

SLAC-PUB-7520
hep-ph/9705369
May 1997

CP Violation in Neutral Kaon Decays *

G. BUCHALLA

*Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309, U.S.A.*

Abstract

A brief review of the theoretical status of CP violation in decays of neutral kaons is presented. We focus on three important topics: ϵ , ϵ'/ϵ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$.

*Invited Talk presented at the Symposium on
Flavor-Changing Neutral Currents, Santa Monica, California, February 19–21, 1997*

hep-ph/9705369 20 May 1997

*Work supported by the Department of Energy under contract DE-AC03-76SF00515.

1 Introduction

The phenomenon of CP violation is of great current interest in particle physics. It defines an absolute, physical distinction between matter and antimatter and is a necessary condition for the generation of a baryon asymmetry in the universe. Furthermore, studying CP violation tests our understanding of flavordynamics. This sector of the Standard Model (SM) is clearly the most complicated, involving spontaneous breaking of electroweak symmetry and containing most of the free parameters of the model, including quark masses, CKM angles and the CP violating phase. In order to address these fundamental questions experimentally, neutral kaons have proved to be a key tool. In fact, until today CP violation has been exclusively observed in a few decay modes of the long lived neutral kaon ($K_L \rightarrow \pi\pi, \pi l\nu, \pi^+\pi^-\gamma$). All of the observed effects are accounted for by a single complex parameter ε , consistent with CP violation in the mass matrix only. The unambiguous demonstration of direct CP violation in $K_L \rightarrow \pi\pi$, measured by ε'/ε , is so far still elusive. The topic is currently still under active investigation. Beyond that, the field of neutral kaon CP violation will continue to offer interesting opportunities in the future. Among the possibilities that have been discussed are the rare decay $K_L \rightarrow \pi^0 e^+ e^-$, muon polarization in $K_L \rightarrow \mu^+ \mu^-$, and in particular the ‘gold-plated’ mode $K_L \rightarrow \pi^0 \nu \bar{\nu}$. The latter is theoretically extremely clean and offers excellent prospects for high precision flavor physics. In the following we will briefly summarize the theoretical status of ε , ε'/ε and $K_L \rightarrow \pi^0 \nu \bar{\nu}$, three main topics in the study of CP violation with neutral kaons. A more complete account and detailed references may be found in [1].

2 Indirect CP Violation in $K^0 \rightarrow \pi\pi$: ε

The parameter ε is determined by the imaginary part of the element M_{12} in the neutral kaon mass matrix, which in turn is generated by the usual $\Delta S = 2$ box-diagrams. The low energy effective Hamiltonian contains a single operator $(\bar{d}s)_{V-A}(\bar{d}s)_{V-A}$ in this case and one obtains

$$\varepsilon = e^{i\frac{\pi}{4}} \frac{G_F^2 M_W^2 f_K^2 m_K}{12\pi^2 \sqrt{2} \Delta M_K} B_K \cdot \text{Im} \left[\lambda_c^{*2} S_0(x_c) \eta_1 + \lambda_t^{*2} S_0(x_t) \eta_2 + 2\lambda_c^* \lambda_t^* S_0(x_c, x_t) \eta_3 \right] \quad (1)$$

Here $\lambda_i = V_{is}^* V_{id}$, $f_K = 160 \text{ MeV}$ is the kaon decay constant and the bag parameter B_K is defined by

$$B_K = B_K(\mu) [\alpha_s^{(3)}(\mu)]^{-2/9} \left[1 + J_3 \alpha_s^{(3)}(\mu) / (4\pi) \right] \quad (2)$$

where $\langle K^0 | (\bar{d}s)_{V-A} (\bar{d}s)_{V-A} | \bar{K}^0 \rangle \equiv 8/3 B_K(\mu) f_K^2 m_K^2$. The index (3) in eq. (2) refers to the number of flavors in the effective theory and $J_3 = 307/162$ (in the NDR scheme).

The Wilson coefficient multiplying B_K in (1) consists of a charm contribution, a top contribution and a mixed top-charm contribution. It depends on the quark masses, $x_i \equiv m_i^2/M_W^2$, through the functions S_0 . The η_i are the corresponding short-distance QCD correction factors (which depend only slightly on quark masses). Numerical values for η_1 , η_2 and η_3 are summarized in Table 1.

ε is dominated by the top contribution ($\sim 70\%$). It is therefore rather satisfying that the related short distance part $\eta_2 S_0(x_t)$ is theoretically extremely well under control, as can be seen in Table 1. Note in particular the very small scale ambiguity at NLO, $\pm 0.4\%$ (for

Table 1: *NLO results for η_i with $\Lambda_{\overline{MS}}^{(4)} = (325 \pm 110) \text{ MeV}$, $m_c(m_c) = (1.3 \pm 0.05) \text{ GeV}$, $m_t(m_t) = (170 \pm 15) \text{ GeV}$. The third column shows the uncertainty due to the errors in $\Lambda_{\overline{MS}}$ and quark masses. The fourth column indicates the residual renormalization scale uncertainty at NLO in the product of η_i with the corresponding mass dependent function from eq. (1). The central values of the QCD factors at LO are also given for comparison.*

| | NLO(central) | $\Lambda_{\overline{MS}}, m_q$ | scale dep. | NLO ref. | LO(central) |
|----------|--------------|--------------------------------|-------------|----------|-------------|
| η_1 | 1.38 | $\pm 35\%$ | $\pm 15\%$ | [2] | 1.12 |
| η_2 | 0.574 | $\pm 0.6\%$ | $\pm 0.4\%$ | [3] | 0.61 |
| η_3 | 0.47 | $\pm 3\%$ | $\pm 7\%$ | [4] | 0.35 |

$100 \text{ GeV} \leq \mu_t \leq 300 \text{ GeV}$). This intrinsic theoretical uncertainty is much reduced compared to the leading order result where it would be as large as $\pm 9\%$.

The η_i factors and the hadronic matrix element are not physical quantities by themselves. When quoting numbers it is therefore essential that mutually consistent definitions are employed. The factors η_i described here are to be used in conjunction with the so-called scheme- (and scale-) invariant bag parameter B_K introduced in (2). The last factor on the rhs of (2) enters only at NLO. As a numerical example, if the (scale and scheme dependent) parameter $B_K(\mu)$ is given in the NDR scheme at $\mu = 2 \text{ GeV}$, then (2) becomes $B_K = B_K(\text{NDR}, 2 \text{ GeV}) \cdot 1.31 \cdot 1.05$.

The quantity B_K has to be calculated by non-perturbative methods. Large N_C expansion techniques for instance find values $B_K = 0.75 \pm 0.15$. The results obtained in other approaches are reviewed in [1]. Ultimately a first principles calculation should be possible within lattice gauge theory. Ref. [5] quotes an estimate of $B_K(\text{NDR}, 2 \text{ GeV}) = 0.66 \pm 0.02 \pm 0.11$ in full QCD. The first error is the uncertainty of the quenched calculation. It is quite small already and illustrates the progress achieved in controlling systematic uncertainties in lattice QCD [5, 6]. The second error represents the uncertainties in estimating the effects of quenching and non-degenerate quark masses.

Phenomenologically ε is used to determine the CKM phase δ . The relevant input parameters are B_K , m_t , V_{cb} and $|V_{ub}/V_{cb}|$. A typical analysis of constraints on CKM parameters from ε can be found for instance in [1].

3 Direct CP Violation in $K^0 \rightarrow \pi\pi$: ε'/ε

The theoretical expression for ε'/ε can be written as

$$\frac{\varepsilon'}{\varepsilon} = \frac{\omega G_F}{2|\varepsilon| \text{Re}A_0} \text{Im}\lambda_t \left(y_6 \langle Q_6 \rangle_0 - \frac{1}{\omega} y_8 \langle Q_8 \rangle_2 + \dots \right) \quad (3)$$

where, for the purpose of illustration we kept only the numerically dominant terms. Here y_i are Wilson coefficients, $\langle Q_i \rangle_{0,2} \equiv \langle \pi\pi(I=0,2) | Q_i | K^0 \rangle$, $A_{0,2}$ are the $K^0 \rightarrow \pi\pi(I=0,2)$ amplitudes and $\omega = \text{Re}A_2/\text{Re}A_0$. The operator Q_6 originates from gluonic penguin diagrams and Q_8 from electroweak contributions. The matrix elements of Q_6 and Q_8 have the form $\langle Q_6 \rangle_0 \sim B_6/m_s^2$ and $\langle Q_8 \rangle_2 \sim B_8/m_s^2$, where B_6 and B_8 are bag parameters. $y_6 \langle Q_6 \rangle_0$ and

$y_8\langle Q_8\rangle_2$ are positive. The value for ε'/ε in (3) is thus characterized by a cancellation of competing contributions. Since the second contribution is an electroweak effect, suppressed by $\sim \alpha/\alpha_s$ compared to the leading gluonic penguin $\sim \langle Q_6\rangle_0$, it could appear at first sight that it should be altogether negligible for ε'/ε . However, a number of circumstances actually conspire to systematically enhance the electroweak effect so as to render it a very important contribution. First, unlike Q_6 , which is a pure $\Delta I = 1/2$ operator, Q_8 can give rise to the $\pi\pi(I = 2)$ final state and thus yield a nonvanishing isospin-2 component in the first place. Second, the $\mathcal{O}(\alpha/\alpha_s)$ suppression is largely compensated by the factor $1/\omega \approx 22$ in (3), reflecting the $\Delta I = 1/2$ rule. Third, $\langle Q_8\rangle_2$ is somewhat enhanced relative to $\langle Q_6\rangle_0$, which vanishes in the chiral limit. Finally, $-y_8\langle Q_8\rangle_2$ gives a negative contribution to ε'/ε that strongly grows with m_t [7, 8]. For the actual top mass value it is quite substantial. The Wilson coefficients y_i have been calculated at NLO [9, 10]. The short-distance part is therefore quite well under control. The remaining problem is then the computation of matrix elements, in particular $\langle Q_6\rangle_0$ and $\langle Q_8\rangle_2$. The cancellation between their contributions enhances the relative sensitivity of ε'/ε to the anyhow uncertain hadronic parameters which makes a precise calculation of ε'/ε impossible at present. In a recent analysis Buras et al. [11] find for $(\varepsilon'/\varepsilon)/10^{-4}$

$$7.4 \pm 8.6 \text{ (s)} \quad 3.6 \pm 3.4 \text{ (g)} \quad m_s(2\text{GeV}) = (129 \pm 18)\text{MeV} \quad (4)$$

$$21.5 \pm 21.5 \text{ (s)} \quad 10.4 \pm 8.3 \text{ (g)} \quad m_s(2\text{GeV}) = (86 \pm 17)\text{MeV} \quad (5)$$

Here (g) refers to the assumption of a Gaussian distribution of errors in the input parameters, (s) to the more conservative ‘scanning’ of parameters over their full allowed ranges. The lower values for the strange quark mass m_s in (5) correspond to recent lattice results [15, 16]. Within the rather large uncertainties (5) is compatible with experiment, which gives $(23 \pm 7) \cdot 10^{-4}$ (CERN-NA31) and $(7.4 \pm 5.9) \cdot 10^{-4}$ (FNAL-E731). On the other hand, (4) is consistent with E731, but somewhat low compared to NA31. Similar results have been obtained by other authors [12, 13, 14].

In conclusion, the SM prediction for ε'/ε suffers from large hadronic uncertainties, reinforced by substantial cancellations between the $I = 0$ and $I = 2$ contributions. Despite this problem, the characteristic pattern of CP violation observed in $K \rightarrow \pi\pi$ decays, namely $\varepsilon = \mathcal{O}(10^{-3})$ and $\varepsilon' = \mathcal{O}(10^{-6})$ (or below), is well accounted for by the standard theory, which can be considered a non-trivial success of the model.

On the experimental side a clarification of the current situation is to be expected from the upcoming new round of ε'/ε experiments conducted at Fermilab (E832), CERN (NA48) and Frascati (KLOE). The goal is a measurement of ε'/ε at the 10^{-4} level. The demonstration that $\varepsilon' \neq 0$ would constitute a qualitatively new feature of CP violation and as such be of great importance. However, due to the large uncertainties in the theoretical calculation, a quantitative use of this result for the extraction of CKM parameters will unfortunately be severely limited. For this purpose one has to turn to theoretically cleaner observables.

4 $K_L \rightarrow \pi^0 \nu \bar{\nu}$

The rare decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is a very attractive probe of flavordynamics. In particular, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is a manifestation of *large direct CP violation* in the SM. A small effect from in-

direct CP violation related to the kaon ε -parameter contributes below $\sim 1\%$ in the branching ratio and is therefore negligible.

In addition to having this phenomenologically interesting feature, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ can be calculated as a function of fundamental SM parameters with exceptionally small theoretical error. The main reasons are the hard GIM suppression of long distance contributions [17], and the semileptonic character, which allows us to extract the hadronic matrix element $\langle \pi^0 | (\bar{s}d)_V | K^0 \rangle$ from $K^+ \rightarrow \pi^0 e \nu$ decay using isospin symmetry. As a consequence $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is based on a purely short-distance dominated flavor-changing neutral current, which is reliably calculable in perturbation theory. The CP properties help to further improve the theoretical accuracy, rendering even the charm contribution completely negligible so that the clean top contribution fully dominates the decay. Next-to-leading QCD effects have been calculated and reduce the leading order scale ambiguity of $\sim \pm 10\%$ to an essentially negligible $\sim \pm 1\%$ [18]. Isospin breaking corrections in the extraction of the matrix element have also been evaluated. They lead to an overall reduction of the branching ratio by 5.6% [19]. As a result of all these developments, the theoretical uncertainty in $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is safely below 2%.

The quantity $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ offers probably the best accuracy in determining $\text{Im}V_{ts}^* V_{td}$ or, equivalently, the Jarlskog parameter $J_{CP} = \text{Im}(V_{ts}^* V_{td} V_{us} V_{ud}^*)$. The prospects here are even better than for B physics at the LHC, assuming a $\pm 10\%$ measurement of $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ at about the central value of SM predictions [20]. The SM expectation for the branching ratio [21] is $(2.8 \pm 1.7) \cdot 10^{-11}$, where the uncertainty is due to our imprecise knowledge of CKM parameters. The current upper bound from direct searches [22] is $5.8 \cdot 10^{-5}$. An indirect upper bound, using the current limit on $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ [23] and isospin symmetry, can be placed [24] at $1.1 \cdot 10^{-8}$. Several activities are under way aiming for an actual measurement of $K_L \rightarrow \pi^0 \nu \bar{\nu}$. A proposal exists at Brookhaven (BNL E926) to measure this decay at the AGS with a sensitivity of $\mathcal{O}(10^{-12})$. There are furthermore plans to pursue this mode with comparable sensitivity at Fermilab and KEK. More details can be found in the contributions by D. Bryman, T. Nakaya, K. Arisaka and T. Inagaki to these proceedings.

5 Summary

Decays of neutral kaons provide the only instance where CP violation has been observed to date. The quantity ε measures indirect CP violation, it is experimentally very precisely known and leads to important constraints on CKM parameters. The search for direct CP violation in $K_L \rightarrow \pi \pi$, measured by ε'/ε , is still ongoing. The SM prediction for this quantity is plagued by large hadronic uncertainties, which severely limits the possibility of extracting useful information on CKM quantities from this observable. Eventually, a measurement of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ would open up exciting prospects for precision studies in flavor physics, complementary and competitive to CP violation studies in B decays. Theoretical progress has been achieved on all three topics, ε , ε'/ε and $K_L \rightarrow \pi^0 \nu \bar{\nu}$, in particular through the calculation of next-to-leading order QCD effects.

The study of CP violation in neutral kaon decays has yielded crucial insight into fundamental physics in the past and it is still under active investigation at present. Excellent opportunities, as those provided by $K_L \rightarrow \pi^0 \nu \bar{\nu}$, continue to exist for the future.

References

- [1] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996).
- [2] S. Herrlich and U. Nierste, *Nucl. Phys.* **B419**, 292 (1994).
- [3] A.J. Buras, M. Jamin and P.H. Weisz, *Nucl. Phys.* **B347**, 491 (1990).
- [4] S. Herrlich and U. Nierste, *Nucl. Phys.* **B476**, 27 (1996).
- [5] S.R. Sharpe, Talk given at Lattice 96, St. Louis, MO, hep-lat/9609029.
- [6] S. Aoki *et al.*, Talk given at Lattice 96, St. Louis, MO, hep-lat/9608134.
- [7] J.M. Flynn and L. Randall, *Phys. Lett.* **B224**, 221 (1989); erratum *ibid.* **B235**, 412 (1990).
- [8] G. Buchalla, A.J. Buras and M.K. Harlander, *Nucl. Phys.* **B337**, 313 (1990).
- [9] A.J. Buras *et al.*, *Nucl. Phys.* **B400**, 37 (1993); A.J. Buras, M. Jamin and M.E. Lautenbacher, *Nucl. Phys.* **B400**, 75 (1993).
- [10] M. Ciuchini *et al.*, *Nucl. Phys.* **B415**, 403 (1994).
- [11] A.J. Buras, M. Jamin and M.E. Lautenbacher, *Phys. Lett.* **B389**, 749 (1996).
- [12] M. Ciuchini *et al.*, *Z. Phys.* **C68**, 239 (1995).
- [13] S. Bertolini, J.O. Eeg and M. Fabbrichesi, *Nucl. Phys.* **B476**, 225 (1996).
- [14] J. Heinrich *et al.*, *Phys. Lett.* **B279**, 140 (1992); E.A. Paschos, DO-TH 96/01, talk presented at the 27th Lepton-Photon Symposium, Beijing, China (1995).
- [15] R. Gupta and T. Bhattacharya, hep-lat/9605039.
- [16] B.J. Gough *et al.*, hep-lat/9610223.
- [17] D. Rein and L.M. Sehgal, *Phys. Rev.* **D39**, 3325 (1989).
- [18] G. Buchalla and A.J. Buras, *Nucl. Phys.* **B400**, 225 (1993).
- [19] W. Marciano and Z. Parsa, *Phys. Rev.* **D53**, R1 (1996).
- [20] G. Buchalla and A.J. Buras, *Phys. Rev.* **D54**, 6782 (1996).
- [21] A.J. Buras, M. Jamin and M.E. Lautenbacher, to appear.
- [22] M. Weaver *et al.*, *Phys. Rev. Lett.* **72**, 3758 (1994).
- [23] S. Adler *et al.*, *Phys. Rev. Lett.* **76**, 1421 (1996).
- [24] Y. Grossman and Y. Nir, *Phys. Lett.* **B398**, 163 (1997); Y. Grossman, these proceedings.