MENLO_PARC, a program for $e^+e^- \to 4$ jets at next-to-leading order

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Abstract

MENLO_PARC is a general purpose Monte Carlo program for calculating the next-to-leading order $\mathcal{O}(\alpha_s^3)$ corrections to four-jet quantities in electron-positron annihilation. Any quantity which can be constructed out of the four jet momenta and which is insensitive to the quark helicities and the beam orientation can be computed. Both, virtual photon and Z intermediate states (and $\gamma\ Z$ interference) are included, hence the quantities can be computed for arbitrary center of mass energies. The cancellation of real and virtual singularities has been achieved using a general version of the subtraction method. The only approximation which has been made is the neglect of the light quark masses and terms which are suppressed by $1/m_{\mathrm{top}}^4$ or higher powers of the top quark mass.

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PROGRAM SUMMARY

Title of program: MENLO_PARC[1]

Catalogue number: ??

Program obtainable from: CPC Program Library, Queen's University of Belfast, N. Ireland (see application form in this issue)

Licensing provisions: none

Computer and operating system: Any system with a standard Fortran 90 compiler

Programming language used: Fortran 90

Memory required to execute with typical data: 1028 Kbytes

Number of bits in a word: ??

Has the code been vectorised ?: No

Number of lines in distributed program, including test data, etc: 10926

Keywords: electron-positron annihilation, next-to-leading order corrections, four jet, subtraction method

Nature of physical problem: The expansion in the strong coupling constant of any four-jet quantity in electron-positron annihilation starts only at $\mathcal{O}(\alpha_s^2)$. As a result, the dependence of the theoretical predictions on the renormalization scale is very large. The only reliable way to reduce this dependence and, therefore, get more precise predictions is to include the full next-to-leading order correction.

Method of solution: Recently, the calculations of the one-loop amplitudes for electron-positron annihilation into four partons have been completed, using the helicity method and color ordering [1,2]. Since the tree-level amplitudes with five partons in the final state have also been calculated by several groups a few years ago, all amplitudes which contribute to the next-to-leading order cross section are now available. A general version [3] of the subtraction method has been used to cancel the real and virtual singularities for arbitrary four-jet quantities. The integrand has been split into pieces which contain only a very limited number of square root singularities. Each of these contributions can then be reliably integrated.

Typical running time: Between hours and months, depending on the nature of the calculation and the desired accuracy of the result.

References: [1] Z. Bern, L. Dixon and D. Kosower, preprint SLAC–PUB–7529
[2] Z. Bern, L. Dixon, D. Kosower and S. Weinzierl, Nucl. Phys. B489 (1997) 3
[3] S. Frixione, Z. Kunszt and A. Signer, Nucl. Phys. B467 (1996) 399

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[1]Matrix Elements for Next-to-Leading Order PARton Calculations

LONG WRITE-UP

Several general purpose Monte Carlo programs have been written for the calculation of next-to-leading order corrections to three-jet events in electron-positron annihilation [1]. They have proven to be extremely useful for comparing theory with experiment. For instance, several measurements of the strong coupling constant α_s would have been impossible without these programs.

In the case of four-jet events, the situation was much less satisfactory. Until recently, only tree-level predictions were available [2]. However, since the perturbative expansion of these quantities starts only at $\mathcal{O}(\alpha_s^2)$ these results depend strongly on the choice of the renormalization scale. Thus, only for normalized quantities such as angular distributions [3] could more or less reliable theoretical predictions be obtained. These quantities do not suffer from the scale dependence, since the overall normalization does not enter the prediction.

In view of the importance of four-jet events at LEP2 and of the existing large four-jet data samples at the Z-pole it is certainly highly desirable to be able to get next-to-leading order predictions. First results have been presented in ref.[4]. They have been obtained with an earlier version of MENLO_PARC, which neglected subleading in color contributions. In the current version of the program all these subleading pieces are implemented and some result have been reported in ref.[5].

In theories with massless gauge bosons and in particular in QCD, the calculation of next-to-leading order corrections generally requires the cancellation of real and virtual singularities. It has become standard to regularize these singularities with dimensional regularization, i.e. by performing the calculation in $D \equiv 4 - 2\epsilon$ space-time dimensions. As a result, the infrared singularities show up as poles $1/\epsilon^i$ and after the cancellation of these poles, the limit $\epsilon \to 0$ can be safely made and, thus, the physical result is recovered.

One possible way to carry through this program is to use the subtraction [6] method. Several variations [7, 8] of this method have been developed which allow the computation of arbitrary next-to-leading order corrections once the corresponding amplitudes are known. The basic idea is the following. In order to make the phase-space integration of the real contribution finite, all singular limits are subtracted from the initial integrand. Of course, these subtracted pieces have to be added back. But since their analytic structure is simple, the phase space integration of these pieces can be done partly analytically. Thereby, the poles in ϵ are obtained and can be cancelled against the virtual poles. The remaining finite integrations can all be done numerically.

It is important to note that although the remaining integrals are finite, the integrands are not. In fact, one is usually left with square root singularities in the integrand. Such square root singularities can appear in many phase space points and, thus, require some care upon numerical integration. In the method used in MENLO_PARC [7] this has been achieved by decomposing the integrand into pieces, which contain only a very limited number of these singularities. Then, each contribution can be integrated separately in a reliable way. It has to be stressed that only the general structure of an infrared safe quantity has to be used to obtain this partitioning and that there is no loss in generality! In section 1 a brief overview of this method will be given.

The amplitudes which are needed for the calculation of four-jet quantities in electron-positron annihilation at tree level are $e^+e^- \to \bar{q}qgg$ and $e^+e^- \to \bar{q}q\bar{q}'q'$ [6]. For the one-loop

corrections to four-jet events we need not only the one-loop corrections to the amplitudes mentioned above [9, 10, 11], but also the tree-level amplitudes of the processes $e^+e^- \to \bar{q}qggg$ and $e^+e^- \to \bar{q}q\bar{q}'q'g$ [12, 13]. For the tree-level amplitudes, MENLO_PARC uses the results of ref. [12] and for the one-loop amplitudes the results of refs. [10, 11] are used. All these results have been obtained with the help of color ordering and the helicity method. The squaring of the amplitudes will always be done numerically.

1 A general version of the subtraction method

There are several general methods available [14, 7, 8] to obtain the cancellation of the soft singularities for next-to-leading order Monte Carlo programs. By general we mean that as soon as all needed amplitudes are known, the application of these algorithms is straightforward and in particular, no integrals have to be computed any more. This section gives a very short summary of the method developed in [7], specified to $\ell\bar{\ell}$ processes since MENLO_PARC is a straightforward implementation of this method. The reader is referred to [7] or the documentation of the program for more details.

Any next-to-leading order jet cross section is a sum of real and virtual contributions

$$d\sigma^{(1)} = d\sigma^{(r)} + d\sigma^{(v)} \tag{1}$$

where

$$d\sigma^{(r)}(p_1, p_2; \{J\}_{1,m}) = \sum_{I_R} \frac{1}{s_{I_R}} \mathcal{M}^{(m+1,0)}(p_1, p_2; \{k\}_{1,m+1}; \{a\}_{1,m+1}) \tag{2}$$

$$\mathcal{S}^{(m+1)}(\{k\}_{1,m+1}; \{J\}_{1,m}) \, d\phi_{m+1}(\{k\}_{1,m+1})$$

$$d\sigma^{(v)}(p_1, p_2; \{J\}_{1,m}) = \sum_{I_V} \frac{1}{s_{I_V}} \mathcal{M}^{(m,1)}(p_1, p_2; \{k\}_{1,m}; \{a\}_{1,m}) \tag{3}$$

$$\mathcal{S}^{(m)}(\{k\}_{1,m}; \{J\}_{1,m}) \, d\phi_m(\{k\}_{1,m})$$

In these equations $\{k\}_{1,m+1} \equiv \{k_1 \ldots k_{m+1}\}$ and $\{a\}_{1,m+1}$ denote the momenta and flavors of the outgoing partons respectively. The momenta of the incoming leptons p_1 and p_2 and the jets $\{J\}_{1,m}$ are often suppressed in the notation. The $m+1$-parton final state squared tree-level matrix elements, summed (averaged) over final (initial) spin and including the flux factor are denoted by $\mathcal{M}^{(m+1,0)}$. Similarly, $\mathcal{M}^{(m,1)}$ denotes the next-to-leading order contribution to the m-parton squared matrix elements, i.e. the interference term of the tree-level and one-loop amplitude. The sum is over all real (I_R) and virtual (I_V) processes and the dependence of $\mathcal{M}$ on the process is shown via its dependence on the final state flavor, while the symmetry factors s_{I_R} and s_{I_V} are shown explicitly. Finally, $\mathcal{S}$ denotes the measurement function, which will be discussed in the following subsection and by $d\phi_m$ we denote the m-body phase space in $D = 4 - 2\epsilon$ dimensions,

$$d\phi_m \equiv (2\pi)^D \delta^{(D)}\left(p_1 + p_2 - \sum_{l=1}^{m+1} k_l\right) \prod_{i=1}^{m} d\phi(i) \tag{4}$$

$$\equiv (2\pi)^D \delta^{(D)}\left(p_1 + p_2 - \sum_{l=1}^{m+1} k_l\right) \prod_{i=1}^{m} \frac{d^{D-1}k_i}{(2\pi)^{D-1} 2E_i}$$

$E_i = k_i^0$ denotes the energy of parton i.

1.1 The measurement function

The measurement function $\mathcal{S}^{(m)}$ specifies which quantity is calculated. The superscript (m) indicates how many partons enter this function. For an m-jet quantity at next-to-leading order only $\mathcal{S}^{(m)}$ and $\mathcal{S}^{(m+1)}$ play a role. In order to get the cancellation of singularities between the real and virtual contributions the measurement function has to fulfill the requirements of infrared safety. That is, $\mathcal{S}$ should be independent of whether or not a final state parton is split into two collinear partons and it should be independent of whether or not a soft parton is added to the final state. These requirements translate into

$$\lim_{i \to 0} \mathcal{S}^{(m+1)}(\{k\}_{1,m+1}) = \mathcal{S}^{(m)}(\{k\}_{1,m+1}^{[i]}) \tag{5}$$

$$\lim_{i \| j} \mathcal{S}^{(m+1)}(\{k\}_{1,m+1}) = \mathcal{S}^{(m)}(\{k\}_{1,m+1}^{[ij]}, k_{ij}) \tag{6}$$

where we introduced the notation $\{k\}_{1,m+1}^{[i]} \equiv \{k_1 \ldots k_{m+1}\} \backslash \{k_i\}$. Note that eqs.(5) and (6) imply that

$$\lim_{i \to 0} \mathcal{S}^{(m)}(\{k\}_{1,m}) = \lim_{i \| j} \mathcal{S}^{(m)}(\{k\}_{1,m}) = 0 \tag{7}$$

for any m-jet quantity.

Obviously, any measurement function can be decomposed according to

$$\mathcal{S}^{(m+1)} = \sum_{i,j} \mathcal{S}_{ij}^{(m+1)} \tag{8}$$

where the sum is over all pairs $\{ij\}$, $i \neq j$. $\mathcal{S}_{ij}^{(m+1)}$ vanishes in all phase space regions where the squared amplitude has a singularity except for $i \| j$ and $i \to 0$. This decomposition of the measurement function leads to a decomposition of the real cross section

$$d\sigma^{(r)} = \sum_{i,j} d\sigma_{ij}^{(r)} \tag{9}$$

with

$$d\sigma_{ij}^{(r)}(p_1, p_2; \{J\}_{1,m}) = \sum_{I_R} \frac{1}{s_{I_R}} \mathcal{M}^{(m+1,0)} \, \mathcal{S}_{ij}^{(m+1)} \, d\phi_{m+1} \tag{10}$$

where $d\sigma_{ij}^{(r)}$ is singular only for $i \| j$ and $i \to 0$.

1.2 The virtual cross section

The calculation of the virtual contributions requires the knowledge of the one-loop m-parton matrix elements. In spite of the complexity of these amplitudes, the structure of the divergent terms is very simple,

$$\mathcal{M}^{(m,1)}(\{k\}_{1,m}; \{a\}_{1,m}) = \tag{11}$$

$$\frac{\alpha_s}{2\pi} \, c_\Gamma \left[- \left(\frac{1}{\epsilon^2} \sum_{p=1}^{m} C(a_p) + \frac{1}{\epsilon} \sum_{p=1}^{m} \gamma(a_p) \right) \mathcal{M}^{(m,0)}(\{k\}_{1,m}; \{a\}_{1,m}) \right.$$

$$\left. + \frac{1}{2\epsilon} \sum_{p,q=1}^{m} \ln \frac{2k_p \cdot k_q}{\mu^2} \mathcal{M}_{pq}^{(m,0)}(\{k\}_{1,m}; \{a\}_{1,m}) + \mathcal{M}_{NS}^{(m,1)}(\{k\}_{1,m}; \{a\}_{1,m}) \right]$$

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In this equation, μ is the renormalization scale and the factor $c_\Gamma \equiv (4\pi)^\epsilon \, \Gamma(1 + \epsilon)\Gamma(1 - \epsilon)^2/\Gamma(1 - 2\epsilon)$ naturally results from the one-loop integrals. The $\mathcal{M}_{pq}^{(m,0)}$, usually called color-linked Born squared amplitudes[2], are symmetric in p and q. In eq. (11) we also made use of the flavor dependent quantities $C(a_p)$ and $\gamma(a_p)$; for the $SU(N_c)$ color group, they are

$$C(g) = C_A = N_c \qquad\qquad \gamma(g) = \frac{11C_A - 4T_F N_f}{6} \qquad\qquad (12)$$

$$C(q) = C_F = \frac{N_c^2 - 1}{2N_c} \qquad\qquad \gamma(q) = \frac{3}{2}C_F \qquad\qquad (13)$$

where $T_F = 1/2$ and N_f is the number of quark flavors. All the non-divergent terms in eq. (11) were collected in $\mathcal{M}_{NS}^{(m,1)}$. Usually, the calculations are carried out using helicity amplitude methods in the dimensional reduction (DR) scheme and, thus, they have to be converted into the conventional dimensional regularization (CDR) scheme [15].

1.3   Decomposition of the real cross section

The decomposition of the real contribution into single singular pieces, as discussed above, allows for a very clear disentangling of the soft and collinear singularities. To illustrate this point, consider a certain real process I_R. The contribution of this process to $d\sigma_{ij}^{(r)}$, as defined in eq.(10), is given by

$$d\sigma_{ij}^{I_R} = \frac{1}{s_{I_R}}\mathcal{M}^{(m+1,0)}(\{k\}_{1,m+1}; \{a\}_{1,m+1}) \, \mathcal{S}_{ij}^{(m+1)}(\{k\}_{1,m+1}) \, d\phi_{m+1}(\{k\}_{1,m+1}) \qquad (14)$$

As a first step, $d\sigma_{ij}^{I_R}$ can be decomposed into a piece $d\sigma_{ij}^{I_R,(s)}$ containing the soft singularity $i \to 0$, a piece $d\sigma_{ij}^{I_R,(c)}$ containing the collinear singularity $i\|j$ and a piece $d\sigma_{ij}^{I_R,(f)}$, which is free of any singularity. However, since $d\sigma_{ij}^{I_R,(f)}$ contains subtractions, it will generally have square root singularities. Thus, the numerical integration can be quite non-trivial. In fact, the whole purpose of splitting up the full cross section into single singular pieces $d\sigma_{ij}^{I_R}$ is done in order to be able to control the numerical integration of this part of the cross section.

1.3.1   Soft contribution

For the separation of the soft singularity it is convenient to use a set of independent variables which contains the energy scale ξ_i of the parton i. The phase space factor in $4-2\epsilon$ dimensions will then contain a factor $\xi_i^{1-2\epsilon}$ and the squared matrix element will have a singularity in the limit $\xi_i \to 0$. The basic idea is to shuffle this singularity into the phase space by multiplying the squared matrix element by ξ_i^2 and, therefore, render it finite in the soft limit. Obviously, this has to be corrected by replacing $\xi_i^{1-2\epsilon} \to \xi_i^{-1-2\epsilon}$ in the phase space. Then, by making use of the identity

$$\xi_i^{-1-2\epsilon} = -\frac{\xi_{\text{cut}}^{-2\epsilon}}{2\epsilon}\delta(\xi_i) + \left(\frac{1}{\xi_i}\right)_c - 2\epsilon\left(\frac{\ln \xi_i}{\xi_i}\right)_c + \mathcal{O}(\epsilon^2) \qquad (15)$$

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[2]Note that the normalization used here is different from [7].

the desired separation has been achieved. Indeed, we only have to define $d\sigma_{ij}^{I_R,(s)}$ to be this part of $d\sigma_{ij}^{I_R}$ which is obtained by taking the first term in eq.(15). The remaining part of $d\sigma_{ij}^{I_R}$, i.e. the one which is obtained by taking the last two terms of eq.(15) will be called $d\sigma_{ij}^{I_R,(ns)}$ and still contains the collinear singularity. In eq.(15), ξ_{cut} is an arbitrary parameter satisfying the condition $0 < \xi_{\text{cut}} \leq 1$, and the distributions on the r.h.s. are defined as follows:

$$\left\langle \left(\frac{1}{\xi_i}\right)_c, f \right\rangle = \int_0^1 d\xi_i \frac{f(\xi_i) - f(0)\theta(\xi_{\text{cut}} - \xi_i)}{\xi_i} \tag{16}$$

$$\left\langle \left(\frac{\ln \xi_i}{\xi_i}\right)_c, f \right\rangle = \int_0^1 d\xi_i \left[f(\xi_i) - f(0)\theta(\xi_{\text{cut}} - \xi_i)\right] \frac{\ln \xi_i}{\xi_i} \tag{17}$$

Thanks to the $\delta(\xi_i)$ function, all phase space integrations associated with parton i can be done analytically in $d\sigma_{ij}^{I_R,(s)}$. Performing these integrations and summing over the labels i and j we end up with

$$d\sigma^{I_R,(s)} = \frac{\alpha_s}{2\pi} \, \mathcal{S}^{(m)}(\{k\}_{1,m}) \, d\phi_m(\{k\}_{1,m}) \tag{18}$$
$$\frac{1}{s_{I_V}} \frac{1}{2} \sum_{pq} \left(\mathcal{I}_{pq}^{(\text{div})} + \mathcal{I}_{pq}^{(\text{reg})}\right) \, \mathcal{M}_{pq}^{(m,0)}(\{k\}_{1,m}; \{a\}_{1,m})$$

Here, s_{I_V} is the symmetry factor of the process I_V which is defined to be the process I_R with one gluon removed. The explicit form of the integrals is [7]

$$\mathcal{I}_{pq}^{(\text{div})} = \frac{c_\Gamma}{\epsilon^2} - \frac{c_\Gamma}{\epsilon} \left(\ln \frac{2k_q \cdot k_p}{\mu^2} - \ln \frac{4E_q E_p}{\xi_{\text{cut}}^2 S}\right) \tag{19}$$

$$\mathcal{I}_{pq}^{(\text{reg})} = \frac{1}{2} \ln^2 \frac{\xi_{\text{cut}}^2 S}{\mu^2} + \ln \frac{\xi_{\text{cut}}^2 S}{\mu^2} \ln \frac{k_q \cdot k_p}{2E_q E_p} - \text{Li}_2\left(\frac{k_q \cdot k_p}{2E_q E_p}\right)$$
$$+ \frac{1}{2} \ln^2 \frac{2k_q \cdot k_p}{E_q E_p} - \ln\left(4 - \frac{2k_q \cdot k_p}{E_q E_p}\right) \ln \frac{k_q \cdot k_p}{2E_q E_p} - 2\ln^2 2 \tag{20}$$

where S is the squared center of mass energy. Note that besides the expected soft poles there are some spurious poles, proportional to $\ln \xi_{\text{cut}}$ which will get cancelled by similar poles coming from the collinear region.

After the cancellation of the singularities, there remains a finite contribution from $d\sigma^{I_R,(s)}$. It can be obtained from eq.(18) by simply setting $\mathcal{I}_{pq}^{(\text{div})}$ to zero. After summation over all processes we find

$$d\sigma_{\text{fin}}^{(s)} = \frac{\alpha_s}{2\pi} \sum_{I_V} \frac{1}{s_{I_V}} \sum_{p<q} \mathcal{I}_{pq}^{(\text{reg})} \, \mathcal{M}_{pq}^{(m,0)}(I_V) \, \mathcal{S}^{(m)}(\{k\}_{1,m}; \{J\}_{1,m}) \, d\phi_m(\{k\}_{1,m}) \tag{21}$$

1.3.2 Collinear contribution

We still have to separate the collinear singularity $i\|j$ in $d\sigma_{ij}^{I_R,(ns)}$. This is done in a similar way as in the soft case. We choose a set of independent variables which contains y_i, the cosine of the angle between partons i and j. The collinear singularity of the squared matrix

6

element in the limit $y_i \to 1$ is moved into the phase space by multiplying the matrix element with $(1 - y_i)$. As a result, the phase space contains a factor $(1 - y_i)^{-1-\epsilon}$. Using the identity

$$(1 - y_j)^{-1-\epsilon} = -\frac{\delta^{-\epsilon}}{\epsilon}\delta(1 - y_j) + \left(\frac{1}{1 - y_j}\right)_\delta + \mathcal{O}(\epsilon) \tag{22}$$

which holds for $0 < \delta \leq 2$, we end up with $d\sigma_{ij}^{IR,(ns)} = d\sigma_{ij}^{IR,(c)} + d\sigma_{ij}^{IR,(f)}$.

Again, the integrations associated with the partons i and j in $d\sigma_{ij}^{IR,(c)}$ can be done analytically. Performing this integration and summing over all pairs i, j and all real processes we find the expected collinear singularities which cancel against the virtual poles as well as spurious poles which cancel against those spurious poles which come from the soft part. After this cancellation we are left with the finite piece

$$d\sigma_{\text{fin}}^{(c)} = -\frac{\alpha_s}{2\pi}\sum_{I_V}\sum_p \frac{\tilde{\mathcal{Z}}(a_p)}{s_{I_V}}\mathcal{M}^{(m,0)}(I_V)\,\mathcal{S}^{(m)}(\{k\}_{1,m}; \{J\}_{1,m})\,d\phi_m(\{k\}_{1,m}) \tag{23}$$

where

$$\tilde{\mathcal{Z}}(a_j) \equiv \gamma(a_j)\,\ln\frac{S\delta\xi_j^2}{2\mu^2} - \gamma'(a_j) - 2C(a_j)\,\ln\frac{\xi_j}{\xi_{\text{cut}}}\,\ln\frac{S\delta\xi_j\xi_{\text{cut}}}{2\mu^2} \tag{24}$$

We defined $\xi_j = 2E_j/\sqrt{S}$ and

$$\gamma'(g) = \frac{67}{9}C_A - \frac{2\pi^2}{3}C_A - \frac{23}{9}T_F N_f \tag{25}$$

$$\gamma'(q) = \frac{13}{2}C_F - \frac{2\pi^2}{3}C_F \tag{26}$$

1.3.3 The remaining finite contribution

Collecting all terms which are neither included in $d\sigma_{ij}^{IR,(s)}$ nor in $d\sigma_{ij}^{IR,(c)}$ we find

$$d\sigma_{ij}^{IR,(f)} = \left(\frac{1}{\xi_i}\right)_c\left(\frac{1}{1 - y_j}\right)_\delta\left(\xi_i^2(1 - y_j)\frac{1}{s_{I_R}}\mathcal{M}^{(m+1,0)}(\{k\}_{1,m+1}; \{a\}_{1,m+1})\right) \tag{27}$$

$$\times \mathcal{S}_{ij}^{(m+1)}(\{k\}_{1,m+1})\,d\tilde{\Phi}_{m+1}^{[ij]}$$

where $d\tilde{\Phi}_{m+1}^{[ij]}$ is a slightly modified integration over the phase space. The detailed form of this phase-space integration is not needed for the present discussion. The only important fact is that the integral is well defined and finite.

1.4 Numerical implementation

Finally we are in the position to put everything together and give a recipe for the calculation of the next-to-leading order corrections to any jet quantity. As already mentioned several times, there are several contribution which have to be included, namely $d\sigma_{\text{fin}}^{(v)}$, $d\sigma_{\text{fin}}^{(c)}$, $d\sigma_{\text{fin}}^{(s)}$ and $d\sigma^{(f)}$. Of course, all four parts are finite. Also, they may depend on the unphysical parameters δ and ξ_{cut}, however, this dependence has to cancel in the final answer. In the remainder of this subsection we briefly comment on all four parts.

1.4.1 The virtual part

Once the one-loop amplitudes are known, this part is simple to handle. All we have to calculate is the one-loop correction to the matrix element squared. This is the interference of the born amplitude with the one-loop amplitude. Thus, we need to numerically calculate

$$d\sigma_{\text{fin}}^{(v)} = \frac{\alpha_s}{2\pi} \sum_{I_V} \frac{1}{s_{I_V}} \mathcal{M}_{NS}^{(m,1)}(I_V) \, \mathcal{S}^{(m)}(\{k\}_{1,m}; \{J\}_{1,m}) \, d\phi_m(\{k\}_{1,m}) \tag{28}$$

where the sum is over all virtual processes and $\mathcal{M}_{NS}^{(m,1)}$ has been defined in eq.(11). Thanks to the measurement function, eq.(7), the integrand is finite over the whole phase space and, therefore, the numerical integration can easily be done.

1.4.2 The soft and collinear part

Our starting point is eq.(21). In order to get the soft part we have to integrate $d\sigma_{\text{fin}}^{(s)}$ over the phase space. Again, the measurement function makes this expression finite and thus the integration trivial. Very much the same comments apply to the collinear part. We merely have to use eq.(23) instead of eq.(21).

Since the structure of the virtual, soft and collinear part is very similar, it is convenient to put these three contributions together. Thus we define

$$
\begin{aligned}
d\sigma_{\text{fin}}^{(vsc)} &\equiv d\sigma_{\text{fin}}^{(v)} + d\sigma_{\text{fin}}^{(s)} + d\sigma_{\text{fin}}^{(c)} \\
&= \frac{\alpha_s}{2\pi} \sum_{I_V} \frac{1}{s_{I_V}} \mathcal{S}^{(m)}(\{k\}_{1,m}; \{J\}_{1,m}) \, d\phi_m(\{k\}_{1,m}) \\
&\quad \times \left[\mathcal{M}_{NS}^{(m,1)}(I_V) - \sum_p \tilde{\mathcal{Z}}(a_p) \mathcal{M}^{(m,0)}(I_V) + \sum_{p<q} \mathcal{I}_{pq}^{(\text{reg})} \, \mathcal{M}_{pq}^{(m,0)}(I_V) \right]
\end{aligned} \tag{29}
$$

1.4.3 The finite part

This is the only part which is problematic – the splitting of the cross section into several pieces has been performed solely to be able to control the numerics in this part of the computation. We are aiming to integrate numerically

$$d\sigma^{(f)} = \sum_{I_R} \sum_{i,j} d\sigma_{ij}^{I_R,(f)} \equiv \sum_{i,j} d\sigma_{ij}^{(f)} \tag{30}$$

where $d\sigma_{ij}^{I_R,(f)}$ is given in eq.(27). As noted previously, this integrand contains many square root singularities, but each single $d\sigma_{ij}^{I_R,(f)}$ has at most two problematic regions, namely when parton i becomes soft or collinear to parton j. Writing out explicitly the distributions which enter into eq.(27) we obtain

$$
\begin{aligned}
\left(\frac{1}{\xi_i}\right)_c \left(\frac{1}{1-y_j}\right)_\delta f(\xi_i, y_j, \ldots) = & \\
f(\xi_i, y_j, \ldots) &- \theta(\xi_{\text{cut}} - \xi_i) f(0, y_j, \ldots) \\
- \; \theta(\delta - 1 + y_j) f(\xi_i, 1, \ldots) &+ \theta(\xi_{\text{cut}} - \xi_i)\theta(\delta - 1 + y_j) f(0, 1, \ldots)
\end{aligned} \tag{31}
$$

This defines the subtracted function (squared matrix element). Since the problematic regions are well known and their number is very limited an adaptive Monte Carlo can do these integrations in a reliable way.

Now we are prepared to write the full next-to-leading order correction to a jet cross section:

$$d\sigma^{(1)} = d\sigma_{\text{fin}}^{(vsc)} + d\sigma^{(f)} \tag{32}$$

As a final remark we note that usually not all $d\sigma_{ij}^{I_R,(f)}$ have to be calculated. In fact, due to Bose symmetry $d\sigma_{ij}^{I_R,(f)}$ depends only on the flavors of the partons i and j. This results in a big reduction of computing time. Indeed, if we consider e.g. the real process $\bar{\ell}\ell \to \bar{q}(3)q(4)g(5)g(6))g(7)$ we have to compute only $d\sigma_{34}^{I_R,(f)}$, $d\sigma_{35}^{I_R,(f)}$, $d\sigma_{53}^{I_R,(f)}$ and $d\sigma_{56}^{I_R,(f)}$.

2 Organization and usage of the program

In this section a brief overview of the potential uses and restrictions of the program is given. For proper documentation and detailed instructions on how to use it, the reader is referred to the documentation.

2.1 The modules

The whole program is split up into several modules and two main programs, MENLO_PARC and MENLO. The former should be used for the calculation of single numbers (e.g. the four jet fraction for a certain jet definition and a certain choice of y_{cut}) whereas the latter has to be used for the calculation of distributions. In Fig. 1 a schematic diagram of the program is shown.

The module GLOBAL_DEF contains parameters and variables which should be easily accessible.

In addition to various auxiliary functions which are needed for the calculation of the tree-level and one-loop amplitudes, the module FUNCTIONS also contains the definition of the measurement functions.

The routines for the generation of the phase space for an arbitrary number of massless final state partons are programmed in the module PHASE_SPACE.

The purpose of the modules TREE_MAT_EL and LOOP_MAT_EL is to calculate all needed (squared) amplitudes. In the former are the functions which are needed for the calculation of the four and five-parton tree-level matrix elements (taken from ref. [12]), as well as their soft and collinear limits. The latter contains the one-loop corrections of the amplitudes (taken from refs. [10, 11]).

In the module LIMITS we find the needed soft, collinear and soft-collinear limits of the squared matrix elements and the measurement functions.

The previous modules contain all necessary procedures for constructing an arbitrary four-jet quantity or distribution. The construction of this function should be done in the module USER. This is the only module which has to be touched by the user of the program.

Finally, the module INTEGRANDS stores the basic functions which have to be integrated, i.e. $d\sigma_{\text{fin}}^{(vsc)}$ and $d\sigma^{(f)}$. These integrands are functions of a list of random numbers

and are ready to be integrated with VEGAS [16]. Thus the main result of all of the above modules is to provide us with these integrands.

Besides these modules the program contains a file with the integration routine VEGAS and two files with the main programs, MENLO_PARC and MENLO.

2.2 How to use the program

In electron-positron annihilation it has become standard to use cluster algorithms for defining jets. There one has to make two choices: (1) a cluster definition which is a measure of closeness d_{ij} that determines whether two partons i and j will be merged, and (2) a recombination scheme which is a description of how to construct the momentum of a proto-jet from the momenta of the two merging partons. Most of the common choices are predefined in the program (in the module FUNCTIONS). They are Durham, Jade, Geneva and Invariant Mass for the cluster definition and E, E0 and P for the recombination scheme.

There are basically two classes of applications. Either the user wants to compute a single number (e.g. the four-jet fraction, a moment of a certain distribution, etc.) or a binned distribution (in the Nachtmann-Reiter angle, etc.). In the former case he should use MENLO_PARC, in the latter case MENLO. In any case, the only module which has to be touched by the user (if he is using a predefined jet definition) is the module USER.

These two classes of applications are treated separately because the user is advised to use MENLO_PARC whenever possible. Since the adaption of the grid in the Monte Carlo integration is much more efficient if just a single number is computed, the results obtained with MENLO_PARC are generally more precise and more reliable than the results obtained with MENLO.

In order to tell the program what to compute the user has to do two things. First, he has to specify which quantity he wants to calculate. This is done by programming a certain function (QUANT for a single number, i.e. for use in MENLO_PARC or QUANTITY for one or more distributions, i.e. for use in MENLO) in the module USER. Second, the user has to set some input variables.

For constructing the quantity to be calculated, the program provides the four momenta of the jets, `jet1 ... jet4`. They are energy ordered, i.e. the energy of `jet1` (`jet4`) is the largest (smallest). Thus, the user can construct any jet quantity he desires and add any cut he wants. As a jet quantity it will automatically be infrared safe.

There are quantities whose perturbative expansion starts also at $\mathcal{O}(\alpha_s^2)$ but which can not be constructed out of the four jet momenta such as the D-parameter or the C-parameter for $C > 3/4$. These quantities have to be expressed directly through the parton momenta. In the current version of MENLO_PARC such quantities can no be calculated without modifying several modules.

2.3 Input variables

This subsection is not meant to contain a precise explanation of how to set all input variables. Its only purpose is to give the reader an overview of what can be done with the program. For more detailed instructions the user is referred to the documentation.

In order to define precisely what four-jet means, the user can choose the cluster algorithm (input variable jet_scheme) and recombination scheme (input variable rec_scheme) as well as the y_{cut} value (input variable ycut). Furthermore, an additional variable ycut_five has to be set. This is the y_{cut} variable which will be used for the first step in the cluster algorithm, i.e. for going from five to four partons. By setting ycut_five equal to one all events are forced to contain at most four jets, that is there are by definition no five jet events. In the original definition of cluster algorithms ycut_five is set equal to ycut, however, certain experimental analyses have made different choices.

Any four-jet quantity can be split into various color pieces. The input variable color_part specifies which color part will be calculated. At tree level, MENLO_PARC organizes this decomposition via the two different processes, i.e. $2q2g$ and $4q$ final state. Other possible choices are ertE to get the Pauli exchange terms, i.e. the E-terms of ref. [6] and axal to get the terms proportional to the axial coupling of the Z-boson. The one-loop correction to a four-jet cross section can be written as

$$
\sigma_{4-\text{jet}}^{(1)} = N_c^2(N_c^2 - 1)\Big[\sigma_4^{\text{nc}+2} + \frac{1}{N_c^2}\,\sigma_4^{\text{nc}\_0} + \frac{1}{N_c^4}\,\sigma_4^{\text{nc}-2}
$$
$$
+ \frac{N_f^2}{N_c^2}\,\sigma_4^{\text{nf}+2} + \frac{N_f}{N_c}\,\sigma_4^{\text{nf}+1} + \frac{N_f}{N_c^3}\,\sigma_4^{\text{nf}-1}
$$
$$
+ \sigma_4^{\text{ertE}} + \sigma_4^{\text{vect}} + \sigma_4^{\text{axal}}\Big]
\tag{33}
$$

where N_c (N_f) is the number of colors (light quark flavors). With the input variable color_part it is possible to pick each contribution separately. σ_4^{vect} and σ_4^{axal} contain the "light-by-glue" terms which are proportional to the vector and axial coupling of the Z-boson respectively. Some generic cut diagrams for these terms are shown in Fig. 2. Note that in the Born case, due to Furry's theorem the vect part vanishes under integration if a charge-blind quantity is computed.

As discussed in section 1, eq.(30) the real contribution to (a certain color part of) any four-jet like quantity can be decomposed into single singular pieces $d\sigma_{ij}^f$. The input variables i_jet and j_jet in principle allow one to choose each of these pieces separately although only the sum of all contributions has a physical meaning. In addition to the standard choices $3 \le i, j \le 7$; $i \ne j$ there are some special choices available which allow one to compute the sum of all contributions at once, as well as four- and five-jet tree-level quantities.

Furthermore, the user can choose the renormalization scale with musq_fac, the seed for the random number generator with ran_seed and the number of points and iterations for the adaption of the grid and the actual calculation. Finally, the unphysical parameters δ and ξ_{cut} as defined in eqs.(15) and (22) can be specified. This allows one to check the independence of the final result on these two parameters and thus gives a hint about the reliability of the error estimate given by the program.

2.4 Approximations and restrictions

The whole calculation is done on the purely partonic level, i.e. parton showers and hadronization effects have not been included. Furthermore, the masses of the light quarks (u, d, s, c, b)

have been neglected. For running at or above the Z-pole this is usually a very good approximation although the inclusion of the b-quark mass at tree-level can give corrections which are not always completely negligible [17]. In the axial part there are terms which are suppressed by the mass of the top quark m_{top}. In MENLO_PARC terms which are suppressed by $1/m_{\text{top}}^2$ are included but $1/m_{\text{top}}^4$ terms and higher powers are neglected, assuming that all other kinematic invariants are smaller than m_{top}. Besides neglecting the quark masses, there is no approximation at all associated with the phase space integration.

There are some restrictions which have to be made for the quantity to be computed. First of all it is assumed that it does not depend on the helicities of the quarks. Secondly, it should not be sensitive to correlations between the beam directions and the final state hadrons. These restrictions are made primarily so that the electroweak structure can be factored out.

The coupling of the intermediate gauge boson to the primary quarks or the leptons is of the form $\gamma^\mu(v + a\gamma_5) = \gamma^\mu(v_L\mathcal{P}_L + v_R\mathcal{P}_R)$. Here $v_L(v_R)$ is the left(right) handed coupling and $\mathcal{P}_{L/R} = \frac{1}{2}(1 \mp \gamma_5)$. The couplings are:

$$
e_L = \frac{-1 + 2\sin^2\theta_W}{\sin 2\theta_W} \qquad e_R = \frac{2\sin^2\theta_W}{\sin 2\theta_W}
$$

$$
u_L = \frac{1 - \frac{4}{3}\sin^2\theta_W}{\sin 2\theta_W} \qquad u_R = \frac{-\frac{4}{3}\sin^2\theta_W}{\sin 2\theta_W}
$$

$$
d_L = \frac{-1 + \frac{2}{3}\sin^2\theta_W}{\sin 2\theta_W} \qquad d_R = \frac{\frac{2}{3}\sin^2\theta_W}{\sin 2\theta_W}
$$

(34)

To understand the structure of the squared matrix elements better, consider the hadronic tensor $H^{\mu\nu}$. It has to fulfill $H^{\mu\nu} = H^{*\,\nu\mu}$ and, thus, can schematically be written as

$$
H^{\mu\nu} \sim v_q^2 H_v^{\mu\nu} + a_q^2 H_a^{\mu\nu} + v_q a_q i H_2^{\mu\nu}
$$

(35)

The H_i are functions of the momenta p_i of the final state partons and $g^{\mu\nu}$ and the Levi-Civita tensor. H_v and H_a are symmetric and H_2 is antisymmetric under the exchange of μ and ν. The explicit factor i in front of $H_2^{\mu\nu}$ comes from the fact that in diagrams contributing to $H_2^{\mu\nu}$ there is exactly one γ_5 matrix present.

Neglecting electroweak corrections and lepton polarization, the leptonic tensor is

$$
L^{\mu\nu} \sim (v_l^2 + a_l^2)(p_\ell^\mu p_\ell^\nu + p_\ell^\nu p_\ell^\mu - g^{\mu\nu}p_{\bar\ell} \cdot p_\ell)\, L_1 + 2v_l a_l\, i\epsilon^{\mu\nu\rho\sigma}p_\ell^\rho p_\ell^\sigma L_2
$$

(36)

Thus, multiplying the two tensors with each other results in three non-vanishing terms,

$$
T_1 \;\sim\; v_q^2(v_l^2 + a_l^2)H_v^{\mu\nu}L_1
$$

(37)

$$
T_2 \;\sim\; a_q^2(v_l^2 + a_l^2)H_a^{\mu\nu}L_1
$$

(38)

$$
T_3 \;\sim\; a_q v_q a_l v_l \epsilon^{\mu\nu\rho\sigma}p_{\bar\ell}^\rho p_\ell^\sigma H_2^{\mu\nu}L_2
$$

(39)

The term T_3 contains a factor $\epsilon^{\mu\nu\rho\sigma}p_{\bar\ell}^\rho p_\ell^\sigma$, which is odd under $\ell \leftrightarrow \bar\ell$. As long as the physical quantity under consideration is independent under $\ell \leftrightarrow \bar\ell$, T_3 does not contribute to the cross section and, therefore, these terms are not implemented in MENLO_PARC.

Furthermore, we observe that $T_1/v_q^2 = T_2/a_q^2$ as long as both intermediate gauge bosons couple to the same quark loop. This can easily be seen by considering a generic cut diagram as shown in Fig. 3, anticommuting one γ_5 matrix over to the other γ_5 and using $\gamma_5^2 = 1$. If we restrict ourselves to quantities which do not depend on the quark helicities nor on the beam orientation, (i.e. quantities which are invariant under $\ell \leftrightarrow \bar{\ell}$) we can simply multiply the squared matrix element with the overall coupling factor

$$f_{\text{standard}} = \left(\sum Q_q^2\right) + \frac{1}{4}(e_L^2 + e_R^2)(N_u(u_L^2 + u_R^2) + N_d(d_L^2 + d_R^2)) \, |P_Z(s)|^2 \tag{40}$$
$$- \frac{1}{2}(e_L + e_R)\left(\frac{2}{3}N_u(u_L + u_R) - \frac{1}{3}N_d(d_L + d_R)\right) \, Re(P_Z(s))$$

The first two terms correspond to the photon and Z-boson exchange respectively, whereas the third term comes from the *gamma Z* interference term, and

$$P_Z(s) = \frac{s}{s - M_Z^2 + i\Gamma_Z M_Z} \tag{41}$$

where M_Z and Γ_Z are the mass and the width of the Z-boson. The number of light up (down) type flavors is denoted by N_u (N_d).

A special role is played by diagrams where the two intermediate gauge bosons couple to different quark loops ("light-by-glue" terms). Some examples are shown in Fig. 2. First of all, these are the only diagrams where the axial part differs from the vector part. Note that in the axial case, we get a cancellation between the two massless members of an iso-doublet. As a result, only the b-t doublet contribution survives, thanks to its large mass splitting. As mentioned previously, we keep terms proportional to $1/m_{\text{top}}^2$, but neglect terms which are suppressed by higher powers of m_{top}. These "light-by-glue" terms get a different coupling constant prefactor. In the case of T_1 it is

$$f_{\text{vector}} = \left(\sum Q_q\right)^2 + \frac{1}{8}(e_L^2 + e_R^2)\left(N_u(u_L + u_R) + N_d(d_L + d_R)\right)^2 \, |P_Z(s)|^2 \tag{42}$$
$$- \frac{1}{2}(e_L + e_R)\sum Q_q\left(N_u(u_L + u_R) + N_d(d_L + d_R)\right) \, Re(P_Z(s))$$
$$\tag{43}$$

whereas in the case of T_2 the prefactor is

$$f_{\text{axial}} = \frac{1}{8}\frac{e_L^2 + e_R^2}{\sin^2 2\theta_W} \, |P_Z(s)|^2 \tag{44}$$

The coupling factors are slightly modified in the case of polarized electron beams. In fact, the only modification in this case is

$$(e_L^2 + e_R^2) \rightarrow ((1 - \wp)e_L^2 + (1 + \wp)e_R^2) \tag{45}$$
$$(e_L + e_R) \rightarrow ((1 - \wp)e_L + (1 + \wp)e_R)$$

where $\wp$ denotes the polarization of the electrons, i.e. $\wp = 1$ (-1) for right (left)-handed polarization, and for unpolarized electrons we have $\wp = 0$.

Acknowledgement

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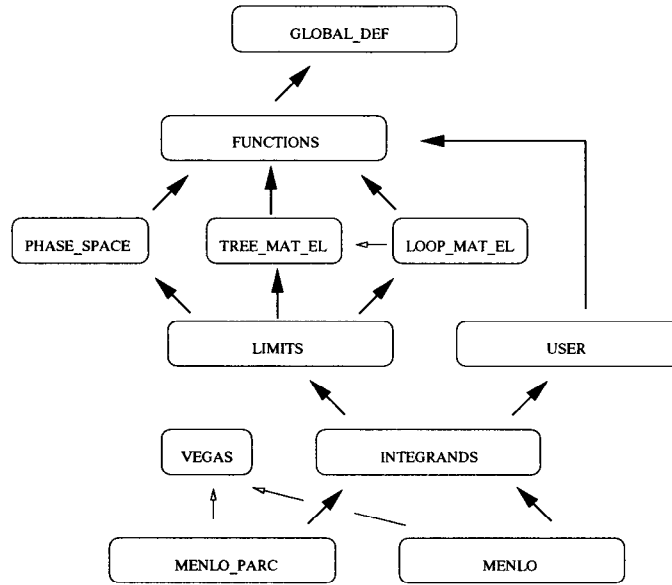
Structure of the program

Figure 1: Structure of the program MENLO_PARC and MENLO. Bold arrows indicate the "using" of other modules, small arrows indicate partial "using".

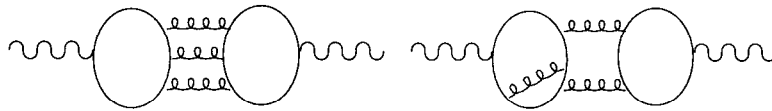

Figure 2: "Light-by-glue" cut diagrams whose axial part differs from the vector part.
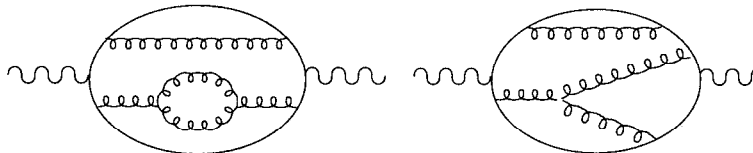

Figure 3: Some generic cut diagrams for next-to-leading order four-jet production. Each diagram can be cut in several ways.