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The Light–Cone Fock State Expansion and Hadron Physics Phenomenology *

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1 Introduction

The concept of the "number of constituents" of a relativistic bound state, such as a hadron in quantum chromodynamics, is not only frame-dependent, but its value can fluctuate to an arbitrary number of quanta. Thus when a laser beam crosses a proton at fixed "light-cone" time $\tau = t + z/c = x^0 + x^z$, an interacting photon can encounter a state with any given number of quarks, anti-quarks, and gluons in flight (as long as $n_q - n_{\overline{q}} = 3$). The probability amplitude for each such n-particle state of on-mass shell quarks and gluons in a hadron is given by a light-cone Fock state wavefunction $\psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$, where the constituents have longitudinal light-cone momentum fractions

$$x_i = \frac{k_i^+}{p^+} = \frac{k^0 + k_i^z}{p^0 + p^z} \ , \quad \sum_{i=1}^{n} x_i = 1 \ , \tag{1}$$

relative transverse momentum

$$\vec{k}_{\perp i} \ , \quad \sum_{i=1}^{n} \vec{k}_{\perp i} = \vec{0}_{\perp} \ , \tag{2}$$

and helicities λ_i. The ensemble $\{\psi_{n/H}\}$ of such light-cone Fock wavefunctions is a key concept for hadronic physics, providing a conceptual basis for representing physical hadrons (and also nuclei) in terms of their fundamental quark and gluon degrees of freedom [1].

The light-cone Fock expansion is defined in the following way: one first constructs the light-cone time evolution operator $P^- = P^0 - P^z$ and the invariant mass operator $H_{LC} = P^- P^+ - P_\perp^2$ in light-cone gauge $A^+ = 0$ from the QCD Lagrangian. The total longitudinal momentum $P^+ = P^0 + P^z$ and transverse momenta $\vec{P}_\perp$ are conserved, *i.e.* are independent of the interactions. The matrix elements of H_{LC} on the complete orthonormal basis $\{|n>\}$ of the free theory $H^0_{LC} = H_{LC}(g = 0)$ can then be constructed. The matrix elements $\langle n \,|\, H_{LC} \,|\, m\rangle$ connect Fock states differing by 0, 1, or 2 quark or gluon quanta, and they include the instantaneous quark and gluon contributions imposed by eliminating dependent degrees of freedom in light-cone gauge.

In practice it is essential to introduce an ultraviolet regulator in order to limit the total range of $\langle n \,|\, H_{LC} \,|\, m\rangle$, such as the "global" cutoff in the invariant mass of the free Fock states:

$$\mathcal{M}_n^2 = \sum_{i=1}^{n} \frac{k_\perp^2 + m^2}{x} < \Lambda^2_{\rm global} \ . \tag{3}$$

One can also introduce a "local" cutoff to limit the change in invariant mass $|\mathcal{M}_n^2 - \mathcal{M}_m^2| < \Lambda^2_{\rm local}$ which provides spectator-independent regularization of the sub-divergences associated with mass and coupling renormalization.

The natural renormalization scheme for the coupling is $\alpha_V(Q)$, the effective charge defined from the scattering of two infinitely-heavy quark test charges. The renormalization scale can then be determined from the virtuality of the exchanged momentum, as in the BLM and commensurate scale methods [2, 3, 4].

In the discretized light-cone method (DLCQ) [5, 6] the matrix elements $\left\langle n \mid H_{LC}^{(\Lambda)} \mid m \right\rangle$, are made discrete in momentum space by imposing periodic or anti-periodic boundary conditions in $x^- = x^0 - x^z$ and $\vec{x}_\perp$. Upon diagonalization of H_{LC}, the eigenvalues provide the invariant mass of the bound states and eigenstates of the continuum. The projection of the hadronic eigensolutions on the free Fock basis define the light-cone wavefunctions. For example, for the proton,

$$\begin{aligned} \mid p\rangle &= \sum_n \langle n \mid p \rangle \ \mid n \rangle \\ &= \psi^{(\Lambda)}_{3q/p}(x, \vec{k}_{\perp i}, \lambda_i) \ \mid uud\rangle \\ &\quad + \psi^{(\Lambda)}_{3qg/p}(x_i, \vec{k}_{\perp i}, \lambda_i) \ \mid uudg\rangle + \cdots \end{aligned} \tag{4}$$

The light-cone formalism has the remarkable feature that the $\psi^{(\Lambda)}_{n/H}(x_i, \vec{k}_{\perp i}, \Lambda_c)$ are invariant under longitudinal boosts; *i.e.*, they are independent of the total momentum P^+, $\vec{P}_\perp$ of the hadron. Given the $\psi^{(\Lambda)}_{n/H}$, we can construct any electromagnetic or electroweak form factor from the diagonal overlap of the LC wavefunctions[7]. Similarly, the matrix elements of the currents that define quark and gluon structure functions can be computed from the integrated squares of the LC wavefunctions [8].

In general, any hadronic amplitude such as quarkonium decay, heavy hadron decay, or any hard exclusive hadron process can be constructed as the convolution of the light-cone Fock state wavefunctions with quark-gluon matrix elements [9]

$$\begin{aligned} \mathcal{M}_{\rm Hadron} &= \prod_H \sum_n \int \prod_{i=1}^{n} d^2k_\perp \prod_{i=1}^{n} dx \, \delta\left(1 - \sum_{i=1}^{n} x_i\right) \, \delta\left(\sum_{i=1}^{n} \vec{k}_{\perp i}\right) \\ &\quad \times \psi^{(\Lambda)}_{n/H}(x_i, \vec{k}_{\perp i}, \Lambda_i) \, \mathcal{M}^{(\Lambda)}_{q,g} \ . \end{aligned} \tag{5}$$

Here $\mathcal{M}_{q,g}^{(\Lambda)}$ is the underlying quark-gluon subprocess scattering amplitude, where the (incident or final) hadrons are replaced by quarks and gluons with momenta $x_i p^+$, $x_i \vec{p}_\perp + \vec{k}_{\perp i}$ and invariant mass above the separation scale $\mathcal{M}_n^2 > \Lambda^2$. The LC ultraviolet regulators thus provide a factorization scheme for elastic and inelastic scattering, separating the hard dynamical contributions with invariant mass squared $\mathcal{M}^2 > \Lambda^2_{\rm global}$ from the soft physics with $\mathcal{M}^2 \leq \Lambda^2_{\rm global}$ which is incorporated in the nonperturbative LC wavefunctions. The DGLAP evolution of parton distributions can be derived by computing the variation of the Fock expansion with respect to $\Lambda^2_{\rm global}$ [9].

The simplest, but most fundamental, characteristic of a hadron in the light-cone representation, is the hadronic distribution amplitudes [9], defined as the integral over transverse momenta of the valence (lowest particle number) Fock wavefunction; *e.g.* for the pion

$$\phi_\pi(x_i, Q) \equiv \int d^2k_\perp \, \psi^{(Q)}_{q\bar{q}/\pi}(x_i, \vec{k}_{\perp i}, \lambda) \tag{6}$$

where the global cutoff Λ_{global} is identified with the resolution Q. The distribution amplitude controls leading-twist exclusive amplitudes at high momentum transfer, and it can be related to the gauge-invariant Bethe-Salpeter wavefunction at equal light-cone time $\tau = x^+$. The $\log Q$ evolution of the hadron distribution amplitudes $\phi_H(x_i, Q)$ can be derived from the perturbatively-computable tail of the valence light-cone wavefunction in the high transverse momentum regime [9].

Light-cone quantization methods have had remarkable success in solving quantum field theories in one-space and one-time dimension—virtually any (1+1) quantum field theory can be solved using DLCQ. A beautiful example is "collinear" QCD: a variant of $QCD(3+1)$ defined by dropping all of interaction terms in H^{QCD}_{LC} involving transverse momenta [10]. Even though this theory is effectively two-dimensional, the transversely-polarized degrees of freedom of the gluon field are retained as two scalar fields. Antonuccio and Dalley [11] have used DLCQ to solve this theory. The diagonalization of H_{LC} provides not only the complete bound and continuum spectrum of the collinear theory, including the gluonium states, but it also yields the complete ensemble of light-cone Fock state wavefunctions needed to construct quark and gluon structure functions for each bound state. Although the collinear theory is a drastic approximation to physical $QCD(3+1)$, the phenomenology of its DLCQ solutions demonstrate general gauge theory features, such as the peaking of the wavefunctions at minimal invariant mass, color coherence and the helicity retention of leading partons in the polarized structure functions at $x \to 1$.

2 Applications of Light-Cone Methods to QCD Phenomenology

Regge behavior.
The light-cone wavefunctions $\psi_{n/H}$ of a hadron are not independent of each other, but rather are coupled via the equations of motion. Recently Antonuccio, Dalley and I [12] have used the constraint of finite "mechanical" kinetic energy to derive"ladder relations" which interrelate the light-cone wavefunctions of states differing by 1 or 2 gluons. We then use these relations to derive the Regge behavior of both the polarized and unpolarized structure functions at $x \to 0$, extending Mueller's derivation of the BFKL hard QCD pomeron from the properties of heavy quarkonium light-cone wavefunctions at large N_C QCD [13].

High momentum transfer exclusive reactions.
Given the solution for the hadronic wavefunctions $\psi_n^{(\Lambda)}$ with $\mathcal{M}_n^2 < \Lambda^2$, one can construct the wavefunction in the hard regime with $\mathcal{M}_n^2 > \Lambda^2$ using projection operator techniques [9]. The construction can be done perturbatively in QCD since only high invariant mass, far off-shell matrix elements are involved. One can use this method to derive the physical properties of the LC wavefunctions and their matrix elements at high invariant mass. Since $\mathcal{M}_n^2 = \sum_{i=1}^n \left(\frac{k_\perp^2+m^2}{x}\right)_i$, this method also allows the derivation of the asymptotic behavior of light-cone wavefunctions at large $k_\perp$, which in turn leads to predictions for the fall-off of form factors and other exclusive matrix elements at large momentum transfer, such as the quark counting rules for predicting the nominal power-law fall-off of two-body scattering amplitudes at fixed θ_{cm}. The phenomenological successes of these rules can be understood within QCD if the coupling $\alpha_V(Q)$ freezes in a range of relatively small momentum transfer [14].

Analysis of diffractive vector meson photoproduction.
The light-cone Fock wavefunction representation of hadronic amplitudes allows a simple eikonal analysis of diffractive high energy processes, such as $\gamma^*(Q^2)p \to \rho p$, in terms of the virtual photon and the vector meson Fock state light-cone wavefunctions

convoluted with the $gp \to gp$ near-forward matrix element [15]. One can easily show that only small transverse size $b_\perp \sim 1/Q$ of the vector meson wavefunction is involved. The hadronic interactions are minimal, and thus the $\gamma^*(Q^2)N \to \rho N$ reaction can occur coherently throughout a nuclear target in reactions such as without absorption or shadowing. The $\gamma^* A \to \phi A$ process thus provides a natural framework for testing QCD color transparency [16].

Structure functions at large x_{bj}.
The behavior of structure functions where one quark has the entire momentum requires the knowledge of LC wavefunctions with $x \to 1$ for the struck quark and $x \to 0$ for the spectators. This is a highly off-shell configuration, and thus one can rigorously derive quark-counting and helicity-retention rules for the power-law behavior of the polarized and unpolarized quark and gluon distributions in the $x \to 1$ endpoint domain. It is interesting to note that the evolution of structure functions is minimal in this domain because the struck quark is highly virtual as $x \to 1$; *i.e.* the starting point Q_0^2 for evolution cannot be held fixed, but must be larger than a scale of order $(m^2 + k_\perp^2)/(1-x)$ [8].

Intrinsic gluon and heavy quarks.
The main features of the heavy sea quark-pair contributions of the Fock state expansion of light hadrons can also be derived from perturbative QCD, since $\mathcal{M}_n^2$ grows with m_Q^2. One identifies two contributions to the heavy quark sea, the "extrinsic" contributions which correspond to ordinary gluon splitting, and the "intrinsic" sea which is multi-connected via gluons to the valence quarks. The intrinsic sea is thus sensitive to the hadronic bound state structure [17]. The maximal contribution of the intrinsic heavy quark occurs at $x_Q \simeq m_{\perp Q}/\sum_i m_\perp$ where $m_\perp = \sqrt{m^2 + k_\perp^2}$; *i.e.* at large x_Q, since this minimizes the invariant mass $\mathcal{M}_n^2$. The measurements of the charm structure function by the EMC experiment are consistent with intrinsic charm at large x in the nucleon with a probability of order $0.6 \pm 0.3\%$ [18]. Similarly, one can distinguish intrinsic gluons which are associated with multi-quark interactions and extrinsic gluon contributions associated with quark substructure [19]. One can also use this framework to isolate the physics of the anomaly contribution to the Ellis-Jaffe sum rule.

Rearrangement mechanism in heavy quarkonium decay.
It is usually taken for granted that a heavy quarkonium state such as the J/ψ decays to

light hadrons via the annihilation of the heavy quark constituents to gluons. However, as Karliner and I [20] have recently shown, the transition $J/\psi \to \rho\pi$ can also occur by the rearrangement of the $c\overline{c}$ from the J/ψ into the $|q\overline{q}c\overline{c}>$ intrinsic charm Fock state of the ρ or π. On the other hand, the overlap rearrangement integral in the decay $\psi' \to \rho\pi$ will be suppressed since the intrinsic charm Fock state radial wavefunction of the light hadrons will evidently not have nodes in its radial wavefunction. This observation provides a natural explanation of the long-standing puzzle why the J/ψ decays prominently to two-body pseudoscalar-vector final states, whereas the ψ' does not.

Asymmetry of Intrinsic heavy quark sea.
As Ma and I have noted [21], the higher Fock state of the proton $|uuds\overline{s}>$ should resemble a $|K\Lambda>$ intermediate state, since this minimizes its invariant mass $\mathcal{M}$. In such a state, the strange quark has a higher mean momentum fraction x than the $\overline{s}$. [22, 21] Similarly, the helicity intrinsic strange quark in this configuration will be anti-aligned with the helicity of the nucleon [21]. This $Q \leftrightarrow \overline{Q}$ asymmetry is a remarkable, striking feature of the intrinsic heavy-quark sea.

Direct measurement of the light-cone valence wavefunction.
Diffractive multi-jet production in heavy nuclei provides a novel way to measure the shape of the LC Fock state wavefunctions. For example, consider the reaction [23, 24]

$$\pi A \to \mathrm{Jet}_1 + \mathrm{Jet}_2 + A' \tag{7}$$

at high energy where the nucleus A' is left intact in its ground state. The transverse momenta of the jets have to balance so that $\vec{k}_{\perp i} + \vec{k}_{\perp 2} = \vec{q}_\perp < \mathcal{R}_A^{-1}$, and the light-cone longitudinal momentum fractions have to add to $x_1 + x_2 \sim 1$ so that $\Delta p_L < \mathcal{R}_A^{-1}$. The process can then occur coherently in the nucleus. Because of color transparency; *i.e.* the cancellation of color interactions in a small-size color-singlet hadron; the valence wavefunction of the pion with small impact separation will penetrate the nucleus with minimal interactions, diffracting into jet pairs [23]. The $x_1 = x$, $x_2 = 1-x$ dependence of the di-jet distributions will thus reflect the shape of the pion distribution amplitude; the $\vec{k}_{\perp 1} - \vec{k}_{\perp 2}$ relative transverse momenta of the jets also gives key information on the underlying shape of the valence pion wavefunction. The QCD analysis can be confirmed by the observation that the diffractive nuclear amplitude extrapolated to $t = 0$ is linear in nuclear number A, as predicted by QCD color transparency. The integrated diffractive rate should scale as $A^2/\mathcal{R}_A^2 \sim A^{4/3}$. A diffractive experiment of this type is now in progress at Fermilab using 500 GeV incident pions on nuclear targets [25].

Data from CLEO for the $\gamma\gamma^* \to \pi^0$ transition form factor favor a form for the pion distribution amplitude close to the asymptotic solution [9] $\phi_\pi^{\rm asympt}(x) = \sqrt{3} f_\pi x(1-x)$ to the perturbative QCD evolution equation [26, 27, 14] It will be interesting to see if the diffractive pion to di-jet experiment also favors the asymptotic form.

It would also be interesting to study diffractive tri-jet production using proton beams $pA \to {\rm Jet}_1 + {\rm Jet}_2 + {\rm Jet}_3 + A'$ to determine the fundamental shape of the 3-quark structure of the valence light-cone wavefunction of the nucleon at small transverse separation. Conversely, one can use incident real and virtual photons: $\gamma^* A \to {\rm Jet}_1 + {\rm Jet}_2 + A'$ to confirm the shape of the calculable light-cone wavefunction for transversely-polarized and longitudinally-polarized virtual photons. Such experiments will open up a remarkable, direct window on the amplitude structure of hadrons at short distances.

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