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SPECTRAL FUNCTION CALCULATION OF ANGLE WAKES, WAKE MOMENTS, AND MISALIGNMENT WAKES FOR THE SLAC DAMPED DETUNED STRUCTURES (DDS)[1]

R.M Jones[†], N.M. Kroll[†‡] & R.H. Miller[†]

†*Stanford Linear Accelerator Center, M/S 26,*
P.O Box 4349, Stanford, CA 94309
‡*University of California, San Diego,*
La Jolla, CA 92093-0319

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R.M Jones†, N.M. Kroll†‡ & R.H. Miller†

†Stanford Linear Accelerator Center, M/S 26, P.O Box 4349, Stanford, CA 94309

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Transverse wake functions so far reported for the SLAC DDS have been limited to those caused by uniform offset of the drive beam in a straight perfectly aligned structure. The complete description of the betatron oscillations of wake coupled bunches requires an array of wake functions, referred to as moments in [1]. Modifications of these arrays induced by structure misalignments are also of interest. In this paper we express the array elements in terms of a spectral function array. Examples are given based upon DDS1.

1. INTRODUCTION

Adolphsen [1] has pointed out that an analysis of transverse wakefield coupled multibunch motion in a FODO array requires an array of wake functions instead of the single function usually defined. Oide [2] has emphasized the need to compute these wake functions and also to measure them where possible. The offset wake function, which we here designate by W_{00} instead of the usual W, gives the angular deviation $\Delta\theta$ of a witness bunch trailing a uniformly offset drive beam at a distance s that has passed through a structure of length L_s via:

$$\Delta\theta = (q_w\, q_d\, L_s/E)\, W_{00}(s)\, r_d \tag{1.1}$$

Here q_w and q_d are the charges of the witness bunch and drive bunch respectively, r_d the drive bunch offset, and E the witness bunch energy. As a result of betatron motion through the FODO array, however, the drive bunch motion may be at an angle with respect to the structure axis. Choosing the structure center as the fiducial point from which to define the offset we expect an additional angular deviation $\Delta\theta$ proportional to θ_d, the drive beam angle, which we express as:

$$\Delta\theta = (q_w\, q_d\, L_s/E)\, W_{01}(s)\, (L_s/2)\, \theta_d, \tag{1.2}$$

the total being the sum of (1.1) and (1.2). Furthermore, for sufficiently large L_s one cannot neglect the effect of the wake induced offset of the witness beam on its betatron motion. This leads us to define two more elements of the wakefield array (called the moment array in [1]), W_{10} and W_{11} via:

$$\Delta x = -(q_w\, q_d\, L_s/E)\, W_{10}(s)\, (L_s/2)\, r_d - (q_w\, q_d\, L_s/E)\, W_{11}(s)\, (L_s/2)^2\, \theta_d. \tag{1.3}$$

For consistency with our treatment of the drive beam, Δx is defined with respect to the center of the structure by projecting the angle and displacement on emergence from the structure back to its center, assuming rectilinear motion. Expressions for the above array elements based upon the independent oscillator model, a model in which W_{10} and W_{01} are equal, are given in [1]. Finally we note that structure misalignments generate offset patterns of the drive beam different from those discussed above. Some are of a sufficiently general nature (*eg* bowing) that it is useful to define wake functions for them as well (we designate the bowed case by W_{02} and W_{12}). The same general methods which we will apply here apply to more irregular cases such as those discusssed in [3], and for each such case there are two functions that need to be computed, one for $\Delta\theta$ and one for Δx.

2. SPECTRAL FUNCTIONS FOR THE WAKE FUNCTION ARRAY

The extension of the formalism introduced in [4] is straightforward although, as we shall see, some elements of the array require spectral function integrals involving both the sine and cosine of ($2\pi sf/c$) instead of merely the sine as is the case for W_{00}. The wake function W_{00} is in the equivalent circuit theory expressed as:

$$W_{00}(s) = \sum_{n,m=1}^{N} w_{nm}(s) \tag{2.1}$$

where $(q_w\, q_d\, L_s/E)w_{nm}$ is the angular kick experienced by the witness bunch at cell n due to a unit displacement of the drive bunch at cell m. Hence the angular kick received by the witness due to a set of displacements $d\chi(m)$ at cell m is:

$$\Delta\theta = (q_w q_d L_s/E)d \sum_{n,m=1}^{N} w_{nm}\chi(m) \tag{2.2}$$

The various W_{0x} referred to above are determined by specification of the dimensionless $\chi(m)$, with an appropriately defined length scale d. To obtain the displacement associated with the above we write (2.2) as:

$$\Delta\theta(s) = \sum_{n=1}^{N} \delta\theta_n \tag{2.3}$$

Then:

$$\begin{aligned}\Delta x &= L_s \sum_{n=1}^{N} \delta\theta_n \left(\tfrac{N-n}{N-1} - \tfrac{1}{2}\right) \\ &= -\tfrac{1}{2} L_s \sum_{n=1}^{N} \delta\theta_n \left(\tfrac{2n-N-1}{N-1}\right)\end{aligned} \tag{2.4}$$

Comparing with eq. (1.3), we see that

$$W_{1x} = \sum_{n,m=1}^{N} w_{nm} \chi_1(n)\chi(m) \tag{2.5}$$

where $\chi_1(n)$ = (2n-N-1)/(N-1). For the angle wake the displacement of the drive bunch at cell m is just $(L_s/2)\theta_d\chi_1(m)$ so that comparing with eq. (1.1), (1.2) and (1.3) we have the general form:

$$W_{ij} = \sum_{n,m=1}^{N} w_{nm} \chi_i(n)\chi_j(m) \tag{2.6}$$

where i, j equals zero or one, and χ_0 equals one .

For a bowed structure the displacement of the drive bunch at cell m is given by d $\chi_2(m)$ where d is the span of the bow and χ_2 is given by:

$$\chi_2(m) = [(2n\text{-}N\text{-}1)/N\text{-}1)]^2 - 1/3 \tag{2.7}$$

In parallel with the W_{ij} array we require a wake impedence array Z_{ij}. Since the wake impedance is simply the fourier transform of the wake function, we have for the array:

$$W_{ij}(s) = \int Z_{ij}(f - j\varepsilon)\exp[2\pi js / c(f - j\varepsilon)]df \tag{2.8}$$

where ε is a positive infinitesimal quantity, and we write the impedences schematically as:

$$Z_{ij}(f) = \sum_{n.m=1}^{N} \kappa_{nm}(f)\exp[2\pi jLf / c(n-m)]\chi_i(n)\chi_j(m) \tag{2.9}$$

where L is the structure period. It will be sufficient for our purposes to specify the needed properties of κ_{nm}. More detailed information is given in [4]. The κ_{nm} are four valued analytic functions of f with branch points on the real axis located at the propagation band edges of both ends of the manifold. The cuts connecting the four sheets are on the real axis wherever either end of the manifold is propagating. The segments where both ends of the manifold are nonpropagating are called gaps. The integral in eq. (2.8) is carried out on the physical sheet, a sheet on which κ_{nm} has no singularities except poles and branch points on the real axis and vanishes at infinity. κ_{nm} also satisfies the symmetry relations:

$$\kappa_{nm}(f) = \kappa^*_{nm}(f^*) = \kappa_{nm}(-f) = \kappa_{mn}(f) \tag{2.10}$$

with f, f* and, -f all on the physical sheet. An important consequence of these relations is that κ is real on the gaps; the real part is continuous and the imaginary part changes sign across the cuts.

To proceed to the spectral function representation it turns out that one must split Z (and correspondingly W) into even and odd parts thus:

$$\begin{aligned}Z^e_{ij}(f) &= \sum_{n,m=1}^{N} \kappa_{nm}\cos[2\pi Lf / c(n-m)]\chi_i(n)\chi_j(m) \\ Z^o_{ij}(f) &= \sum_{n,m=1}^{N} \kappa_{nm}\sin[2\pi Lf / c(n-m)]\chi_i(n)\chi_j(m)\end{aligned} \tag{2.11}$$

with:

$$Z_{ij} = Z^e_{ij} + jZ^o_{ij} \tag{2.12}$$

The expressions for W_e and W_o are reduced to spectral function form by making use of the fact in that in general the precursor term $\theta(s)W(-s)$ is negligeably small [4]. Noting that the Fourier transform of $W^{e,o}(-s)$ is just $\pm$, respectively, that of $W^{e,o}(s)$ except that it is evaluated just above the real axis instead of below, we are led to write:

$$W^{e,o}_{ij} = \theta(s)\left[W^{e,o}_{ij}(s) \mp W^{e,o}_{ij}(-s)\right] \tag{2.13}$$

$$W_{ij} = W^e_{ij} + W^o_{ij} \tag{2.14}$$

These lead to integrals over positive frequency cuts and sums over positive frequency poles as in [4]. We find

$$\begin{aligned}W^e_{ij} &= \theta(s)\int S^e_{ij}\sin[2\pi Lf / c(n - m)]df \\ W^o_{ij} &= \theta(s)\int S^o_{ij}\cos[2\pi Lf / c(n - m)]df\end{aligned} \tag{2.15}$$

$$S^{e,o}_{ij}(f) = -4\,\mathrm{Im}\left\{Z^{e,o}_{ij}(f - j\varepsilon)\right\} \tag{2.16}$$

Pole terms are included in the spectral functions as δ functions as in [4].

We note that the even wake functions vanish (by construction) at zero s as expected from causality while the odd ones do not. Some sense of the importance of the precursor term can be obtained by comparing the wake envelope functions of the even and odd parts at zero s.

3. EXAMPLES OF SPECTRAL FUNCTION AND WAKE FUNCTION ARRAY ELEMENTS

We conclude by presenting a number of examples of the spectral function array elements and their associated wake envelope functions. These are useful for assessing the relative importance of the various components. It should be recognized, however, that the envelope functions cannot be combined linearly.

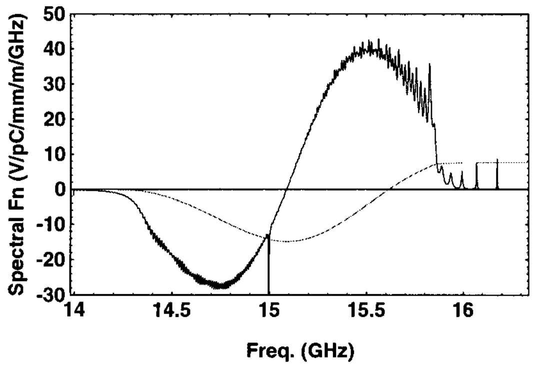

Fig 1: Even spectral function, W^c_{01} associated with the sine wake function, and its integral for a matched HOM coupler with a beam transiting a DDS at an angle

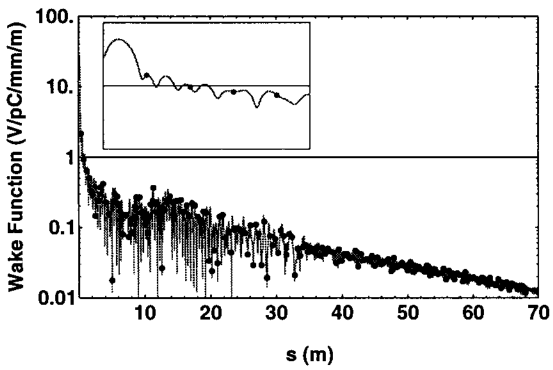

Fig 2: Sine wake envelope function resulting from the spectral function of fig 1.

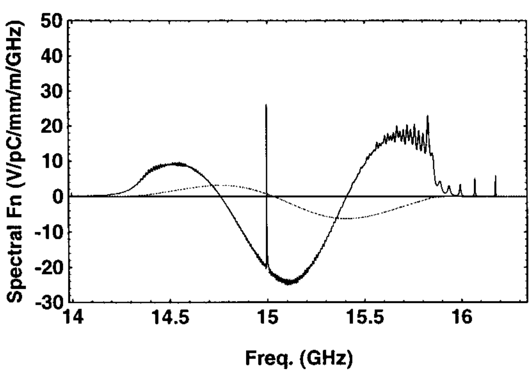

Fig 3: Even spectral function, W^c_{01} for matched HOM coupler with a bowed DDS.

For any given situation the wake functions themselves must be linearly combined and the result put in envelope form if desired. In general, it is our experience that the cosine wake terms are far smaller than the sine wake terms, and they are especially small as s vanishes, consistent with the expectation that precursor terms are small. Since W_{01} and W_{10} differ only in the sign of these otherwise equal cosine terms, they are very nearly equal.

Illustrated in fig 1. is the spectral function corresponding to Z_{01} for a beam travelling through the DDS at an angle. The features of the W_{00} are evident, namely under-coupled modes in the upper frequency end.

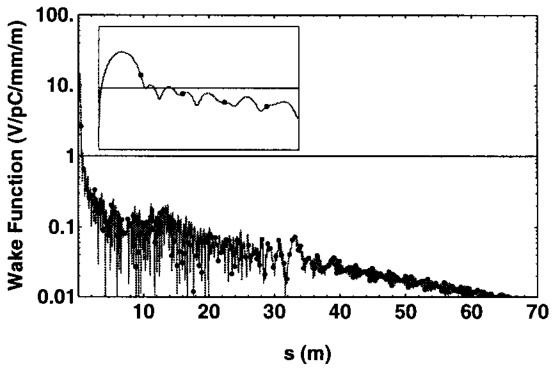

Fig 4: Sine wake envelope function for a bowed DDS

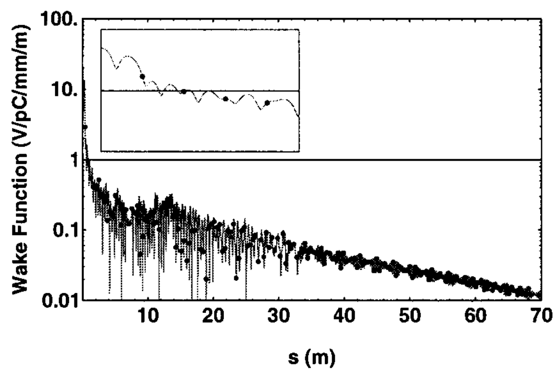

Fig 5: Sine wake envelope function corresponding to S_{11} for a beam travelling at an angle through the DDS

4. ACKNOWLEDGMENTS

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