showing this is by considering the first order Born approximation to the OPEP scattering amplitude  $f_B$

$$f_B \propto \frac{3\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + m_\pi^2}, \quad (3)$$

where  $\vec{q}$  is the momentum transfer. The quantity  $f_B$  vanishes in the forward direction  $\vec{q} = 0$ . Taking the chiral limit of a  $m_\pi \rightarrow 0$ , causes a non-vanishing amplitude which is independent of the magnitude of  $\vec{q}$ , and therefore contains significant scattering at all partial waves.

In the  $^3P_0$  channel the expectation value of  $S_{12}$  has the relatively large magnitude of 4, and the OPEP is attractive. This causes us to examine the possibility that the OPEP interaction in the chiral limit could lead to a bound state. In this channel the tensor potential is given by

$$V(r, ^3P_0) = -\frac{g^2}{4\pi m^2} \left[ \frac{1}{r^3} - \left( \frac{1}{r^3} + \frac{\Lambda}{r^2} + \frac{\Lambda^2}{2r} + \frac{\Lambda^3}{6} \right) \exp(-\Lambda r) \right]. \quad (4)$$

The effective potential,  $V_{eff}$ , defined by adding the centrifugal barrier of  $2/(m r^2)$  to the potential of Eq. (4) is shown in Fig. 1 (for  $\Lambda = 1.8$  GeV). This quantity has a broad minimum which is deep enough to support a bound state. However, one must also include the effects of the short range potential. Fig. 1 shows the effects of using the Argonne V18 potential[8] to represent the short distance interaction. This repulsive core causes the minimum to be eliminated and there will be no bound state.

Even if no bound  $NN$  state with quantum numbers  $T = 1$  and  $^3P_0$  exists, the chiral limit interaction in this channel is significantly stronger and has a significantly longer range than the usual OPEP potential. Thus we consider consequences for nuclear physics. In nuclear matter only the short range part of the  $NN$  interaction is strongly renormalized [9, 10] (the healing length is rather short), while the long

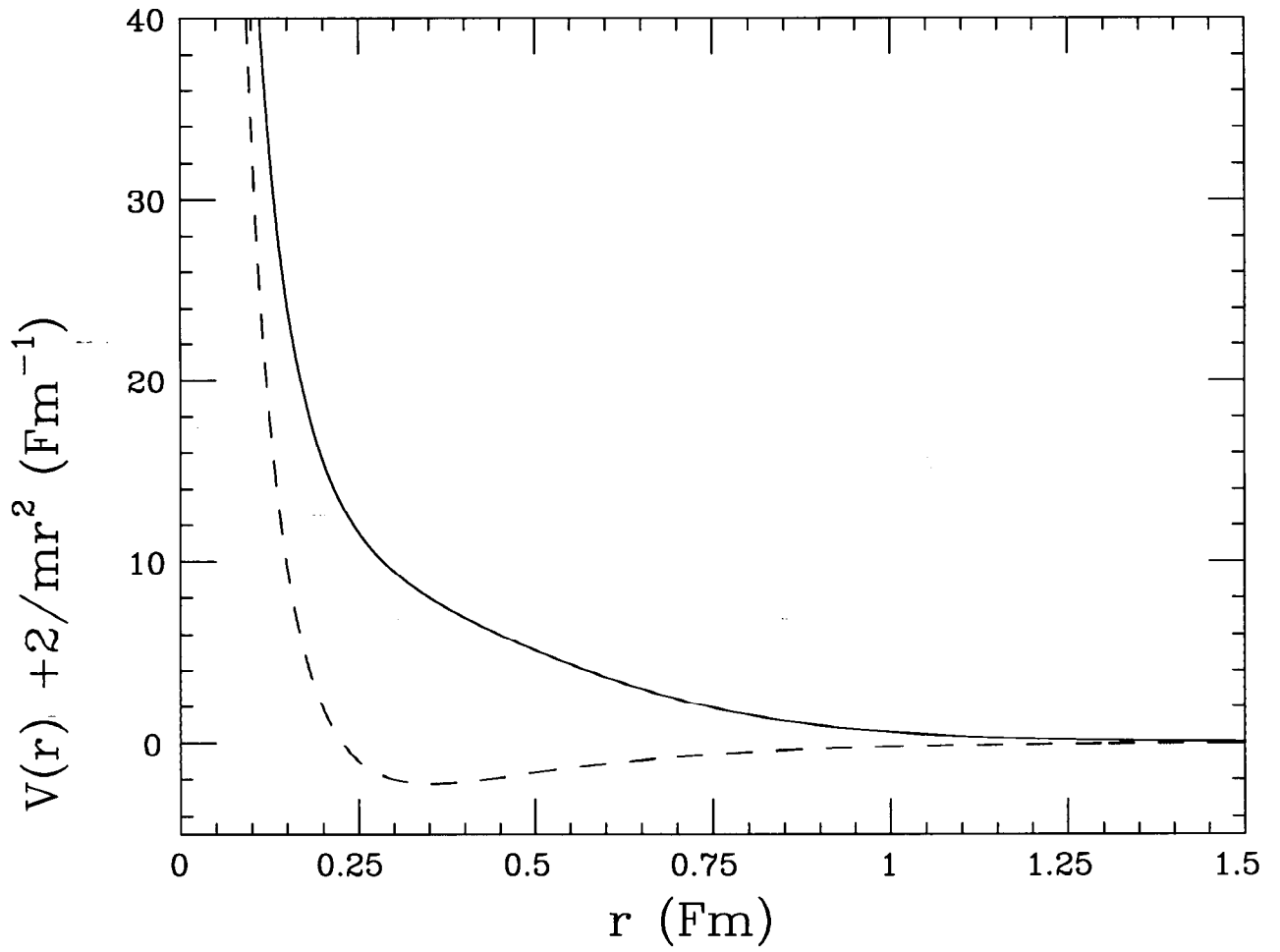


Figure 1:  $V_{eff} = V(r) + 2/mr^2$  for the  $^3P_0$  channel. Dashed curve –  $V(r)$  from Eq. (4).

range part survives. Therefore, the only qualitatively new element is the possibility to form nuclear matter with pairing properties analogous to bulk  ${}^3\text{He}$  [11], namely  $0^-$  nucleon pairs.  ${}^3P_0$  pairing is in some respects analogous to usual  ${}^1S_0$  pairing [11], as it leads to a isotropic momentum space gap and the condition for its appearance is formally similar to the usual BCS pairing gap equation.

We take up the notion of this new kind of condensation for infinite nuclear matter. For that system the gap equation is given by

$$\Delta(k) = - \int_0^\infty p^2 dp \frac{\langle k|V({}^3P_0)|p\rangle\Delta(p)}{2\sqrt{(\frac{p^2}{2M} - \varepsilon_F)^2 + \Delta^2(p)}}, \quad (5)$$

where  $\langle k|V({}^3P_0)|p\rangle$  is the momentum-space version of the potential of Eq. (4). The use of the interaction of Eq. (4), leads to nontrivial solutions for a range of values of  $\Lambda$ . The value  $\Delta(k)$  is typically several MeV for  $k \approx k_F$  for  $\Lambda \approx 1000$  MeV/c and significantly larger for larger values of  $\Lambda$ . It is amusing that in the chiral limit nuclei could become unstable with respect to the formation of a condensate of  $(T, J^\pi) = (1, 0^-)$  pairs, with the same quantum numbers as the real pions, which lead to this instability.

However, including the effects of the short ranged repulsive interaction causes the nontrivial solutions to be eliminated, such that  $\Delta(k) = 0$ .

We also examine some other situations for which the chiral limit has little impact. First we take up question of whether the increased attraction is sufficient to cause nuclei to condense into a crystalline solid[12]. This would occur for potentials that are strong enough to overcome the zero-point motion of nucleons about a possible lattice site. However, in the chiral limit the OPEP potential is not strong enough to lead to additional bound states. If more bound states than the deuteron would have appeared, and if the short range repulsion were absent, one could have expected that nuclear matter and nuclei in particular would have had a crystalline structure in their ground state. For potentials of the strength that we find, the zero point oscillations would have an amplitude of the same order of magnitude as the size of the deuteron, so we don't expect crystallization in the chiral limit.

Another possibility to consider is the formation of a condensate of real pions. In the chiral limit the pion mass is zero, and apparently there is no energy cost to generate real pions and thus create a "real pion condensate", which differs from the



condensate of virtual pions suggested by Migdal [13]. The energy penalty to create a zero mass pion, localized inside a large nucleus is of the order of  $\pi/R_A \approx 100$  MeV and thus a “real pion condensate” is extremely unlikely. We also find, by explicitly using  $m_\pi = 0$  in the usual equations[7, 13], that the possibility of a Migdal condensate is not enhanced in the chiral limit.

Finally, we consider the effects of the increased range of the tensor interaction for the binding energy of infinite nuclear matter. This enters in second-order in a non-relativistic calculation[9]. Explicit evaluate of this shows that the effects are negligible.

The net result is that the physics of nuclei in a universe where  $m_\pi = 0$  would be similar to what is actually observed. The QCD sum rule calculations of nuclear matter properties[1, 2] are not rendered erroneous through their use of the chiral limit.

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## References

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