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Ideas for Fast Accelerator Model Calibration*

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Abstract

With the advent of a simple matrix inversion technique, measurement-based storage ring modeling has made rapid progress in recent years. Using fast computers with large memory, the matrix inversion procedure typically adjusts up to 10^3 model variables to fit the order of 10^5 measurements. The results have been surprisingly accurate. Physics aside, one of the next frontiers is to simplify the process and to reduce computation time. In this paper, we discuss two approaches to speed up the model calibration process: recursive least-squares fitting and a piecewise fitting approach.

1 CONVENTIONAL APPROACH

Some of the first accelerator model fitting routines were based on numerical adjustment of model parameters to make simulated orbit perturbation data match the measured data [1]. Originally known as the 'GOLD' method [2], model calibration concentrated on one section of the lattice at a time. With the GOLD method, the operator first identified parts of the lattice where the model agreed with the measured data. The next step was to vary model parameters near any discontinuities until the model-simulated trajectories matched the measurements. The power of the GOLD method is twofold: (1) hardware errors are quickly identified, and (2) once the model fields are found optical parameters can be predicted throughout. The same technique can applied to both perturbation orbit and absolute orbit measurements.

'Multi-track fitting' increases the accuracy of the result. With multi-track fitting, a set of orbit perturbations made by different dipole kicks provides many self-consistent constraints for the analysis. In effect, with each new trajectory the particle beam makes an independent probe of the spatial field structure of the lattice. Multi-track fitting was initially carried out with RESOLVE [3], a graphical interface program where the operator can interactively adjust parameters to find model and/or hardware errors.

In the limit, multi-track fitting can include the complete corrector-to-bpm response matrix with many parameters in the model declared as variables. Unfortunately, the number of operations required to set-up the calculation, the computation time, and interpretation of the results become unwieldy in a graphical interface environment.

To speed up the process, the problem was linearized and statistically correlated solutions were found by matrix inversion in a dedicated code [4]. As with many system identification problems, the non-linear aspect was accommodated by iterative re-expansion around the operating point.

The matrix inversion procedure evolved into a powerful tool that can accurately predict quadrupole strengths, BPM gains, corrector gains, and a variety of other model parameters [5-8]. Although the linearization technique is robust and accurate, the turn-around time required to make the measurements, calculate hundreds of closed orbits, and then invert the matrix can be many hours.

In the following sections, we describe ideas for reducing the model calibration turn-around time. The goal is to predict the model soon after the measurements become available.

2 RECURSIVE LEAST-SQUARES

In order of increasing complication, some of the more useful accelerator model calibration procedures are:

I. Fit single orbits to verify corrector and bpm operation.

II. Fit single quadrupoles or families in-plane.

III. Fit quadrupoles, correctors, bpms in-plane.

IV. Fit quadrupoles, correctors, bpms, coupled-plane.

Depending on the objectives of the analysis, one or more of these procedures might be adequate. For large calculations, it is worthwhile to minimize the number of correctors and bpms needed to realize an accurate fit. Eliminating redundant measurements reduces the time to calculate the closed orbits and to invert the sensitivity matrix.

One way to speed up the process would be to employ a recursive least-squares algorithm. With recursive least-squares, a 'batch' of data is initially processed to arrive at preliminary least-squares answer [9]. Subsequent data is used to update the previous model estimate iteratively. For storage ring model calibration, this translates into choosing a subset of orbit

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perturbations from the response matrix (with appropriate corrector phases) to yield an initial estimate for the model. Each new set of orbit data is then used to update the previous solution on an iterative basis.

In a fast on-line system, the model calculations of the perturbed orbits could be carried out in parallel with the initial measurements. Once a sufficient block of data is acquired, the model is fit with a set of variables chosen by the operator. The new model is used to calculate the next set of closed orbits while more measurements made. Depending on the goal of the analysis, subsequent iterations could include more subtle model variables.

3 PIECEWISE FITTING

In this section, we return to the original 'GOLD' notion of piecewise model calibration, but apply the linearized response-matrix fitting technique. Instead of adjusting many model parameters to agree with the measured response matrix around the entire storage ring, suppose we want to analyze only a part of the ring, the IP or an insertion device for instance. Data is acquired in the same way as before, namely, kick the beam with corrector magnets and measure the beam deflection.

For this application, however, fewer measurements are needed since only a subset of the model parameters are variable. Furthermore, the time required to calculate trajectories from the perturbed model, and the time required to invert the sensitivity matrix is reduced. The main difference is that (similar to the GOLD method) each section of the storage ring is treated like a transmission line.

Mathematically, the procedure becomes clear if we compare to the closed-orbit response matrix method. In that case, the Taylor expansion of the response matrix '$\mathbf{C}$' is

$$\mathbf{C}_{measure} = \mathbf{C}_{model} + (\partial\mathbf{C}/\partial\mathbf{k})_{model} \cdot \Delta\mathbf{k} + \ldots \qquad (1)$$

The column vector of expansion variables, $\Delta\mathbf{k}$, is nominally a set of quadrupole strengths, but corrector gains, BPM gains, energy shifts or other model parameters can be added [5,8]. Any increase in the number of variables will expand the dimensions of the problem. Collecting known quantities in Eq. 1, the parameter vector $\Delta\mathbf{k}$ is found by least-squares inversion of the sensitivity matrix $(\partial\mathbf{C}/\partial\mathbf{k})_{model}$.

Piecewise model calibration proceeds similarly. Expressed in terms of linear transport equations, the displacement $\Delta\mathbf{x}$ of a single trajectory propagates as

$$\frac{\Delta\mathbf{x}}{\Delta\theta} = A\mathbf{R}^{11} + B\mathbf{R}^{12} + \ldots \qquad (2)$$

where {A , B} are the cosine- and sine-like components of each trajectory. The column vectors $\{\mathbf{R}^{11}, \mathbf{R}^{12}\}$

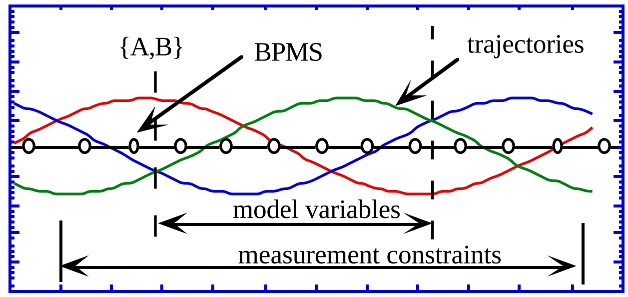

Figure 1: Schematic Piecewise Fitting

are elements of the linear transport matrix evaluated along the beamline. If we Taylor expand the $\mathbf{R}$-matrix elements about the nominal quadrupole strengths, we get an equation for each trajectory:

$$\left(\frac{\Delta\mathbf{x}}{\Delta\theta}\right)_{measure} = A[\mathbf{R}^{11}_{model} + (\partial\mathbf{R}^{11}/\partial\mathbf{k})_{model} \cdot \Delta\mathbf{k}] + B[\mathbf{R}^{12}_{model} + (\partial\mathbf{R}^{12}/\partial\mathbf{k})_{model} \cdot \Delta\mathbf{k}] + \ldots \qquad (3)$$

In this form, $\mathbf{R}^{11}$ and $\mathbf{R}^{12}$ contain model values for elements of $\mathbf{R}$-matrix, and the calculated matrices $\partial\mathbf{R}^{11}/\partial\mathbf{k}$ and $\partial\mathbf{R}^{12}/\partial\mathbf{k}$ contain the sensitivity terms. Although it is desireable to solve for $\Delta\mathbf{k}$ directly, the initial conditions {A, B}† and the column vectors $\{A\Delta\mathbf{k}, B\Delta\mathbf{k}\}$ appear as the variable parameters for each trajectory.

To reduce the dimensions of the problem, and to constrain the solution vector $\Delta\mathbf{k}$, an iterative approach is possible:

I. Solve for {A , B} from Eq. 2 for each trajectory, then

II. Solve for $\Delta\mathbf{k}$ from Eq. 3 with initial conditions {**A** , **B**} as determined in Step I, i.e., solve for $\Delta\mathbf{k}$ from

$$\frac{\Delta\mathbf{x}}{\Delta\theta} - A\mathbf{R}^{11} - B\mathbf{R}^{12} = (\partial\mathbf{R}^{11}/\partial\mathbf{k} + \mathbf{R}^{12}/\partial\mathbf{k}) \cdot \Delta\mathbf{k} \qquad (4)$$

for all trajectories simultaneously. As with the closed orbit case, BPM, corrector, coupling, and energy variations can be added to the parameter vector $\Delta\mathbf{k}$.

The approach of iteratively solving for the initial 'launch' conditions {**A** ,**B**} and then the model parameters $\Delta\mathbf{k}$ is used routinely in RESOLVE [10] analysis (and independently suggested by S. Prabhakar). Other numerical solutions are possible. The difference here is the linearization step that facilitates the analysis.

†With transmission line fitting, the 'beam' can be propagated in either direction, e.g., the initial conditions {A ,B} can be located on the right in Fig. 1.

4 SUMMARY

Two ideas are presented with the intention of speeding up measurement based model calibration for on-line applications. The first method, recursive least-squares analysis, is similar to the method used in practice today, but explores the possibility of 'updating' the solution as new data becomes available without re-analyzing the entire data set.

The second method draws on concepts used before closed-orbit analysis became available, namely piecewise model calibration. Relative to closed-orbit model calibration for an entire storage ring, piecewise analysis involves

- less calculation time for perturbed trajectories,
- fewer BPMs and correctors, and
- fewer variable model parameters.

In practice, once a corrector is kicked the full (closed) orbit can be recorded, but only a section of the accelerator is analyzed at any one time. The model fitting can be 'propagated' along a transmission line, around a storage ring, or focus on a specific region of the accelerator. The time savings for piecewise fitting is derived from elimination of closed orbit calculations (only the coefficients $\mathbf{R}^{11}(s)$ and $\mathbf{R}^{12}(s)$ are needed), and inversion of a smaller sensitivity matrix. After adjusting IP optics in a large collider, for instance, a quick check of quadrupole strengths is possible without analysis of the entire storage ring.

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