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Light Front Treatment of Nuclei: Formalism and Simple Applications

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A relativistic light front treatment of nuclei is developed by performing light front quantization for a chiral Lagrangian. The energy momentum tensor and the appropriate Hamiltonian are obtained. Three illustrations of the formalism are made. (1) Pion-nucleon scattering at tree level is shown to reproduce soft pion theorems. (2) The one boson exchange treatment of nucleon-nucleon scattering is developed and shown (by comparison with previous results of the equal time formulation) to lead to a reasonable description of nucleon-nucleon phase shifts. (3) The mean field approximation is applied to infinite nuclear matter, and the plus momentum distributions of that system are studied. The mesons are found to carry a significant fraction of the plus momentum, but are inaccessible to experiments.

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I. INTRODUCTION

The need for a relativistic methodology that is broadly applicable to nuclear physics has never been more apparent. One of our most important sets of problems involves understanding the transition between the (baryon, meson) and the (quark, gluon) degrees of freedom. Using a relativistic formulation of the hadronic degrees of freedom is necessary to avoid a misinterpretation of a kinematic effect as a signal for the transition.

The goal of understanding future high momentum studies of nuclear targets using exclusive, nearly exclusive or inclusive processes can only be met through using relativistic techniques. The light front approach of Dirac [1] in which the time variable is taken as $t+z$ and the spatial variables are $t-z, x, y$ [2,3] is one of the promising approaches because the momentum canonically conjugate to $t-z$, $p^+ \equiv p^0 + p^3$, is directly related to the observables.

It is worthwhile to begin with a qualitative explanation of the utility of these light cone variables and this light front approach in a qualitative fashion. Consider lepton-nucleus deep inelastic scattering as a first example. The observed structure function depends on the Bjorken variable x_{Bj} which in the parton model is the ratio of the quark plus momentum to that of the target. If one regards the nucleus as a collection of nucleons, $x_{Bj} = p^+/k^+$, where k^+ is the plus momentum of a nucleon bound in the nucleus. Thus, a more direct relationship between the necessary nuclear theory and experiment occurs by using a theory in which k^+ is one of the canonical variables. Since k^+ is conjugate to a spatial variable $x^- \equiv t - z$, it is natural to quantize the dynamical variables at the equal light cone time variable of $x^+ \equiv t + z$. To use such a formalism is to use light front quantization, since the other three spatial coordinates $(x^-, \boldsymbol{x}_\perp)$ are on a plane perpendicular to a light like vector [4]. This use of light front quantization requires a new derivation of the nuclear wave function, because previous work used the equal time formalism.

Are these light cone variables useful only in nuclear deep inelastic scattering? Let's answer this by examining the origin of such coordinates. The four momentum of the incident virtual photon, q, can be said to have the components $q = (\nu, 0, 0, -\nu - Q^2/2\nu)$, with

$q^2 = -Q^2$, and ν, Q^2 very large but Q^2/ν finite (the Bjorken limit). Then $x_{Bj} \equiv \frac{Q^2}{2k\cdot q} = \frac{Q^2}{k^+q^-}$ The condition that the reaction be elastic scattering from the quarks is that $(p+q)^2 = p^2$, or $2p\cdot q = Q^2 = p^+q^-$. Thus $x_{Bj} = p^+/k^+$ results from having only one large momentum in the problem, which can be taken in the negative z-direction, so that minus component is enhanced. More generally, one expects to be able to use light cone coordinates $(p^+, p^-, p_\perp)$ whenever there is such a large momentum in the problem as in any high energy scattering process. Diverse applications are shown in the text by Cheng and Wu [5]. Examples of most relevance include high energy projectile nuclear scattering and high momentum transfer quasi-elastic reactions involving nuclear targets.

Light front techniques have previously been applied to systems of two hadrons. The two main approaches have been the relativistic quantum mechanics of directly interacting particles [6–9] and relativistic field theory [2,10,11]. We choose here to employ specific Lagrangians which embody chiral and other symmetries, and thus use field theory.

The light front quantization procedure necessary to treat nucleon interactions with scalar and vector mesons was derived by Soper [12], and by Yan and collaborators [13,14]. Here we combine the previous formalisms to obtain a light front treatment of a Lagrangian which contains pions, vector and scalar mesons, and which respects the constraints of chiral symmetry.

Here is an outline. The bulk of the formalism is presented in Sect. 2. First, a chiral Lagrangian which includes pions, scalar mesons and neutral vector mesons is presented. The field equations are derived, and the quantization procedure for the free and interacting fields are quantized at the zero of light cone time x^+. The energy momentum tensor, the light front hamiltonian P^- and plus momentum operator P^+ are derived. The necessary contact interactions involving the exchange of instantaneous fermions and vector bosons are obtained. The principal purpose of the present work is to develop a technique that could have wide application in nuclear physics. Thus we study and check the present formalism by applying it to three different examples- πN and NN scattering, and a mean field treatment of infinite nuclear matter- of relevance to nuclear physics. Dealing successfully with each of these subjects is a prerequisite for making progress.

Sect. 3 shows how light front field theory leads to a chiral treatment of low energy pion-nucleon scattering, which is consistent with the results of soft pion theorems. Then nucleon-nucleon scattering is handled in a manifestly covariant manner, within the one-boson approximation, in Sect. 4. A discussion of the impact of chiral symmetry on the two-nucleon intermediate state contribution to the two pion exchange potential is also included. The mean field approximation is applied to infinite nuclear matter in Sect 5. Glazek and Shakin [15] used a Lagrangian containing nucleons and scalar mesons to study infinite nuclear matter. Here vector mesons are included and the rotational-invariance arguments used in Sect. 4 are used to derive the Glazek-Shakin k^+ variable. The energy of nuclear matter is computed and shown to be the same as found in the equal time formalism. The unique feature of the present formalism is the ability to obtain the nuclear and mesonic plus-momentum distributions from the energy momentum tensor. We find that that mesons can carry a significant fraction of the nuclear plus momentum, but have support only at 0 plus-momentum. Some the results for nuclear matter have been presented in an earlier publication [16]; here the calculation is performed in two different ways and explained in more detail. Sect. 6 summarizes the new results, presents a critique and discusses possible future applications. Appendix A contains a summary of notation and some useful equations.

II. LIGHT FRONT QUANTIZATION

A. Lagrangian and Field equations

We use a non-linear chiral model in which the nuclear constituents are nucleons ψ (or ψ'), pions $\boldsymbol{\pi}$ scalar mesons ϕ [17] and vector mesons V^μ. The Lagrangian $\mathcal{L}$ is given by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2) - \frac{1}{4}V^{\mu\nu}V_{\mu\nu} + \frac{m_v^2}{2}V^\mu V_\mu$$
$$+\frac{1}{4}f^2 Tr(\partial_\mu\, U\; \partial^\mu\; U^\dagger) + \frac{1}{4}m_\pi^2 f^2\; Tr(U + U^\dagger - 2) + \bar{\psi}'\left(\gamma^\mu(\frac{i}{2}\overleftrightarrow{\partial}_\mu - g_v\; V_\mu) - MU - g_s\phi\right)\psi' \quad (2.1)$$

where the bare masses of the nucleon, scalar and vector mesons are given by M, m_s, m_v, and $V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. The unitary matrix U can be chosen from amongst three forms U_i:

$$U_1 \equiv e^{i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}/f}, \quad U_2 \equiv \frac{1 + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}/2f}{1 - i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}/2f}, \quad U_3 = \sqrt{1 - \pi^2/f^2} + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}/f, \tag{2.2}$$

which correspond to different definitions of the fields.

The pion-nucleon coupling here is chosen as that of linear representations of chiral symmetry used by Gursey [18], with the the Lagrangian approximately ($m_\pi \neq 0$) invariant under the chiral transformation

$$\begin{aligned} \psi' &\rightarrow e^{i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{a}} \psi' \\ U &\rightarrow e^{-i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{a}}\, U\, e^{-i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{a}}. \end{aligned} \tag{2.3}$$

One may transform the fermion fields, by taking $U^{1/2}\psi'$ as the nucleon field. One then gets Lagrangians of the non-linear representation, as explained by Weinberg [19]. In this case the early soft pion theorems are manifest in the Lagrangian, and the linear pion-fermion coupling is of the pseudovector type. However, the use of light front theory, requires that one find an easy way to solve the constraint equation that governs the fermion field. We shall show that the constraint can be handled in a simple fashion by using the linear representation. Moreover, we shall see that the early soft pion theorems are indeed manifest from the form of the light front Hamiltonian.

The constant $\frac{M}{f}$ plays the role of the bare pion-nucleon coupling constant. If f is chosen to be the pion decay constant, the Goldberger-Trieman relation yields the result that the axial vector coupling constant $g_A = 1$, which would be a problem for the Lagrangian, unless loop effects can make up the needed 25% effect. Corrections of that size are typical of order $(\frac{M}{f})^3$ effects found in the cloudy bag model [20] for many observables, including g_A.

There are no explicit $\Delta's$ in the above Lagrangian. Those will be handled in a future publication. For the moment we note that treating the higher order effects of the pion-nucleon inherent in this Lagrangian is likely to lead to a resonance in the (3,3) channel of pion nucleon scattering. Such effects can be included in the two-pion exchange contribution to nucleon-nucleon scattering. However, such an approach seems cumbersome.

The choice of using an explicit Δ instead of the iterated $\pi - N$ interaction is analogous to our use of a scalar meson even though the effects of $\pi - \pi$ interactions, which could lead to similar effects, are included in the Lagrangian. We follow many authors (see the review [21]) and include a scalar meson to simplify calculations. In this treatment, which follows that of Refs. [22,23], the scalar meson ϕ is not a chiral partner of the pion- the chiral transformation is that of Eq. (2.3).

The present Lagrangian may be thought of as a low energy effective theory for nuclei under normal conditions. A more sophisticated Lagrangian is reviewed in [23] and used in [22]; the present one is used to show that light front techniques can be applied to hadronic theories relevant for nuclear physics. This hadronic model, when evaluated in mean field approximation, gives [21] at least a qualitatively good description of many (but not all) nuclear properties and reactions. There are a variety of problems occuring when higher order terms are included [23]. The aim here is to use a reasonably sophisticated Lagrangian to study the effects that one might obtain by using a light front formulation.

We could also have used the linear sigma model. The light front quantization for that model can be accomplished using a simple generalization of the work of Refs. [12] and [13], and is not shown here. According to the review [23] the use of such a Lagrangean precludes a successful description of nuclei at the mean-field level.

The next step is to examine the field equations. The relevant Dirac equation for the nucleons is

$$\gamma \cdot (i\partial - g_v V)\psi' = (M\ U + g_s\phi)\psi'. \tag{2.4}$$

The field equations for the mesons are

$$\partial_\mu V^{\mu\nu} + m_v^2 V^\nu = g_v \bar{\psi}'\gamma^\nu\psi' \tag{2.5}$$

$$\partial_\mu \partial^\mu \phi + m_s^2 \phi = -g_s \bar{\psi}'\psi'. \tag{2.6}$$

$$\partial_\mu \partial^\mu \pi_i + m_\pi^2 f \sin(\pi/f)\frac{\pi_i}{\pi} + \partial_\mu \left[\frac{\pi_i}{\pi}\partial^\mu \pi (1 - \frac{f^2}{\pi^2}\sin^2\frac{\pi}{f})\right] = -m\bar{\psi}'\frac{\partial U}{\partial \pi_i}\psi', \tag{2.7}$$

where $\pi = (\sum_j \pi_j^2)^{1/2}$.

The next step is obtain the light front Hamiltonian (P^-) as a sum of a free, non-interacting and a set of terms containing all of the interactions. This is accomplished by using the energy momentum tensor as

$$P^\mu = \frac{1}{2}\int dx^- d^2x_\perp \, T^{+\mu}(x^+ = 0, x^-, \boldsymbol{x}_\perp). \tag{2.8}$$

The usual relations determine $T^{+\mu}$, with

$$T^{\mu\nu} = -g^{\mu\nu}\mathcal{L} + \sum_r \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_r)}\partial^\nu \phi_r, \tag{2.9}$$

in which the degrees of freedom are labelled by ϕ_r.

B. Free Meson Fields

It is worthwhile to consider the limit in which the interactions between the fields are removed. This will allow us to define the free Hamiltonian P_0^- and to display the necessary commutation relations. The energy momentum tensors of the non-interacting fields are defined as as $T_0^{\mu\nu}(\phi), T_0^{\mu\nu}(V)$, and $T_0^{\mu\nu}(\pi)$. The fermion fields are quantized in the next sub-section. Then the use of Eq.(2.9) leads to the result

$$T_0^{\mu\nu}(\phi) = \partial^\mu\phi\partial^\nu\phi - \frac{g^{\mu\nu}}{2}\left[\partial_\sigma\phi\partial^\sigma\phi - m_s^2\phi^2\right], \tag{2.10}$$

with

$$T^{+-}(\phi) = \frac{1}{2}\boldsymbol{\nabla}_\perp\phi\cdot\boldsymbol{\nabla}_\perp\phi + \frac{1}{2}m_s^2\phi^2. \tag{2.11}$$

The scalar field can be expressed in terms of creation and destruction operators:

$$\phi(x) = \int \frac{d^2k_\perp dk^+ \theta(k^+)}{(2\pi)^{3/2}\sqrt{2k^+}}\left[a(\boldsymbol{k})e^{-ik\cdot x} + a^\dagger(\boldsymbol{k})e^{ik\cdot x}\right], \tag{2.12}$$

where $k \cdot x = \frac{1}{2}(k^- x^+ + k^+ x^-) - \boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp$ with $k^- = \frac{k_\perp^2 + m_s^2}{k^+}$, and $\boldsymbol{k} \equiv (k^+, \boldsymbol{k}_\perp)$. The θ function restricts k^+ to positive values. The commutation relations are

$$[a(\boldsymbol{k}), a^\dagger(\boldsymbol{k}')] = \delta(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp)\delta(k^+ - k'^+) \tag{2.13}$$

with $[a(\boldsymbol{k}), a(\boldsymbol{k}')] = 0$. It is useful to define

$$\delta^{(2,+)}(\boldsymbol{k} - \boldsymbol{k}') \equiv \delta(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp)\delta(k^+ - k'^+), \tag{2.14}$$

which will be used throughout this paper.

The derivatives appearing in the quantity T^{+-} are evaluated and then one sets x^+ to 0 to obtain the result

$$P_0^-(\phi) = \int d^2k_\perp dk^+ \theta(k^+) a^\dagger(\boldsymbol{k}) a(\boldsymbol{k}) \frac{k_\perp^2 + m_s^2}{k^+}, \tag{2.15}$$

which has the interpretation of an operator the counts the light front energy $k^- = \frac{k_\perp^2 + m_s^2}{k^+}$ of all of the particles.

The pion field is treated in a similar manner, with the result

$$\boldsymbol{\pi}(x) = \int \frac{d^2k_\perp dk^+ \theta(k^+)}{(2\pi)^{3/2}\sqrt{2k^+}} \left[\boldsymbol{a}(\boldsymbol{k}) e^{-ik\cdot x} + \boldsymbol{a}^\dagger(\boldsymbol{k}) e^{ik\cdot x}\right], \tag{2.16}$$

and

$$P_0^-(\pi) = \int d^2k_\perp dk^+ \theta(k^+) \boldsymbol{a}^\dagger(\boldsymbol{k}) \cdot \boldsymbol{a}(\boldsymbol{k}) \frac{k_\perp^2 + m_\pi^2}{k^+}, \tag{2.17}$$

with commutation relations analogous to that of Eq.(2.13).

The energy momentum tensor for the free vector meson field is obtained directly from the defining relation (2.9) as

$$T_0^{\mu\nu}(V) = V^{\alpha\mu}\partial^\nu V_\alpha + g^{\mu\nu}\left[\frac{1}{4}V^{\alpha\beta}V_{\alpha\beta} - \frac{m_v^2}{2}V_\alpha V^\alpha\right]. \tag{2.18}$$

It is desirable to obtain the symmetric energy momentum tensor. This is done by using $\partial^\nu V^\alpha = \partial^\alpha V^\nu + V^{\nu\alpha}$, subtracting a total divergence and using the free field equations. The result is

$$T_0^{\mu\nu}(V) = V^{\alpha\mu}V^{\nu\beta}g_{\beta\alpha} + m_v^2 V^\mu V^\nu + g^{\mu\nu}\left[\frac{1}{4}V^{\alpha\beta}V_{\alpha\beta} - \frac{m_v^2}{2}V_\alpha V^\alpha\right]. \tag{2.19}$$

The component relevant for the light front Hamiltonian can be shown to be

$$T_0^{+-}(V) = \frac{1}{2}V^{\alpha+}\partial^- V_\alpha - V^{\alpha+}\partial_\alpha V_\alpha + \frac{m_v^2}{2}V^k V^k. \tag{2.20}$$

The expression for the vector meson field operator is

$$V^\mu(x) = \int \frac{d^2k_\perp dk^+\theta(k^+)}{(2\pi)^{3/2}\sqrt{2k^+}} \sum_{\omega=1,3} \epsilon^\mu(\boldsymbol{k},\omega)\left[a(\boldsymbol{k},\omega)e^{-ik\cdot x} + a^\dagger(\boldsymbol{k},\omega)e^{ik\cdot x}\right], \tag{2.21}$$

where the polarization vectors are the usual ones:

$$\begin{aligned} k^\mu \epsilon_\mu(\boldsymbol{k},\omega) = 0, \quad \epsilon_\mu(\boldsymbol{k},\omega)\epsilon^\mu(\boldsymbol{k},\omega') &= -\delta_{\omega\omega'} \\ \sum_{\omega=1,3} \epsilon^\mu(\boldsymbol{k},\omega)\epsilon^\nu(\boldsymbol{k},\omega) &= -(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_v^2}). \end{aligned} \tag{2.22}$$

Once again the four momenta are on-shell with $k^- = \frac{\boldsymbol{k}_\perp^2 + m_v^2}{k^+}$. The light front commutation relations:

$$[a(\boldsymbol{k},\omega), a^\dagger(\boldsymbol{k}',\omega')] = \delta_{\omega\omega'}\delta^{(2,+)}(\boldsymbol{k} - \boldsymbol{k}'), \tag{2.23}$$

with the others vanishing, lead to commutation relations amongst the field operators that are the same as in Ref. [14]. The expression for $P_0^-(V)$ can now be obtained from Eqs.(2.20) and (2.21) as

$$P_0^-(V) = \sum_{\omega=1,3} \int d^2k_\perp dk^+\theta(k^+)\frac{\boldsymbol{k}_\perp^2 + m_v^2}{k^+} a^\dagger(\boldsymbol{k},\omega)a(\boldsymbol{k},\omega). \tag{2.24}$$

C. Interacting Fields

This subject is complicated by the presence of massive vector meson fields. Various difficulties were handled by Soper [12] and Yan [14]. The key features that we use are summarized here. In particular, the fields V^+, V^{+i} are chosen as the three independent fields, with the others expressible in terms of these. We shall need only one of these relationships in which the plus component of Eq.(2.5) can be used to obtain

$$V^{-+} = \frac{2}{\partial^+}\left[g_v\, J^+ - m_v^2 V^+ - \partial_i V^{i+}\right]. \tag{2.25}$$

with

$$J^\mu \equiv \bar{\psi}'\gamma^\mu\psi'. \tag{2.26}$$

and the inverse of ∂^+ is defined in Refs. [12]- [14]. A more recent discussion is given by Harindrinath and Zhang [24], and the essentials are presented here in Appendix A,

We turn to the case of spin 1/2 fermions. Although described by four-component spinors, these fields have only two independent degrees of freedom. The light front formalism allows a convenient separation of dependent and independent variables via the projection operators $\Lambda_\pm \equiv \gamma^0\gamma^\pm/2$ [12], with $\psi'_\pm \equiv \Lambda_\pm\psi'_\pm$. The independent Fermion degree of freedom is chosen to be ψ'_+. The properties of the projection operators are discussed in Appendix A. One gets two coupled equations for $\psi'_\pm$ by multiplying Eq.(2.4) by Λ_+ and Λ_-:

$$\begin{aligned}(i\partial^- - g_v V^-)\psi'_+ &= (\boldsymbol{\alpha}_\perp\cdot(\boldsymbol{p}_\perp - g_v\boldsymbol{V}_\perp) + \beta(MU + g_s\phi))\psi'_- \\ (i\partial^+ - g_v V^+)\psi'_- &= (\boldsymbol{\alpha}_\perp\cdot(\boldsymbol{p}_\perp - g_v\boldsymbol{V}_\perp) + \beta(MU + g_s\phi))\psi'_+.\end{aligned} \tag{2.27}$$

The relation between ψ'_- and ψ'_+ is very complicated unless one may set the plus component of the vector field to zero [2]. This is a matter of a choice of gauge for QED and QCD, but the non-zero mass of the vector meson prevents such a choice here. Instead, one simplifies the equation for ψ'_- by [12,14] transforming the Fermion field according to

$$\psi' = e^{-ig_v\Lambda(x)}\psi \tag{2.28}$$

with

$$\partial^+\Lambda = V^+. \tag{2.29}$$

This transformation leads to the result

$$\begin{aligned}(i\partial^- - g_v\bar{V}^-)\psi_+ &= (\boldsymbol{\alpha}_\perp\cdot(\boldsymbol{p}_\perp - g_v\bar{\boldsymbol{V}}_\perp) + \beta(MU + g_s\phi))\psi_- \\ i\partial^+\psi_- &= (\boldsymbol{\alpha}_\perp\cdot(\boldsymbol{p}_\perp - g_v\bar{\boldsymbol{V}}_\perp) + \beta(MU + g_s\phi))\psi_+\end{aligned} \tag{2.30}$$

where

$$\partial^+ \bar{V}^\mu = \partial^+ V^\mu - \partial^\mu V^+ = V^{+\mu}. \tag{2.31}$$

Note that all of the previously obtained Fermionic sources of meson fields are unchanged by the transformation (2.28):

$$\begin{aligned} \bar{\psi}\psi &= \bar{\psi}'\psi' \\ \bar{\psi}U\psi &= \bar{\psi}'U\psi' \\ \bar{\psi}\gamma^\mu\psi &= \bar{\psi}'\gamma^\mu\psi'. \end{aligned} \tag{2.32}$$

The eigenmode expansion for $\bar{V}^\mu$ is needed to compute the interaction between nucleons. Eqs. (2.21) and (2.31) can be used to obtain

$$\bar{V}^\mu(x) = \int \frac{d^2k_\perp dk^+ \theta(k^+)}{(2\pi)^{3/2}\sqrt{2k^+}} \sum_{\omega=1,3} \bar{\epsilon}^\mu(\boldsymbol{k},\omega) \left[a(\boldsymbol{k},\omega)e^{-ik\cdot x} + a^\dagger(\boldsymbol{k},\omega)e^{ik\cdot x} \right], \tag{2.33}$$

where the polarization vectors $\bar{\epsilon}^\mu(\boldsymbol{k},\omega)$ are given by [14]:

$$\bar{\epsilon}^\mu(\boldsymbol{k},\omega) = \epsilon^\mu(\boldsymbol{k},\omega) - \frac{k^\mu}{k^+}\epsilon^+(\boldsymbol{k},\omega), \tag{2.34}$$

with the properties

$$\begin{aligned} k^\mu \bar{\epsilon}_\mu(\boldsymbol{k},\omega) = -\frac{m_v^2}{k^+}\epsilon^+(\boldsymbol{k},\omega), \quad \bar{\epsilon}_\mu(\boldsymbol{k},\omega)\bar{\epsilon}^\mu(\boldsymbol{k},\omega') &= -\delta_{\omega\omega'} + \frac{{m_v}^2}{k^+}\,\bar{\epsilon}^{\,+}(\boldsymbol{k},\omega)\bar{\epsilon}^{\,+}(\boldsymbol{k},\omega') \\ \sum_{\omega=1,3} \bar{\epsilon}^\mu(\boldsymbol{k},\omega)\bar{\epsilon}^\nu(\boldsymbol{k},\omega) &= -(g^{\mu\nu} - g^{+\mu}\frac{k^\nu}{k^+} - g^{+\nu}\frac{k^\mu}{k^+}). \end{aligned} \tag{2.35}$$

The path towards the light front Hamiltonian proceeds via the energy momentum tensor, which is given by

$$\begin{aligned} T^{\mu\nu} = -g^{\mu\nu}\mathcal{L} \; + \; & V^{\alpha\mu}V^{\nu\beta}g_{\beta\alpha} + m_v^2 V^\mu V^\nu + \frac{1}{2}\bar{\psi}'[\gamma^\mu(i\partial^\nu - g_v V^\nu) + \gamma^\nu(i\partial^\mu - g_v V^\mu)]\psi' \\ & + \partial^\mu\phi\partial^\nu\phi + \partial^\mu\boldsymbol{\pi}\cdot\partial^\nu\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\partial^\nu\boldsymbol{\pi}\frac{\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}}{\pi^2}(1 - \frac{f^2}{\pi^2}\sin^2\frac{\pi}{f}). \end{aligned} \tag{2.36}$$

The use of the Fermion field equation allows one to obtain the light front Hamiltonian density

$$T^{+-} = \nabla_\perp\phi\cdot\nabla_\perp\phi + m_\phi^2\phi^2 + \frac{1}{4}(V^{+-})^2 + \frac{1}{2}V^{kl}V^{kl} + m_v^2V^kV^k$$
$$+(\nabla_\perp\boldsymbol{\pi})^2 + \frac{\left(\frac{1}{2}\nabla_\perp\pi^2\right)^2}{\pi^2}\left(1 - \frac{f^2}{\pi^2}\sin^2\frac{\pi}{f}\right) + m_\pi^2 f^2\sin^2\frac{\pi}{f}$$
$$+2\psi_+^\dagger\left(i\frac{1}{2}\overleftrightarrow{\partial}^- - g_v\bar{V}^-\right)\psi_+. \tag{2.37}$$

It is now worthwhile to discuss a subtle feature regarding chiral symmetry in light front formalisms. Chiral invariance is defined as invariance under the transformation defined by Eq.(2.3) if the equal time formalism is used. Now the independent fermion variable is ψ_+ and ψ_- is a functional of this. Thus chiral invariance is the invariance under the transformation

$$\psi_+ \longrightarrow e^{i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{a}}\psi_+ \tag{2.38}$$

which is not the same as Eq.(2.3) because Eq.(2.38) produces a change in ψ_- that is different than using $\psi_- \longrightarrow e^{i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{a}}\psi_-$ [25,26]. The T^{+-} (or equivalently the light front Hamiltonian) of Eq.(2.37) is invariant under the transformation (2.38) if the pion mass is neglected so the usual chiral properties are obtained in these light front dynamics.

The expression (2.37) is useful for situations, such as in the mean field approximation case for infinite nuclear matter examined below, for which a simple expression for ψ_+ is known. This is not always the case, so it is worthwhile to use the Dirac equation to express T^{+-} in an alternate form:

$$T^{+-} = \nabla_\perp\phi\cdot\nabla_\perp\phi + m_\phi^2\phi^2 + \frac{1}{4}(V^{+-})^2 + \frac{1}{2}V^{kl}V^{kl} + m_v^2V^kV^k$$
$$+(\nabla_\perp\boldsymbol{\pi})^2 + \frac{\left(\frac{1}{2}\nabla_\perp\pi^2\right)^2}{\pi^2}\left(1 - \frac{f^2}{\pi^2}\sin^2\frac{\pi}{f}\right) + m_\pi^2 f^2\sin^2\frac{\pi}{f}$$
$$+\bar{\psi}\left(\boldsymbol{\gamma}_\perp\cdot(\boldsymbol{p}_\perp - g_v\bar{\boldsymbol{V}}_\perp) + (MU + g_s\phi)\right)\psi. \tag{2.39}$$

It is convenient to consider ψ_- as a sum of terms, one ξ_- whose relation with ψ_+ is free of interactions [12], the other η_- containing the interactions. That is, rewrite the second of Eq. (2.30) as [24]

$$\xi_- = \frac{1}{p^+}(\boldsymbol{\alpha}_\perp\cdot\boldsymbol{p}_\perp + \beta M)\psi_+$$
$$\eta_- = \frac{1}{p^+}(-\boldsymbol{\alpha}_\perp\cdot g_v\bar{\boldsymbol{V}}_\perp + \beta(M(U-1) + g_s\phi))\psi_+. \tag{2.40}$$

Furthermore, define $\xi_+(x) \equiv \psi_+(x)$, so that

$$\psi(x) = \xi(x) + \eta_-(x), \tag{2.41}$$

where $\xi(x) \equiv \xi_-(x) + \xi_+(x)$. The purpose of the above decomposition is to separate the dependent and independent parts of ψ and to allow one to expand ξ in terms of eigenstates of momentum.

One may make a similar treatment for the vector meson fields. The operator V^{+-}, determined by Eq (2.25), is relevant for the Hamiltonian. Part of this operator is determined by a constraint equation. To see this examine Eq (2.25), and make a definition:

$$V^{+-} = v^{+-} + \omega^{+-}, \tag{2.42}$$

where

$$\omega^{+-} = \frac{-2}{\partial^+} J^+. \tag{2.43}$$

Next use equations (2.41) and (2.42) to rewrite the Hamiltonian as a sum of a free and interacting terms. The sum of the last term of Eq (2.39) and the terms involving ω^{+-} is the density of the interaction Hamiltonian, P_I^-, plus the free fermion term, $P_0^-(N)$. Use equations (2.41) and (2.42) in the expression (2.39) for T^{+-} along with the field equations and integration by parts to find:

$$P_0^-(N) = \frac{1}{2} \int d^2x_\perp dx^- \bar{\xi} \left(\gamma_\perp \cdot \boldsymbol{p} + M\right) \xi, \tag{2.44}$$

and

$$P_I^- = v_1 + v_2 + v_3, \tag{2.45}$$

with

$$v_1 = \int d^2x_\perp dx^- \bar{\xi} \left(g_v \gamma \cdot \bar{V} + M(U-1) + g_s \phi\right) \xi, \tag{2.46}$$

$$v_2 = \int d^2x_\perp dx^- \bar{\xi} \left(-g_v \gamma \cdot \bar{V} + M(U-1) + g_s \phi\right) \frac{\gamma^+}{2p^+} \left(-g_v \gamma \cdot \bar{V} + M(U-1) + g_s \phi\right) \xi, \tag{2.47}$$

and

$$v_3 = \frac{g_v^2}{32} \int d^2x_\perp dx^- \int dy_1^- \bar{\xi}(\boldsymbol{x}_\perp, y_1^-)\gamma^+\xi(\boldsymbol{x}_\perp, y_1^-)\epsilon(x^- - y_1^-) \int dy_2^- \epsilon(x^- - y_2^-)\bar{\xi}(\boldsymbol{x}_\perp, y_2^-)\gamma^+\xi(\boldsymbol{x}_\perp, y_2^-). \tag{2.48}$$

The term v_1 accounts the emission or absorption of a single vector or scalar meson, as well as the emission or absorption of any number of pions through the operator $U - 1$. The term v_2 includes contact terms in which there is propagation of an instantaneous fermion. The term v_3 accounts for the propagation of an instantaneous vector meson.

We may now quantize the fermion fields using

$$\xi(x) = \int \frac{d^2k_\perp dk^+\theta(k^+)}{(2\pi)^{3/2}\sqrt{2k^+}} \sum_{\lambda=+,-} \left[u(\boldsymbol{k},\lambda)e^{-ik\cdot x}b(\boldsymbol{k},\lambda) + v(\boldsymbol{k},\lambda)e^{+ik\cdot x}d^\dagger(\boldsymbol{k},\lambda)\right], \tag{2.49}$$

where again the momenta are on shell: $k^- = \frac{\boldsymbol{k}_\perp^2+M^2}{k^+}$, and the anti-commutation relations are given by

$$\{b(\boldsymbol{k},\lambda), b^\dagger(\boldsymbol{k}',\lambda')\} = \{d(\boldsymbol{k},\lambda), d^\dagger(\boldsymbol{k}',\lambda')\} = \delta_{\lambda,\lambda'}\delta^{(2,+)}(\boldsymbol{k}-\boldsymbol{k}'),$$
$$\{b(\boldsymbol{k},\lambda), b(\boldsymbol{k}',\lambda')\} = \{d(\boldsymbol{k},\lambda), d(\boldsymbol{k},\lambda')\} = 0. \tag{2.50}$$

The properties of the Dirac spinors are described in Appendix A. The term $P_0^-(N)$ of Eq. (2.44) can now be expressed as

$$P_0^-(N) = \int d^2k_\perp dk^+\theta(k^+)\frac{k_\perp^2 + M^2}{k^+}\sum_\lambda (b^\dagger(\boldsymbol{k},\lambda)b(\boldsymbol{k},\lambda) + d^\dagger(\boldsymbol{k},\lambda)d(\boldsymbol{k},\lambda)). \tag{2.51}$$

The component that is related to the plus momentum is T^{++}. The necessary expression is given by

$$T^{++} = V^{ik}V^{ik} + m_v^2V^+V^+ + \bar{\psi}\gamma^+i\partial^+\psi$$
$$+\partial^+\phi\partial^+\phi + \partial^+\boldsymbol{\pi}\cdot\partial^+\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\partial^+\boldsymbol{\pi}\frac{\boldsymbol{\pi}\cdot\partial^+\boldsymbol{\pi}}{\pi^2}(1 - \frac{f^2}{\pi^2}\sin^2\frac{\pi}{f}). \tag{2.52}$$

III. CHIRAL SYMMETRY AND PION-NUCLEON SCATTERING

We begin by showing that, if one starts with a non-linear representation of chiral symmetry, the requirement of solving the constraint equation for the $-$ component of the fermion field leads one to a Lagrangian of the Gursey-type linear representation.

The focus is on chiral properties and pion-nucleon scattering, so we dispense with the vector and non-chiral ϕ meson fields for this section, and it is sufficient to examine only the following fermion-pion term of a non-linear representation [27]:

$$\mathcal{L}_{N\pi} = \bar{N}\left[\gamma_\mu i\partial^\mu - M + \frac{1}{1+(\pi/2f)^2}\left(\frac{1}{2f}\gamma^\mu\gamma_5\boldsymbol{\tau}\cdot\partial^\mu\boldsymbol{\pi} - (\frac{1}{2f})^2\gamma^\mu\boldsymbol{\tau}\cdot\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi}\right)\right]N. \quad (3.1)$$

Next obtain the fermion field equation and make the usual decomposition: $N_\pm \equiv \Lambda_\pm N$ with

$$\begin{aligned}\left(i\partial^- - O^-\right)N_+ &= [\boldsymbol{\alpha}_\perp\cdot(\boldsymbol{p}_\perp - \boldsymbol{O}_\perp) + \beta M]N_- \\ \left(i\partial^+ - O^+\right)N_- &= [\boldsymbol{\alpha}_\perp\cdot(\boldsymbol{p}_\perp - \boldsymbol{O}_\perp) + \beta M]N_+,\end{aligned} \quad (3.2)$$

where the operator O^μ has been defined as

$$O^\mu \equiv \frac{-1}{1+(\pi/2f)^2}\left(\frac{1}{2f}\gamma_5\boldsymbol{\tau}\cdot\partial^\mu\boldsymbol{\pi} - (\frac{1}{2f})^2\boldsymbol{\tau}\cdot\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi}\right). \quad (3.3)$$

We wish to remove the O^+ term from the left hand side of the equation for N_-. This can be done by defining a unitary operator F and fermion field χ such that

$$N = F\chi \quad (3.4)$$

with

$$i\partial^+ F = O^+ F. \quad (3.5)$$

The identity [18]

$$U_2^{\frac{1}{2}}\partial^\mu U_2^{-\frac{1}{2}} = iO^\mu, \quad (3.6)$$

where U_2 is given in Eq. (2.2), helps a good deal. Its use in Eq. (3.5), combined with the condition $\partial^\mu(U_2U_2^{-1}) = 0$, leads to the result

$$F = U_2^{\frac{1}{2}}, \tag{3.7}$$

so that using Eqs. (3.7) and (3.4) in (3.2) yields

$$\begin{aligned} i\partial^- \chi_+ &= [\boldsymbol{\alpha}_\perp \cdot \boldsymbol{p}_\perp + \beta M U_2]\, \chi_- \\ i\partial^+ \chi_- &= [\boldsymbol{\alpha}_\perp \cdot \boldsymbol{p}_\perp + \beta M U_2]\, \chi_+. \end{aligned} \tag{3.8}$$

This is of the desired form in which no interactions appear on the left-hand-side of the equation for χ_-. Thus the use of light front quantization mandates that the pion-nucleon interactions be of the form of Eq. (2.1).

The first test for any chiral formalism is to reproduce the early soft pion theorems [28]. Here we concentrate on low energy pion nucleon scattering because of its relation to the nucleon-nucleon force. We work to second order in $1/f$ in this first application. In this case, each of the U_i takes the same form:

$$U = 1 + i\gamma_5 \frac{\boldsymbol{\tau} \cdot \boldsymbol{\pi}}{f} - \frac{1}{2f^2}\pi^2. \tag{3.9}$$

This expression is to be used in the potentials v_1 and v_2 of Eqs. (2.46) and (2.47). The second order scattering graphs are of three types and are shown as time x^+ ordered perturbation theory diagrams in Fig. 1. The kinematics are such that $\pi(q)N(k) \to \pi(q')N(k')$, with $P_i = q + k$ and $P_f = q' + k'$. The iteration of v_1 to second order yields the direct and crossed graphs of Fig. 1a. In this formalism v_1 is proportional to the matrix element of γ_5 between u spinors, so it is proportional to the momentum of the absorbed or emitted pion. Thus the terms of Fig. 1a vanish near threshold. The terms of Fig. 1b are generated by the $\bar{u}\gamma_5 v$ terms of v_1. Using the various field expansions in the expression (2.46) for v_1 leads to the result that plus-momentum is conserved and the plus momentum of every particle is greater than zero. This means that the first of Fig.1b vanishes identically and the second vanishes for values of the initial pion plus momentum that are less than twice the nucleon mass. The net result is that only the instantaneous term of v_2 and the π^2 term of v_1 (shown in Fig. 1c) remain to be evaluated.

Proceeding more formally, we evaluate the S-matrix given by

$$S = T_+ e^{-\frac{i}{2}\int_{-\infty}^{\infty} dx^+ \hat{P}_I^-(x^+)} \tag{3.10}$$

where T_+ is the x^+ (light-front time) ordering operator and $\hat{P}_I^-$ is the interaction representation light front Hamiltonian. Then

$$(S-1)_{fi} = -2\pi i \delta(P_i^- - P_f^-) \langle f | T(P_i^-) | i \rangle, \tag{3.11}$$

with

$$T(P_i^-) = P_I^- + P_I^- \frac{1}{P_i^- - P_0^-} T(P_i^-) \tag{3.12}$$

The evaluation proceeds by using the field expansions in the expressions for v_1 and v_2. Integrating over $d^2x_\perp dx^+$ and evaluating the result between the relevant initial and final pion-nucleon states leads to the result that each contribution to the S-matrix is proportional to a common factor,

$$\frac{\delta^{(2,\perp)}(P_i - P_f)}{2(2\pi)^3\sqrt{k'^+k^+q'^+q^+}},$$

which combines with the result of the required integration over the light cone time (x^+) to provide the necessary momentum conservation and flux factors. The remaining factor of each term is its contribution to the invariant amplitude $\mathcal{M}$. The result is

$$\mathcal{M} = \tau_i\tau_f \frac{M^2}{f^2} \frac{\bar{u}(k')\gamma^+ u(k)}{2(k^+ + q^+)} + \tau_f\tau_i \frac{M^2}{f^2} \frac{\bar{u}(k')\gamma^+ u(k)}{2(k^+ - q^+)} - \delta_{if} \frac{M}{f^2} \bar{u}(k') u(k) \tag{3.13}$$

where the three terms here correspond to the three terms of Fig. 1c. The role of cancellations in the reduction of the term proportional to δ_{if} is already apparent. To understand the threshold physics take $k'^+ = k^+ = M$ and $q'^+ = q^+ = m_\pi$. Then one finds

$$\mathcal{M} = \delta_{if} \frac{2m_\pi^2}{f^2} + 2i\epsilon_{fin}\tau_n \frac{m_\pi M}{f^2} \tag{3.14}$$

to leading order in m_π/M. The weak nature of the δ_{if} term and the presence of the second Weinberg-Tomazowa term is the hallmark of chiral symmetry [28].

The same results could be obtained using the linear sigma model, with σ exchange playing the role of the π^2 term of Eq.(3.9).

IV. NUCLEON NUCLEON SCATTERING VIA ONE-BOSON EXCHANGE POTENTIALS

The ultimate aim is to derive the nuclear wave function including correlation effects. The first step is to understand nucleon-nucleon scattering using our light front formalism. We start with the one boson exchange approximation, discuss the light front wave equation and show that this procedure gives the same scattering amplitude as the usual procedure of computing the one-boson exchange contributions to the invariant amplitudes and using the Blankenbeckler-Sugar reduction of the Bethe-Salpeter equation [29,30]. This usual procedure is covariant, so that our construction shows that the light front wave procedure respects rotational invariance. This invariance is the result of Strikman and Frankfurt [10] and others. The present treatment explicitly includes the effects of nucleon spin and the nucleon-nucleon interaction can derived from an underlying chiral Lagrangian.

The starting point is the S matrix of Eqs. (3.10) and (3.11). Here the initial state i consists of nucleons with quantum numbers labeled by 1 and 2, and the state f consists of nucleons 3 and 4. To be definite, we take the plus momentum of nucleon 1 to be greater than that of nucleon 3, and the momentum transfer q to be

$$q \equiv k_1 - k_3, \tag{4.1}$$

so that $q^+ > 0$.

The lowest order contributions to the invariant amplitude are represented by the light-front-time ordered graphs shown in Fig. 2. The graphs of Fig. 2a represent terms of the form $v_1 \frac{1}{P_i^- - P_0^-} v_1$, and that of Fig. 2b accounts for the instantaneous massive vector boson exchange term of v_3. These terms may be evaluated by using the field expansions and doing the relevant integrals over the $d^2x_\perp dx^-$ coordinate space. Each term has a common factor of

$$\frac{4M^2 \delta^{(2,\perp)}(P_i - P_f)}{2\sqrt{k_1^+ k_2^+ k_3^+ k_4^+}},$$

where the factor $4M^2$ in the numerator is compensated by dividing the invariant amplitude by $4M^2$.

It is simplest to consider the effects of scalar ϕ and pseudoscalar $\boldsymbol{\pi}$ exchanges at the same time. The scattering amplitudes $\langle 3,4|\mathcal{K}(\phi,\boldsymbol{\pi})|1,2\rangle$ take the form

$$\langle 3,4|\mathcal{K}(\phi,\boldsymbol{\pi})|1,2\rangle = \frac{\bar{u}(4)\Gamma u(2)\,\bar{u}(3)\Gamma u(1)}{4M^2(2\pi)^3 k^+\left(k_1^- - k_3^- - k^-\right)}, \tag{4.2}$$

where the notation is that $u(i)$ is the spinor for a nucleon of quantum numbers i, and Γ is either of the form g_s or $i\ g\gamma_5$. The momentum of the exchanged meson is k, and it is necessary to realize that

$$k^+ = q^+,\ \boldsymbol{k}_\perp = \boldsymbol{q}_\perp \tag{4.3}$$

but

$$k^- = \frac{k_\perp^2 + \mu^2}{k^+} \neq q^-, \tag{4.4}$$

where μ is the mass of the exchanged scalar meson or pion. The factor $1/k^+$ arises from the denominators of the field expansions and $\left(k_1^- - k_3^- - k^-\right)$ is the result of evaluating the light front energy denominator $P_i^- - P_0^-$. Define the energy denominator of eq.(4.2) to be D so that

$$D = k^+\left(k_1^- - k_3^- - k^-\right) = (k_1^+ - k_3^+)(k_1^- - k_3^-) - k^+k^-. \tag{4.5}$$

Using Eqs. (4.3) and (4.4) immediately yields

$$D = q^2 - \mu^2, \tag{4.6}$$

so the amplitude $\mathcal{K}$ takes the form

$$\langle 3,4|\mathcal{K}(\phi,\pi)|1,2\rangle = \frac{\bar{u}(4)\Gamma u(2)\,\bar{u}(3)\Gamma u(1)}{4M^2(2\pi)^3\,(q^2 - \mu^2)}. \tag{4.7}$$

This is the usual [29–31] expression for a one-boson exchange potential, if no form factor effects are included. Note that the Klein-Gordon propagator is obtained using only a single time-ordered graph. The calculation with the equal-time formulation requires the summation of two time-ordered graphs.

The derivation of the contribution of vector meson exchange proceeds by adding the terms of Fig 2a and 2b. The term of Fig. 2a can immediately seen to be

$$\langle 3,4|\mathcal{K}_{2a}(V)|1,2\rangle = g_v^2 \frac{\bar{u}(4)\gamma_\mu u(2)\ \bar{u}(3)\gamma_\nu u(1)}{4M^2(2\pi)^3\,(q^2-m_v^2)}\left[-g^{\mu\nu}+g^{+\mu}\,\frac{k^\nu}{k^+}+g^{+\nu}\,\frac{k^\mu}{k^+}\right]. \tag{4.8}$$

The factor in brackets arises from the polarization sum, recall Eq. (2.35). It is worthwhile to define the contribution of the second two terms in the bracket, which result from the difference between $\bar{V}^\mu$ and V^μ fields, as $\langle 3,4|\mathcal{K}_{bar}|1,2\rangle$, with

$$\langle 3,4|\mathcal{K}_{bar}|1,2\rangle = g_v^2 \frac{\bar{u}(4)\gamma^+ u(2)\ \bar{u}(3)\gamma\cdot k u(1) + \bar{u}(4)\gamma\cdot k u(2)\ \bar{u}(3)\gamma^+ u(1)}{4M^2(2\pi)^3 q^+\,(q^2-m_v^2)}. \tag{4.9}$$

Next use the relations $\bar{u}(3)\gamma\cdot q\ u(1) = \bar{u}(4)\gamma\cdot q\ u(2) = 0$ and the equality of the $+$ and $\perp$ components of k with those of q to obtain the results

$$\begin{aligned}\bar{u}(3)\gamma\cdot k\ u(1) &= \frac{1}{2}\bar{u}(3)\gamma^+ u(1)(k^- - q^-),\\ \bar{u}(4)\gamma\cdot k u(2) &= \frac{1}{2}\bar{u}(4)\gamma^+ u(2)(k^- - q^-).\end{aligned} \tag{4.10}$$

But $k^- - q^- = \frac{q_\perp^2+m_v^2}{q^+} - q^- = -\frac{q^2-m_v^2}{k^+}$, which leads to a compact rewriting of Eq. (4.9) as

$$\langle 3,4|\mathcal{K}_{bar}|1,2\rangle = -g_v^2 \frac{\bar{u}(4)\gamma^+ u(2)\ \bar{u}(3)\gamma^+ u(1)}{4M^2(2\pi)^3(k^+)^2} \tag{4.11}$$

The term of Fig. 2b is obtained by using the field expansion in the equation for v_3, (2.48), integrating over coordinate space and removing the common factor. The result is

$$\langle 3,4|\mathcal{K}_{2b}(V)|1,2\rangle = g_v^2 \frac{\bar{u}(4)\gamma^+ u(2)\ \bar{u}(3)\gamma^+ u(1)}{4M^2(2\pi)^3(k^+)^2}, \tag{4.12}$$

which exactly cancels the term $\langle 3,4|\mathcal{K}_{bar}|1,2\rangle$. The net result is that the amplitude for vector meson exchange, $\langle 3,4|\mathcal{K}(V)|1,2\rangle = \langle 3,4|\mathcal{K}_{2a}(V)+\mathcal{K}_{2b}|1,2\rangle$, takes the familiar form:

$$\langle 3,4|\mathcal{K}(V)|1,2\rangle = -g_v^2 \frac{\bar{u}(4)\gamma_\mu u(2)\bar{u}(3)\gamma^\mu u(1)}{4M^2(2\pi)^3\,(q^2-m_v^2)}. \tag{4.13}$$

The sum of the amplitudes arising from each of the individual one boson exchange terms:

$$\langle 3,4|\mathcal{K}|1,2\rangle = \langle 3,4|\mathcal{K}(\phi)+\mathcal{K}(\boldsymbol{\pi})+\mathcal{K}(V)|1,2\rangle, \tag{4.14}$$

gives the invariant amplitude to second order in each of the coupling constants.

These amplitudes are strong, so computing the nucleon-nucleon scattering amplitude and phase shifts requires including higher order terms. One may include a sum which gives unitarity by including all iterations of the scattering operator $\mathcal{K}$ through intermediate two-nucleon states:

$$\mathcal{M} = \mathcal{K} + \mathcal{K}\frac{P_{2N}}{P_i^- - P_0^-}\mathcal{M}, \tag{4.15}$$

where P_i^- is the negative-momentum in the initial state and P_{2N} projects on to two-nucleon intermediate states. More explicitly, Eq. (4.15) is given by

$$\langle 3,4|\mathcal{M}|1,2\rangle = \langle 3,4|\mathcal{K}|1,2\rangle + \sum_{\lambda_5,\lambda_6}\int \langle 3,4|\mathcal{K}|5,6\rangle \frac{2M^2}{p_5^+ p_6^+}\frac{d^2p_{5\perp}dp_5^+}{P_i^- - (p_5^- + p_6^-) + i\epsilon}\langle 5,6|\mathcal{M}|1,2\rangle. \tag{4.16}$$

after removing the common factor and accounting for the momentum conserving delta functions. One realizes that this is of the form of the Weinberg equation [32] by expressing the plus-momentum variable in terms of a light-front momentum fraction α such that

$$p_5^+ = \alpha P_i^+, \tag{4.17}$$

and using the relative and total momentum variables:

$$\begin{aligned} \boldsymbol{p}_\perp &\equiv (1-\alpha)\boldsymbol{p}_{5\perp} - \alpha \boldsymbol{p}_{6\perp} \\ \boldsymbol{P}_{i\perp} &= \boldsymbol{p}_{5\perp} + \boldsymbol{p}_{6\perp} \end{aligned} \tag{4.18}$$

Then

$$\langle 3,4|\mathcal{M}|1,2\rangle = \langle 3,4|\mathcal{K}|1,2\rangle + \int \sum_{\lambda_5,\lambda_6} \langle 3,4|\mathcal{K}|5,6\rangle \frac{2M^2}{\alpha(1-\alpha)}\frac{d^2p_\perp d\alpha}{P_i^2 - \frac{p_\perp^2+M^2}{\alpha(1-\alpha)} + i\epsilon}\langle 5,6|\mathcal{M}|1,2\rangle, \tag{4.19}$$

where P_i^2 is square of the total initial four-momentum, otherwise known as the invariant energy s and $\frac{p_\perp^2+M^2}{\alpha(1-\alpha)}$ is the corresponding quantity for the intermediate state. Because the kernal $\mathcal{K}$ is itself an invariant amplitude the procedure of solving this equation to determine observables is manifestly covariant.

Equation (4.19) can in turn can be re-expressed as the Blankenbecler-Sugar (BbS) equation [33] by using the variable transformation [34]:

$$\alpha = \frac{E(p) + p^3}{2E(p)}, \tag{4.20}$$

with $E(p) \equiv \sqrt{\boldsymbol{p} \cdot \boldsymbol{p} + M^2}$. The result is:

$$\langle 3,4|\mathcal{M}|1,2\rangle = \langle 3,4|\mathcal{K}|1,2\rangle + \int \sum_{\lambda_5,\lambda_6} \langle 3,4|\mathcal{K}|5,6\rangle \frac{M}{E(p)} \frac{d^3p}{\frac{p_i^2 - p^2}{M} + i\epsilon} \langle 5,6|\mathcal{M}|1,2\rangle, \tag{4.21}$$

which is the desired equation. The three-dimensional propagator is exactly that of the BbS equation; there is one difference. Our one boson exchange potentials depend on the square of the four momentum q^2 transferred when a meson is absorbed or emitted by a nucleon. Thus the energy difference between the initial and final on-shell nucleons is included and $q^0 \neq 0$. The derivation of the BbS equation from the Bethe-Salpeter equation specifies that $q^0 = 0$ is used in the meson propagator. Including $q \neq 0$ instead of $q^0 = 0$ increases the range of the potential. Such an effect can be hidden in phenomenological potentials by changing the pion-nucleon coupling constant or form factor.

One can easily convert Eq. (4.21) into the Lippman-Schwinger equation of non-relativistic scattering theory by removing the factor $M/E(p)$ with a simple transformation [35].

A. Comparison with Realistic One-Boson Exchange Potentials

The present results are that one can use the light front technique to derive nucleon-nucleon potentials in the one-boson exchange OBE approximation and use these in an appropriate wave equation. Therefore our procedure is directly comparable to the one used in constructing the realistic Bonn one-boson exchange potentials used in momentum space. Those potentials also have a close connection with an underlying Lagrangian. Our purpose here is to argue that the present procedure can yield potentials essentially identical to the Bonn OBEP potentials and therefore would lead to a good description of the NN data.

The Bonn one-boson exchange potentials employ six different mesons $\pi, \eta, \omega, \rho, \sigma$ and the (isovector scalar) δ meson. The present techniques can be used to handle all of these mesons and their couplings, with the possible exception of the tensor $\sigma_{\mu\nu} q_\nu$ part of the ρ-nucleon interaction.

The presence of such a tensor interaction makes it difficult (or impossible) to write the equation for ψ_- as $\psi_- = 1/p^+ \cdots \psi_+$. This is relevant because the standard value of the ratio of the tensor to vector ρ-nucleon coupling f_ρ/g_ρ is 6.1, based upon Ref. [36] and subsequent papers. Reproducing the observed values of ε_1 and P-wave wave phase shifts requires a large value f_ρ/g_ρ; see Ref. [37]. However the Lagrangian compensates for its lack of a ρ-N interaction with tensor coupling by generating such a term via vertex correction diagrams (which are the origin of the anomalous magnetic moment of the electron in QED). Such diagrams probably do not generate the phenomenologically required values of the coupling constants, but all that is needed here is that terms of the correct form be produced. This is because the standard procedure is to choose the values of the coupling constants so as to yield a good description of the NN scattering data. Indeed the potentials A,B, and C are defined by the parameters which account for the mesonic masses, coupling constants and form factors. Thus we end up with the same procedure that is used in the Bonn one boson exchange potentials.

This brings us to the treatment of divergent terms in our procedure. The definition of any effective Lagrangian requires the specification of such a procedure. For the present, it is sufficient to say that we introduce form factors, $F_\alpha(q^2)$ which reduce the strength of the α meson-nucleon coupling for large values of $-q^2$. This is also the procedure of Ref [29,30].

The net result is that the one-boson exchange treatment of the nucleon-nucleon potential and the T-matrix resulting from its use in the BbS equation is essentially the same as the one-boson exchange procedure of Ref. [29,30]. Thus our light front treatment is guaranteed to be consistent with nucleon-nucleon nucleon scattering data measured in the standard energy range. Such a similarity has also been obtained using relativistic Hamiltonian dynamics [9].

B. Nucleonic Contribution to the Two Pion Exchange Potential

The dominant contribution to the two pion exchange potential arises from contributions to intermediate states that include one or two Δ's [29], and a treatment of such effects based on chiral symmetry has been provided by van Kolck and collaborators [38].

Including the effects of Δ's is beyond the scope of the present work, but we are able to discuss the two pion exchange contribution (of order $(M/f)^4$) to the nucleon nucleon potential. The property that a sum of light cone time-ordered diagrams is equal a single Feynman graph can be used to simplify the calculation. The relevant Feynman graphs are displayed in Fig. 3; the terms originating from the linear $\gamma_5\tau \cdot \pi$ coupling (a,b), from the quadratic $\pi^2 - N$ coupling (c) and from a combination of the linear and quadratic interactions (d) are indicated. The line through the two-nucleon intermediate state of Fig. 3a is meant to indicate that the contribution arising from iterating the one pion exchange interaction is removed. This has been a standard procedure since the work of Ref. [35], and will not be discussed further.

The sum of the terms of Fig. 3a and 3b is equal to the Partovi-Lomon two pion exchange potential, as they used the pseudoscalar pion-nucleon interaction. This interaction certainly simplifies the calculation; in particular the diagrams of Fig. 3a, b and d are convergent (whereas they would be strongly divergent if pseudovector coupling were to be used. One can use such a pseudoscalar coupling, and include the effects of chiral symmetry, provided one also includes the effects of the $\pi^2 - N$ coupling shown in Fig. 3c, and the combined effects of the linear and quadratic interactions, Fig. 3d. The quadratic interaction term cancels the large pair terms in pion-nucleon scattering and should also play a significant role here in reducing the size of the computed potential. Thus we expect that the Partovi-Lomon potential contains too large an attraction.

Next turn to the procedure used in constructing the full Bonn potential. This potential is constructed by ignoring all of the Z-graphs and including the the effects of the two-nucleon intermediate states which arise from the crossed graph, Fig. 3b, as well as the parts of

Fig. 3a arising from time ordered terms in which two pions exist at the same time. (For such contributions to the TPEP the linear pseudoscalar and pseudovector interactions are are evaluated between on shell positive energy nucleon spinors, and are therefore equivalent. The resulting contribution to the TPEP is small, but is comparable to that of the iterated OPEP. The neglect of the Z graphs goes a long way towards including the effects of chiral symmetry. However, terms involving the Weinberg-Tomazowa interaction at one or two vertices are ignored. The computation of the graphs of Fig. 3 would include such effects implicitly as well as that of pair suppression. Thus a detailed comparison would be useful. However, the small nature of the effects that we discuss now indicate that the dominance of the TPEP by effects of intermediate Δ's will remain unchallenged.

V. MEAN FIELD APPROXIMATION

The nucleon-nucleon interaction of the previous section can be used as the basis for a light front Brueckner theory of nuclei. We study the mean field approximation for infinite nuclear matter as a first step. The nuclear mean field model- the shell model- occupies pre-eminence in understanding nuclear structure. We need to see if our formalism can describe this physics.

In the mean field approximation [21], the coupling constants are considered strong and the Fermion density large. Then the meson fields can be approximated as classical- the sources of the meson fields are replaced by their expectation values. In this case, the nucleon mode functions will be plane waves and the nuclear matter ground state can be assumed to be a normal Fermi gas, of Fermi momentum k_F, and of large volume Ω in its rest frame. We consider the case that there is an equal number of protons and neutrons.

First we examine the mesonic field equations (2.5)-(2.7). The baryon source of the pion field is a pseudoscalar operator, so its expectation value vanishes in the ground state. Thus this mean field approximation leads to the result that $\pi_i \to 0$. The other meson fields are constants, independent of space and time, given by

$$\phi = -\frac{g_s}{m_s^2}\langle\bar{\psi}\psi\rangle \tag{5.1}$$

$$V^\mu = \frac{g_v}{m_v^2}\langle\bar{\psi}\gamma^\mu\psi\rangle = \delta^{0,\mu}\frac{g_v\rho_B}{m_v^2}, \tag{5.2}$$

where the brackets denote expectation values of the nuclear ground state in its rest frame and the baryon density is

$$\rho_B = 2k_F^3/3\pi^2. \tag{5.3}$$

This result that V^μ is a constant, along with Eqs. (2.31) and (5.2), can be used to determine $\bar{V}^\mu$. In particular, $\bar{V}^+ = 0$ by construction. Furthermore, the conditions that $V^i = 0$ and $\partial^i V^+ = \partial^i V^0 = 0$ tell us that $\bar{V}^i = 0$. Finally $\partial^- V^+ = 0$, so that $\partial^+\bar{V}^- = \partial^+ V^0$, so the net result is that the only non-vanishing component of $\bar{V}^\mu$ is $\bar{V}^- = V^0$.

With this mean field approximation, the fermionic field equations (2.30) can be rewritten as

$$\begin{aligned}(i\partial^- - g_v\bar{V}^-)\psi_+ &= (\boldsymbol{\alpha}_\perp\cdot\boldsymbol{p}_\perp + \beta(MU + g_s\phi))\psi_- \\ i\partial^+\psi_- &= (\boldsymbol{\alpha}_\perp\cdot\boldsymbol{p}_\perp + \beta(MU + g_s\phi))\psi_+.\end{aligned} \tag{5.4}$$

Now ϕ and $\bar{V}^-$ are constants so we expect the mode functions for the field expansion of ψ to be of the plane wave form $\sim e^{ik\cdot x}$ and can be obtained from Eq. (5.4) as [39]

$$(i\partial^- - g_v\bar{V}^-)\psi_+ = \frac{\boldsymbol{k}_\perp^2 + (M + g_s\phi)^2}{k^+}\psi_+. \tag{5.5}$$

The light front eigenenergy ($i\partial^- \equiv k^-$) is the sum of a kinetic energy term in which the mass is shifted by the presence of the scalar field, and an energy arising from the vector field. Comparing this equation with the one for free nucleons, $k^- = \frac{k_\perp^2+M^2}{k^+}$, shows that the nucleons have a mass $M + g_s\phi$ and move in plane wave states. The nucleon field operator is constructed using the solutions of Eq. (5.5) as the plane wave basis states. This means that the nuclear matter ground state, defined by operators that create and destroy baryons in eigenstates of Eq. (5.5), is the correct wave function and that Equations (5.2) and (5.5) represent the solution of the approximate field equations, and the diagonalization of the Hamiltonian.

One question remains. We are going to fill up a Fermi sea, but k_F is the magnitude of a three vector. How is this three vector defined? This was answered in the paper of Glazek and Shakin [15] who showed that rotational invariance is manifest if one uses the definition:

$$k^+ = \sqrt{(M + g_s\phi)^2 + k \cdot k} + k^3, \tag{5.6}$$

which implicitly defines k^3. Using Eq. (5.6) allows one to maintain the equivalence between energies computed in the light front and equal time formulations of scalar field theories [40]. A similar equation has been used to restore manifest rotational invariance in light-front QED [41]. We shall show that this same expression also restores rotational invariance in this mean field problem when vector mesons are included.

Equation (5.6) has the correct form in the limit of non-interacting nucleons and therefore seems natural [42]. We attempt a heuristic derivation of this equation using the requirement that manifest rotational invariance be restored. The starting point is the observation that Eq. (4.20), with its definition of α as the plus momentum fraction carried by a nucleon, restores manifest rotational invariance in the two-nucleon system. Let's consider the mean field approximation as involving an interaction between a nucleon and a very heavy particle containing A-1 nucleons (with $A \to \infty$). Then the variable α_A, which is the fraction of the nuclear plus-momentum carried by a nucleon, is given by

$$\alpha_A = \frac{\sqrt{\boldsymbol{k} \cdot \boldsymbol{k} + (M + g_s\phi)^2} + k^3}{\sqrt{\boldsymbol{k} \cdot \boldsymbol{k} + M_{A-1}^2} + \sqrt{\boldsymbol{k} \cdot \boldsymbol{k} + (M + g_s\phi)^2}}, \tag{5.7}$$

and is a suitable generalization of the variable α. The nucleon mass is taken to be $M + g_s\phi$, because it is this mass that appears in the nucleon field equations. The mass of the $A-1$ body system is dominated by the mass of the (A-1) nucleons mass (the binding energy per particle is 16 MeV ($\equiv \epsilon_B$) compared with 940 MeV), so that we may write

$$\begin{aligned}\alpha_A &= \frac{\sqrt{\boldsymbol{k} \cdot \boldsymbol{k} + (M + g_s\phi)^2} + k^3}{M_A}\left(1 + \epsilon_B/M_A + k_F^2/2(M + g_s\phi)M_A\right) \\ &= \frac{\sqrt{\boldsymbol{k} \cdot \boldsymbol{k} + (M + g_s\phi)^2} + k^3}{M_A},\end{aligned} \tag{5.8}$$

in which the last line results from the limit $A \to \infty$. The key feature is that the variable α_A is defined as a momentum fraction, so that

$$\alpha_A M_A = k^+. \tag{5.9}$$

Comparing Eq. (5.8) and Eq. (5.9) leads to Eq. (5.6).

The computation of the energy and plus momentum distribution proceeds from taking the appropriate expectation values of the energy momentum tensor $T^{\mu\nu}$ discussed in Sect. 2 and

$$P^\mu = \frac{1}{2}\int d^2x_\perp dx^- \langle T^{+\mu}\rangle. \tag{5.10}$$

We are concerned with the light front energy P^- and momentum P^+. The relevant components of $T^{\mu\nu}$ are presented in Eqs. (2.37) and (2.52). Within the mean field approximation(MFA), the derivatives of the meson fields are zero so that one finds

$$\begin{aligned} T^{+-}_{MFA} &= m_s^2\phi^2 + 2\psi_+^\dagger(i\partial^- - g_v\bar{V}^-)\psi_+ \\ T^{++}_{MFA} &= m_v^2 V_0^2 + 2\psi_+^\dagger i\partial^+\psi_+. \end{aligned} \tag{5.11}$$

Taking the nuclear matter expectation value of T^{+-} and T^{++} and performing the spatial integral of Eq. (5.10) leads to the result

$$\frac{P^-}{\Omega} = m_s^2\phi^2 + \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+ \frac{\boldsymbol{k}_\perp^2 + (M+g_s\phi)^2}{k^+} \tag{5.12}$$

$$\frac{P^+}{\Omega} = m_v^2 V_0^2 + \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+ k^+. \tag{5.13}$$

The subscript F denotes that $|\vec{k}| < k_F$ with k^3 defined by the relation (5.6).

Equations (5.12) and (5.13) along with the expression for k^+, (5.6) allow an evaluation of P^- and P^+. This shall be done in two different ways. In the first method we evaluate the energy of the A-nucleon system $E_A = \frac{1}{2}(P^+ + P^-)$ [15], which turns out to be the same as in the usual equal-time treatment [21]. This can be seen by summing equations (5.12) and (5.13) to obtain

$$\frac{E_A}{\Omega} = \frac{1}{2}m_s^2\phi^2 + \frac{1}{2}m_v^2V_0^2 + \frac{4}{(2\pi)^3}\frac{1}{2}\int_F d^2k_\perp dk^+ \left(\frac{\boldsymbol{k}_\perp^2 + (M+g_s\phi)^2}{k^+} + k^+\right) \tag{5.14}$$

Then replace the integration over k^+ by one over k^3, using Eq. (5.6) so that

$$dk^+ \to \frac{k^+}{\sqrt{(M+g_s\phi)^2 + \boldsymbol{k}\cdot\boldsymbol{k}}}dk^3 = \frac{k^+}{E(k)}dk^3. \tag{5.15}$$

where

$$E(k) \equiv \sqrt{\boldsymbol{k}\cdot\boldsymbol{k} + (M+g_s\phi)^2}. \tag{5.16}$$

Then Eq. (5.14) takes the form:

$$\frac{E_A}{\Omega} = \frac{1}{2}m_s^2\phi^2 + \frac{1}{2}m_v^2V_0^2 + \frac{4}{(2\pi)^3}\int_F d^3k\theta(k_F-k)\ E(k),. \tag{5.17}$$

which is the expression familiar from the Walecka model, this confluence of energies is a nice check on the present result because a manifestly covariant solution of the present problem, with the usual energy, has been obtained [43].

We consider the system to be at a fixed large volume, Ω, so that E_A/A depends on ϕ and k_F. The ground state energy is determined by minimizing E_A/A with respect to those two parameters. Setting $\frac{\partial E_A}{\partial\phi}$ [44] to zero reproduces the field equation for ϕ (5.1) as is also the case in the equal-time formalism. The next step is to minimize the energy per particle $E_A/A = E_A/(\rho_B\Omega)$ at fixed volume with respect to k_F. (One may also minimize the energy with respect to the volume [15].) Start this calculation by using

$$\frac{\partial}{\partial k_F}\left(\frac{E_A}{\rho_B}\right) = 0 \tag{5.18}$$

to obtain:

$$\frac{\partial E}{\partial k_F} = 3\frac{E}{k_F}. \tag{5.19}$$

Using Eq. (5.17) followed by Eqs. (5.1) and (5.2) leads to the result

$$\frac{4}{(2\pi)^3}\frac{4\pi}{3}k_f^3E_F = \frac{m_s^2}{2}\phi^2 - \frac{m_v^2}{2}V_0^2 + \frac{4}{(2\pi)^3}\int_F d^3k\theta(k_F-k)E(k), \tag{5.20}$$

where $E_F \equiv E(k_F)$. This is a transcendental equation which determines k_F, so that the calculation of E_A is complete.

It is useful to note that the relation $P^+ = P^-$ (which must hold for a system in its rest frame) also emerges as a result of this minimization. To see this rewrite the left hand side of Eq. (5.20) as

$$\frac{4}{(2\pi)^3}\frac{4\pi}{3}k_f^3 E_F = \frac{4}{(2\pi)^3}\int d^3k\theta(k_F - k)\left(E(k) + \frac{\boldsymbol{k}\cdot\boldsymbol{k}}{3E(k)}\right). \tag{5.21}$$

Using this in Eq. (5.20) leads to

$$\frac{m_s^2}{2}\phi^2 - \frac{m_v^2}{2}V_0^2 = \frac{4}{(2\pi)^3}\int d^3k\theta(k_F - k)\frac{\boldsymbol{k}\cdot\boldsymbol{k}}{3E(k)}, \tag{5.22}$$

which is just the relation that one obtains by setting $P^+ = P^-$ with the versions of Eqs. (5.12) and (5.13) obtained by replacing the variable k^+ by k^3.

Another way to obtain the energy of the ground state is to minimize the value of P^-/A subject to the constraint that $P^- = P^+$, or to minimize the quantity $\mathcal{E}$ with

$$\mathcal{E} \equiv \frac{P^-}{A} - \lambda\left(\frac{P^-}{A} - \frac{P^+}{A}\right), \tag{5.23}$$

where λ is a Lagrange multiplier. Setting $\frac{\partial\mathcal{E}}{\partial\phi}$ leads to [44]

$$\frac{\partial P^-}{\partial\phi}(1-\lambda) + \lambda\frac{\partial P^+}{\partial\phi} = 0 \tag{5.24}$$

But the field equation (5.1) for ϕ can be restated as

$$\frac{\partial P^-}{\partial\phi} = -\frac{\partial P^+}{\partial\phi}. \tag{5.25}$$

Combining Eqs.(5.24) and (5.25) leads to the result that

$$\lambda = \frac{1}{2}, \tag{5.26}$$

so that the minimization of $\mathcal{E}$ with respect to k_F is the same as minimizing E_A/A with respect to k_F. This ends the discussion of how the expressions for $P^\pm$ are used to determine the energy of the system.

Solving the field equations and minimizing the energy density determines the properties of nuclear matter, once the meson-nucleon coupling constants and masses are chosen. One can now discuss the properties of the resulting system. Of course the necessary calculations have been done long ago. In particular, the parameters

$$g_v^2 M^2/m_v^2 = 195.9 \tag{5.27}$$

and

$$g_s^2 M^2/m_s^2 = 267.1 \tag{5.28}$$

have been chosen [45] so as to give the binding energy per particle of nuclear matter as 15.75 MeV with k_F=1.42 Fm^{-1}. In this case, solving the equation for ϕ gives $M + g_s\phi = 0.56\ M$.

A. Nucleon and Meson Plus Momentum and Deep Inelastic Scattering

The light front formalism embodies the use of k^+ as a canonical variable which allows us to study the nucleonic and mesonic contributions to the nuclear plus momentum. The study of the plus momentum content is motivated by the desire to obtain a better understanding of lepton-nucleus deep inelastic scattering. The EMC effect [46] that the structure function of a bound nucleon differs from that of a free one, showed a principal effect that the plus momentum carried by the valence quarks is less for a bound nucleon than for a free one. Many different interesting interpretations and related experiments [47] were stimulated by these experiments. But a correct interpretation requires that the role of conventional effects, such as nuclear binding, be assessed and understood.

Our formalism employs plus component of the momentum so that it's use in assessing the nucleon's (and therefore the valence quark's) plus-momentum is necessary. We therefore examine the ++ component of the energy momentum tensor Eq. (5.13) to determine how much momentum is carried by nucleons and how much by mesons. Rewrite Eq. (5.13) as a sum of mesonic m and nucleonic N terms

$$\frac{P^+}{A} = \frac{P^+_m}{A} + \frac{P^+_N}{A}, \tag{5.29}$$

with

$$\frac{P^+_m}{A} = \frac{m_v^2 V_0^2}{\rho_B} \tag{5.30}$$

and

$$\frac{P^+_N}{A} = \frac{4}{\rho_B (2\pi)^3} \int_F d^2 k_\perp dk^+ k^+. \tag{5.31}$$

The parameters of Eqs. (5.27) and (5.28) leads to

$$\frac{P^+}{A} = M - 15.75 MeV, \tag{5.32}$$

and the use of Eq. (5.27) in Eq. (5.30) gives:

$$\frac{P^+_m}{A} = 329 MeV, \tag{5.33}$$

while performing the integral involved in Eq. (5.31) leads to

$$\frac{P^+_N}{A} = 594 MeV. \tag{5.34}$$

The result is that 65% of the nuclear plus momentum must be carried by the nucleons and the remainder of 35% is carried by the mesons.

How do these numbers relate to experiments? To answer we need to recall that the nuclear structure function F_{2A} can be obtained from the light front distribution function $f(y)$ (which gives the probability for a nucleon to have a plus momentum fraction y) and the nucleon structure function F_{2N} using the relation [48]:

$$\frac{F_{2A}(x)}{A} = \int dy f(y) F_{2N}(x/y), \tag{5.35}$$

where y is the A times the fraction of the nuclear plus-momentum carried by the nucleon, and x is the Bjorken variable computed using the nuclear mass divided by A ($\bar{M}$): $x = Q^2/2\bar{M}\nu$. This formula is the expression of the convolution model in which one means to assess, via

$f(y)$, only the influence of nuclear binding. Other effects such as the nuclear modification of the nucleon structure function (if F_{2N} is obtained from deep inelastic scattering on the free nucleon) and any influence of the final state interaction between the debris of the struck nucleon and the residual nucleus [49] are neglected.

Our formalism enables us to calculate the function $f(y)$ from the integrand of Eq.(5.13). Since the integral gives the total plus-momentum carried by nucleons, the integrand which multiplies the factor k^+ can be interpreted as the necessary probability distribution. Thus:

$$P_N^+/A = \int dk^+ \; k^+ f(k^+), \tag{5.36}$$

$$f(k^+) = \int_F d^2k_\perp. \tag{5.37}$$

The function $f(y)$ can be obtained by replacing k^+ by the dimensionless variable y using $y \equiv \frac{k^+}{\bar{M}}$ with $\bar{M} \equiv M - 15.75$ MeV. Then using Eq.(5.37) leads to the result

$$f(y) = \frac{3}{4}\frac{\bar{M}^3}{k_F^3}\theta(y^+ - y)\theta(y - y^-)\left[\frac{k_F^2}{\bar{M}^2} - (\frac{E_F}{\bar{M}} - y)^2\right], \tag{5.38}$$

where $y^\pm \equiv \frac{E_F \pm k_F}{\bar{M}}$ and $E_F \equiv \sqrt{k_F^2 + (M + g_s\phi)^2}$. Knowing the following integrals is useful:

$$\int dy f(y) = 1 \tag{5.39}$$

$$\int dy \; y \; f(y) = 0.65, \tag{5.40}$$

with the 0.65 representing the earlier 65% result.

We may now assess the implications of the statement that nucleons carry only 65% of the nuclear plus-momentum. This number is to be compared with the value obtained by Frankfurt and Strikman [11]. They used data for F_{2A} and F_{2N} along with Eq. (5.35) to determine the average value of y required by experiments with the result that

$$\int dy \; y \; f_{exp}(y) = 0.95. \tag{5.41}$$

This means that nucleons carrying 95% of the nuclear plus momentum (a 5% depletion effect) is sufficient to explain the 10-15% depletion effect observed for the Fe nucleus. Our

35% depletion seems to be rather large, but one must remember that it results from nuclear matter. A result that compares more closely with experiment could be obtained in a version of the present model for which the value of $M + g_s\phi$ is closer to M. However, determining specific features of the present model is not the goal of the present work. Instead we wish to demonstrate that the light front formalism can be used to obtain a nuclear wave function expressed in terms of the plus-momentum variable which is closely related to experiment.

Indeed we can verify that the ability to obtain the nucleonic plus momentum, feature that requires the use of a light front front formalism, instead of an equal time formalism. To do this compare the 0.65 fraction with the result of a relativistic calculation using the equal time (et) formalism [50]. In this calculation, which uses Eq. (2.1)) and for which the scalar and vector fields are the same as here, the plus momentum of a nucleon was chosen as the sum of the Dirac eigenenergy and k^3:

$$k^+_{et} \equiv E_{Dirac} + k^3 = \sqrt{(M + g_s\phi)^2 + \boldsymbol{k}^2} + g_v V^0 + k^3. \tag{5.42}$$

Using this leads to an average nucleon plus momentum fraction $\langle y \rangle_{et} = (E_F + g_v V^0)/\bar{M}$, which when evaluated with our parameters for k_F, ϕ and $\bar{V}^-$, leads to $\langle y \rangle_{et} = 1.00$! The big difference between our result and the earlier equal time result -compare Eqs. (5.6) and (5.42)- arises from our use of the plus momentum as a canonical momentum variable and the consequent use of $T^{+\mu}$ to construct the light front momentum and energy density. In particular, the first line of Eq.(5.42) is only a reasonable guess.

We note also that the baryon number distribution $f_B(y)$ (number of baryons per y, normalized to unity) can be determined from the expectation value of $\psi^\dagger\psi$. The result is

$$f_B(y) = \frac{3}{8}\frac{\bar{M}^3}{k_F^3}\theta(y^+ - y)\theta(y - y^-)\left[(1 + \frac{E_F^2}{\bar{M}^2 y^2})(\frac{k_F^2}{\bar{M}^2} - (\frac{E_F}{\bar{M}} - y)^2) - \frac{1}{2y^2}(\frac{k_F^4}{\bar{M}^4} - (\frac{E_F}{\bar{M}} - y)^4)\right]. \tag{5.43}$$

Some phenomenological models treat the two distributions $f(y)$ and $f_B(y)$ as identical. The distributions have the same normalization, but they are different as shown by Eqs. (5.38) and (5.43).

Consider that the average value of y equal to 0.65 represents a very strong binding effect on lepton-nucleus deep inelastic scattering. One might think that the mesons, which cause this binding, would also have huge effects on deep inelastic scattering. It is therefore certainly necessary to determine the momentum distributions of the mesons. The mesons contribute 0.35 of the total nuclear plus momentum, but we need to know how this is distributed over different individual values. The paramount feature is that ϕ and V^μ are the same constants for any and all values of the spatial coordinates $x^-, \boldsymbol{x}_\perp$ (and also x^+). This means that the related momentum distribution can only be proportional to a delta function setting both the plus and $\perp$ components of the momentum to zero. This result is attributed to the mean field approximation for infinite nuclear matter, in which the meson fields are treated as classical quantitates. Thus the finite plus momentum can be thought of as coming from an infinite number of quanta, each carrying an infinitesimal amount of plus momentum. A plus momentum of 0 can only be accessed experimentally at $x_{Bj} = 0$, which requires an infinite amount of energy. Thus, in the mean field approximation, the scalar and vector mesons can not contribute to deep inelastic scattering. The usual term for a field that is constant over space is a zero mode, and the present Lagrangian provides a simple example. For finite nuclei, in the mean field approximation, the mesons would carry a very small momentum of scale given by the inverse of the nuclear radius, under the mean field approximation. If fluctuations were to be included, the relevant momentum scale would be of the order of the inverse of the average distance between nucleons (about 2 Fm).

We can understand the significance of the presence of components of a wave function that carry plus momentum but do not participate deep inelastic processes by reviewing a bit of history. The nuclear binding effect is that the plus momentum of a bound nucleon is reduced by the binding energy, and so is that of its confined quarks. Conservation of momentum implies that if nucleons lose momentum, other constituents such as nuclear pions [51], must gain momentum. This partitioning of the total plus momentum amongst the various constituents is called the momentum sum rule. Pions are quark anti-quark pairs so that a specific enhancement of the nuclear antiquark momentum distribution, mandated by momentum conservation, is a testable [52] consequence of this idea. A nuclear Drell Yan experiment [53], in which a quark from a beam proton annihilates with a nuclear antiquark to form a $\mu^+\mu^-$ pair, was performed. No influence of nuclear pion enhancement was seen, leading Bertsch et al. [54] to question the idea that the pion is a dominant carrier of the nuclear force. In the present situation, we have a huge depletion effect of 35%, but with no consequence for either the nuclear deep inelastic scattering or Drell-Yan experiments.

We hasten to add that the Lagrangian of Eq. (2.1) and its evaluation in mean field approximation for nuclear matter have been used to provide a simple but semi-realistic example. It would be premature to compare the present results with data before obtaining light front dynamics for a model in which the correlational corrections to the mean field approximation are included, and which treats finite nuclei. Thus the specific numerical results of the present work are far less relevant than the central feature that the mesons responsible for nuclear binding need not be accessible in deep inelastic scattering.

However, the present model may be regarded as being one of a class of models in which the mean field plays an important role [55]. For such models nuclei would have constituents that contribute to the momentum sum rule but do not contribute to deep inelastic scattering. In particular, a model can have a large binding effect, nucleons can carry a significantly less fraction of P^+ than unity, and it is not necessary to include the influence of mesons that could be ruled out in a Drell-Yan experiment.

VI. SUMMARY AND DISCUSSION

The present paper shows how the light front quantization of a chiral Lagrangian can be accomplished. The resulting formalism can be applied to many problems of interest to nuclear physics. In particular, pion-nucleon, nucleon-nucleon scattering, and infinite nuclear matter (in the mean field approximation) are presented here. Soft pion theorems for pion-nucleon scattering are reproduced. The treatment of nucleon-nucleon scattering is shown to be manifestly covariant in the one boson exchange approximation. The implications of chiral symmetry for the two-nucleon intermediate state contribution to the two pion exchange potential are discussed. The present results mainly constitute a feasibility study, in that the emphasis here is on checking the formalism by reproducing known results. But this light front treatment does allow the effects of chiral symmetry to be incorporated within a relativistic formalism, and therefore should have a broad applicability in the future. One remaining technical problem is to provide a light front quantization of a Lagrangian for spin 3/2 particles.

The special feature of the light front formalism is its use of the plus momentum as one of the canonical variables. This enables a close contact with the experimental variables used to analyse deep inelastic scattering and any experiment in which there is one large momentum. This feature is exploited here in the derivation (within the mean field approximation) of the nucleonic and mesonic distribution functions for infinite nuclear matter. The mesons are shown to carry a significant fraction of the nuclear plus momentum, but only a zero plus-momentum (a zero mode), and therefore do not participate in nuclear deep inelastic scattering or Drell-Yan experiments.

The ultimate validity of the above (perhaps startling) statement depends on whether or not the dominance of mesonic zero modes survives calculations performed for finite nuclei and calculations which include the effects of nucleon-nucleon correlations. There seems to be no technical barrier precluding such calculations.

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APPENDIX A: NOTATION, CONVENTIONS, AND USEFUL RELATIONS

This is patterned after the review of Harindrinath [3] The light-front variables are defined by

$$x^+ = x^0 + x^3 \ , \quad x^- = x^0 - x^3, \tag{A1}$$

so the four-vector x^μ is denoted

$$x^\mu = (x^+, x^-, \boldsymbol{x}^\perp). \tag{A2}$$

With this notation the scalar product is denoted by

$$x \cdot y = \frac{1}{2}x^+y^- + \frac{1}{2}x^-y^+ - \boldsymbol{x}^\perp \cdot \boldsymbol{y}^\perp. \tag{A3}$$

The metric tensor $g^{\mu\nu}$ with $\mu = (+,-,1,2)$ is obtained from the usual one by using (A1) (i.e. $g^{0\mu} = g^{0\mu} + g^{3\mu}$). Then $g^{+-} = g^{-+} = 2, g^{ij} = -1$, with the other elements vanishing. The term $g_{\mu\nu}$ is obtained from the condition that $g^{\alpha\beta}g_{\beta\gamma} = \delta_{\alpha\gamma}$. Its elements are the same as those of $g^{\mu\nu}$ except for $g_{-+} = g_{+-} = 1/2$. Thus

$$x_- = \frac{1}{2}x^+, \ \ x_+ = \frac{1}{2}x^-, \tag{A4}$$

and the partial derivatives are similarly given by

$$\partial^+ = 2\partial_- = 2\frac{\partial}{\partial x^-} \quad \partial^- = 2\partial_+ = 2\frac{\partial}{\partial x^+}. \tag{A5}$$

The step function is defined as $\theta(x) = 1$ for $x > 0$, and $\theta(x) = 0$ for $x \leq 0$. The antisymmetric step function is given by

$$\epsilon(x) = \theta(x) - \theta(-x), \tag{A6}$$

with

$$\frac{\partial \epsilon}{\partial x} = 2\ \delta(x). \tag{A7}$$

In this notation, $\mid x \mid \; = \; x\,\epsilon(x)$. The above few definitions allow us to express the inverse operators appearing in the text in terms of integrals:

$$\frac{1}{\partial^+} f(x^-) = \frac{1}{4} \int dy^- \epsilon(x^- - y^-)\, f(y^-), \tag{A8}$$

$$(\frac{1}{\partial^+})^2 f(x^-) = \frac{1}{8} \int dy^- \mid x^- - y^- \mid f(y^-). \tag{A9}$$

The Bjorken and Drell [56] convention for gamma matrices is used and

$$\gamma^\pm \; \equiv \gamma^0 \pm \gamma^3. \tag{A10}$$

The relations

$$\gamma^\pm \gamma^\pm = 0, \quad \gamma^+ \gamma^- \gamma^+ = 4\gamma^+, \quad \gamma^- \gamma^+ \gamma^- = 4\gamma^- \tag{A11}$$

can be used to simplify various computations.

The hermitian projection operators $\Lambda_\pm$ are given by

$$\Lambda_\pm \; = \; \frac{1}{4}\gamma^\mp \gamma^\pm = \frac{1}{2}\gamma^0 \gamma^\pm \; = \; \frac{1}{2}(I \pm \alpha^3), \tag{A12}$$

and obey the following relations

$$(\Lambda_\pm)^2 \; = \; \Lambda_\pm, \qquad \gamma^\perp \, \Lambda_\pm, \; = \; \Lambda_\pm \gamma^\perp, \tag{A13}$$

$$\gamma^0 \, \Lambda \pm \; = \; \Lambda_\mp \gamma^0 \qquad \alpha^\perp \, \Lambda_\pm \; = \; \Lambda_\mp \alpha^\perp, \tag{A14}$$

$$\gamma^5 \, \Lambda_\pm \; = \; \Lambda_\pm \gamma^5 \quad \gamma^\mp = 2\Lambda_\pm \gamma^0 = \gamma^\mp \Lambda_\mp, \tag{A15}$$

$$\gamma^i \Lambda_\mp = \frac{1}{2}\gamma^i \pm i\frac{1}{2}\epsilon^{ij}\gamma^j \gamma^5, \tag{A16}$$

and

$$\alpha^j \gamma^i \Lambda_+ = \frac{i}{2}\epsilon^{ij}\gamma^+ \gamma^5. \tag{A17}$$

The Dirac spinors are given by

$$u_\lambda(k) = \sqrt{\frac{2}{k^+}}[M\ \Lambda_- \ + \ (k^+ \ + \boldsymbol{\alpha}^\perp \cdot \boldsymbol{k}^\perp)\ \Lambda_+]\ \chi_\lambda, \tag{A18}$$

where

$$\chi_\uparrow^\dagger = (1,0,0,0), \quad \chi_\downarrow^\dagger = (0,1,0,0). \tag{A19}$$

The anti-particle spinors are given by $v_\lambda(k) = C\ (\bar{u}_\lambda(k))^T$ where $C = i\gamma^2\gamma^0$ is the charge conjugation operator, so that

$$v_\lambda(k) = \sqrt{\frac{2}{k^+}}[M\ \Lambda_- \ + \ (k^+ \ + \boldsymbol{\alpha}^\perp \cdot \boldsymbol{k}^\perp)\ \Lambda_+]\ \eta_\lambda \tag{A20}$$

with

$$\eta_\uparrow^\dagger = (0,0,0,1), \quad \eta_\downarrow^\dagger = (0,0,-1,0). \tag{A21}$$

Other useful relations are

$$\bar{u}(\boldsymbol{k},\lambda)u(\boldsymbol{k},\lambda') = 2M\delta_{\lambda,\lambda'}, \quad \bar{v}(\boldsymbol{k},\lambda)v\boldsymbol{k},\lambda') = -2M\delta_{\lambda,\lambda'}, \tag{A22}$$

$$\bar{u}(\boldsymbol{k},\lambda)\gamma^\mu u(\boldsymbol{k},\lambda') = 2p^\mu\delta_{\lambda,\lambda'}, \quad \bar{v}(\boldsymbol{k},\lambda)\gamma^\mu v\boldsymbol{k},\lambda') = -2p^\mu\delta_{\lambda,\lambda'}, \tag{A23}$$

and

$$\sum_\lambda u(\boldsymbol{k},\lambda)\bar{u}(\boldsymbol{k},\lambda) = \gamma\cdot k + m, \quad \sum_\lambda v(\boldsymbol{k},\lambda)\bar{v}(\boldsymbol{k},\lambda) = \gamma\cdot k - m. \tag{A24}$$

Note that in the above three equations k^μ is an on-shell four vector with $k^- = \frac{\boldsymbol{k}_\perp^2+M^2}{k^+}$ and $\boldsymbol{k} = (k^+, \boldsymbol{k}_\perp)$.

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Figure Captions

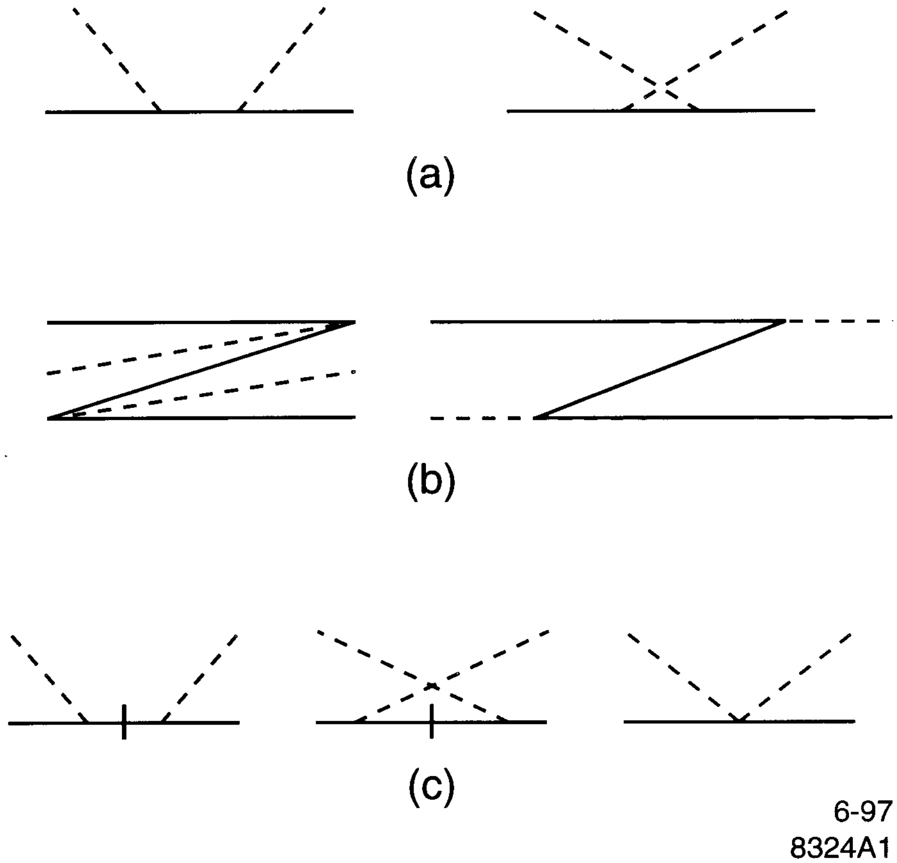

Fig. 1 x^+-ordered graphs for low energy pion-nucleon scattering. (a) Second-order effects of the $\bar{u}\gamma_5 u$ term v_1. (b) Second-order effects of the $\bar{u}\gamma_5 v$ and $\bar{v}\gamma_5 u$ terms of v_1. (c) Effects of the instantaneous fermion propagation terms of v_2, and of the π^2 term of v_1. The terms v_i are defined in Eqs, (2.46)-(2.48).

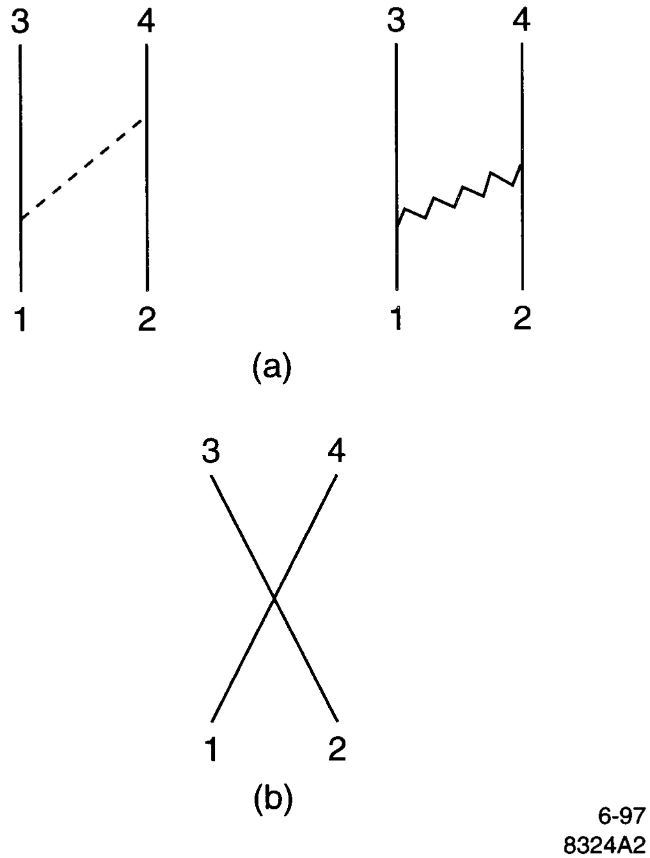

Fig. 2. x^+-ordered graphs for one boson exchange contributions to nucleon-nucleon scattering. The numbers 1-4 represent the momentum, spin and charge states of the nucleons. Here $k_1^+ > k_3^+$. (a) meson propagation terms (b) instantaneous vector meson exchange of v_3, Eq. (2.48)

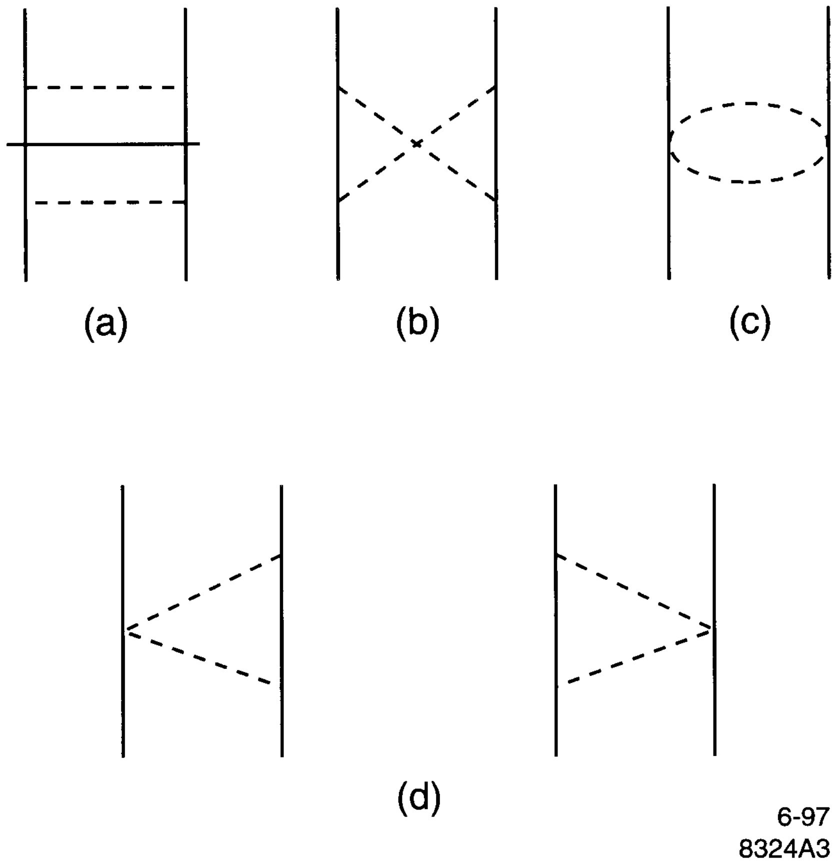

Fig. 3 Feynman graphs for the two-pion exchange potential (a) uncrossed box diagram- the horizontal line represents the subtraction of the contribution arising from the iterated one pion exchange potential. (b) Crossed box diagram (c) Second order effect of the π^2 term of v_1, Eq. (2.46) (d) Terms with one π^2 term.