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Estimate of the Impedance Due to Wall Surface Roughness*

K.L.F. Bane, C.K. Ng, and A.W. Chao
Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

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SLAC, Stanford, CA 94309, USA

1 INTRODUCTION

In the Next Linear Collider (NLC) after being accelerated the beam is collimated to remove tail particles. Wakefields generated in the collimator section, however, can significantly degrade the beam emittance[1]. The collimators are, therefore, carefully designed to balance and minimize the effects of the geometric and the resistive wall wakefields. Recent measurements of collimator wakefields in the Stanford Linear Collider (SLC) linac seem to confirm the geometric wakefield calculations but yield results for the resistive wall wakefield that are 3–4 times as large as expected[2]. One possibility is that this discrepancy is due to the roughness of the collimator surface. In this report we estimate this effect.

The longitudinal impedance of small perturbations on a vacuum chamber pipe is a well studied subject (see *e.g.*, Ref. [3, 4, 5]). Let us limit ourselves to perturbations in a round beam pipe with radius b. In Ref. [3], exact expressions are derived for the impedance of small elliptical holes on such a beam pipe. For the special case of a circular hole[3, 4]

$$Z^{\parallel}(\omega) = -i\frac{\omega}{c}Z_0\frac{a^3}{6\pi^2 b^2}f \quad , \tag{1}$$

with $Z_0 = 377\ \Omega$, $a \ll b$ the hole radius, and the form factor

$$f = \left\{ \begin{array}{c} 1 \\ 0.56 \end{array} \right\} \quad [\text{small hole with} \left\{ \begin{array}{c} \text{thin} \\ \text{thick} \end{array} \right\} \text{wall}] \ . \tag{2}$$

In Ref. [5] exact expressions are derived for small ellipsoidal protrusions on the pipe wall. In the special case of a small hemispherical bump it is shown that Eq. 1 is still valid, if a is taken to represent the radius of the sphere and

$$f = \frac{3\pi}{2} \qquad [\text{small hemispherical bump}] \quad . \tag{3}$$

Note that Eq. 1 is an inductive impedance, *i.e.* it can be written in the form $Z^{\parallel} = -i\omega L$, with L the inductance.

In a round beam tube the transverse impedance of a small perturbation is related to the longitudinal impedance by [6]

$$Z^{\perp}(\omega) = \frac{4c}{\omega b^2}Z^{\parallel}(\omega) = -iZ_0\frac{2a^3}{3\pi^2 b^4}f \quad . \tag{4}$$

This equation assumes a dipole beam on axis, with the dipole moment in the direction of the perturbation. Note that the transverse impedance is related to the same form factor f as in Eq. 1 and is independent of ω at low frequencies.

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Fourier transforming, we find that the transverse kick to the beam is proportional to its longitudinal charge distribution. For a Gaussian beam the average kick is given by

$$\langle \Delta y' \rangle = \frac{4}{3\pi^{\frac{3}{2}}}\frac{N r_0 y_0 a^3}{\gamma \sigma_z b^4}f \quad , \tag{5}$$

with N the number of electrons in the bunch, r_0 the electron classical radius ($= 2.8 \times 10^{-15}$ m), eNy_0 the dipole moment of the beam, γ the relativistic energy factor, and σ_z the rms bunch length. Note that if the directions of the dipole moment and the perturbation are not aligned, then Eqs. 4 and 5 need to be multiplied by $\cos\phi$, with ϕ the angle between the two directions, and the kick direction is toward the perturbation[6].

In Ref. [7] the low frequency impedance of narrow, longitudinal slots in a round chamber wall were studied numerically. It was shown, for example, that with square ends the impedance is 50% larger than with round ends. In this report, we perform similar calculations but applied to small protrusions of differing sizes and orientations, and find the effect on the form factor f. These results are then applied to estimate the importance of surface roughness to the transverse impedance of the collimators in the NLC and the SLC.

2 NUMERICAL RESULTS

We use the 3-dimensional, finite difference computer module of MAFIA, T3[8], to find the wakefield generated by an on-axis Gaussian bunch passing by a small protrusion in the beam tube wall. The geometries that we consider are right rectangular solids and wedges with 45° angles, objects that can be exactly represented by a cubic mesh. The low frequency impedance we are interested in can be obtained by using a relatively long Gaussian bunch in the simulation. We use $\sigma_z = 1$ cm, while the small perturbations we simulate are on the order of 1 mm in size. As beam tube radius we take $b = 1$ cm. A typical result, giving the longitudinal wakefield, is shown in Fig. 1. As expected, the wake is nearly exactly proportional the derivative of the bunch shape, implying that the impedance is inductive. From the MAFIA results we obtain the inductance L using

$$L \approx \hat{W}^{\parallel}\frac{\sqrt{2\pi}\sigma_z^2 e^{1/2}}{c^2} \quad , \tag{6}$$

with $\hat{W}^{\parallel}$ the peak of the longitudinal wake function. Alternatively, we can obtain the low frequency impedance from the transverse wake as obtained by MAFIA. For a small perturbation this function is proportional to the charge distribution itself. Comparing the two results is a consistency check of the MAFIA results, as well as a validation of the applicability of Eq. 4. In all cases to be presented the results of the two methods agree to within 1–2%.

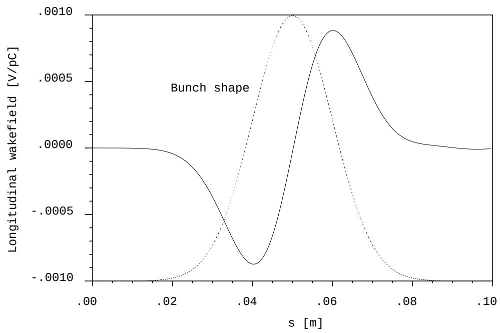


Figure 1: A typical MAFIA result. Plotted is the longitudinal wakefield excited by a gaussian bunch (the solid curve) and the bunch shape (the dots), with the head to the left.

In Fig. 2 we present results for a rectangular post in the beam tube wall, showing what happens when the height h (a), length l (b), and width w (c) are varied. In Fig. 2a, for comparison, the dashed curve gives the inductance of a half-ellipsoid of revolution[5], with radius $a = .5$ mm and height h; at $h = .5$ mm it becomes a hemispherical bump. The MAFIA results vary roughly as h^2. Note that the case with a square base (the $l = 1$ mm curve) has a result that is much larger than that of the half-ellipsoid of the same height, by a factor of 3.7 at $h = .5$ mm, by a factor of 2.5 at $h = 2$ mm. In Figs. 2b-c we note that the dependence of the impedance on l and w is much weaker than the dependence on h.

In Table 1 we present the inductance of 5 selected objects: (1) a hemisphere with radius $a = .5$ mm, (2) a half cube with $w = l = 1$ mm, $h = .5$ mm, (3) a post with $w = l = 1/\sqrt{2}$ mm and $h = .5$ mm, rotated by 45°, (4) a wedge with base dimensions $w = l = 1$ mm, depth $h = .5$ mm, and (5) a cube with $w = l = h = 1$ mm. Also given is the form factor f when comparing to a shallow hole with radius $a = .5$ mm. The longitudinal and transverse profiles are sketched below the table. We note by the first 4 examples that for object that look very similar, the form factor f can vary by a large factor, in this case by a factor 3–4. The fifth example again demonstrates the roughly quadratic dependence on height.

In Fig. 3 we plot the inductance of two cubes, 1 mm on a side, that are longitudinally aligned, as function of the space between them d, and note that when the distance is comparable to the length of a side the result is nearly the same as for two independent cubes. It has been suggested that there might be a partial cancellation of effect when there are many perturbations longitudinally aligned[10]. To test this question we ran an example with five 1-mm cubes, each separated by a distance of 2 mm. The resulting inductance was 5/2 times the corresponding two-cube result,

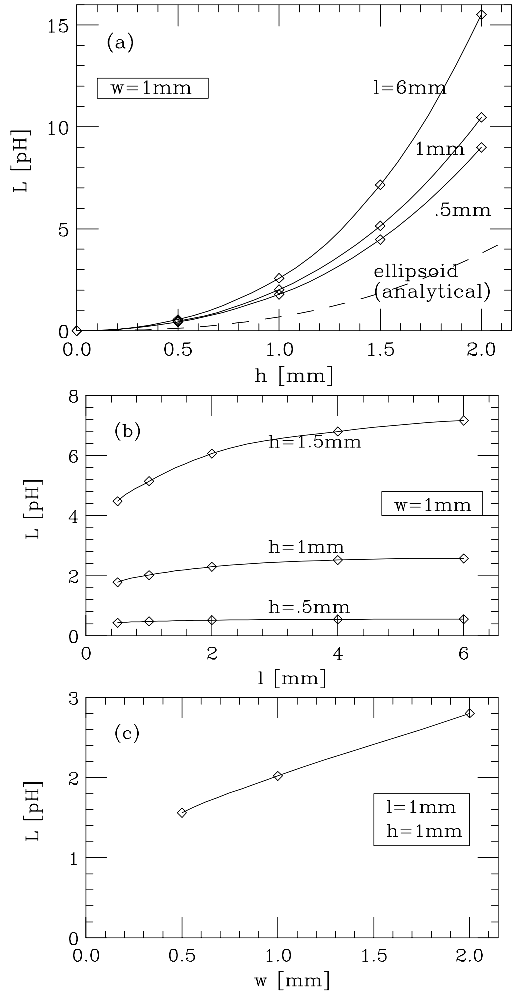


Figure 2: The inductance obtained for a single rectangular post when varying the height h (a), length l (b), and width w (c) (the plotting symbols). The dashed curve in (a) is the analytic result for a half-ellipsoid with radius $a = .5$ mm[5].

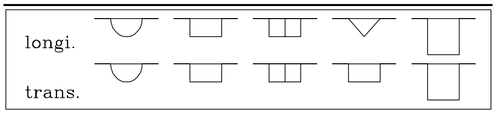
consistent with there being no such cancellation.

3 ROUGH SURFACES

The microstructure of a metallic surface depends on the manufacturing and machining method used to create the surface. For simplicity we here model a rough surface as a random distribution of small bumps and cavities of a certain size—the granularity size—on a smooth surface. Since the impedance of a bump seems to be significantly larger than that of a cavity of similar size, we will neglect the effect of the cavities. We begin with the transverse kick due to a single bump, Eq. 5, and average the effect over a random distribution of such bumps. Note that averaging over all azimuthal angles introduces a factor 1/2 in the amplitude, and the direction of the kick becomes the direction of the beam offset. The average kick to a Gaussian beam which

Table 1: Results for 5 selected objects, whose longitudinal and transverse profile are sketched below. Given is the inductance L, and the form factor f when comparing to a shallow hole with radius $a = .5$ mm.

Case	L [pH]	f
Hemisphere (analytical)	0.13	4.7
Half Cube	0.48	17.4
Rotated Post	0.33	11.9
Wedge	0.21	7.6
Cube	2.02	73.0



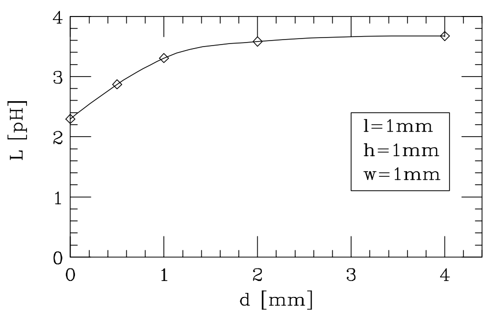


Figure 3: The inductance of two cubes, 1 mm on a side, that are longitudinally aligned, as function of the space between them d.

is displaced by an amount y_0 in the y-direction is given by

$$\langle \Delta y' \rangle = \frac{4}{3\pi^{\frac{3}{2}}} \frac{N r_0 y_0 Z}{\gamma \sigma_z b^3} a f \alpha \quad , \tag{7}$$

with Z the length of the beam pipe, a the typical size of the bumps, and α the surface filling factor of the bumps.

This kick should be compared to that due to the resistive wall wakefield of a smooth pipe of radius b

$$\langle \Delta y' \rangle_{rw} = (0.78) \sqrt{\frac{8}{\pi}} \frac{N r_0 y_0 Z}{\gamma \sigma_z b^3} \delta \quad , \tag{8}$$

with δ the skin depth. For a Gaussian bunch we take $\delta = \sqrt{c\sigma_z/(2\pi\sigma)}$ with σ the conductivity of the metal. Eq. 7 is expected to be valid only if a is large compared to δ. If the two are comparable, and one wants an accurate result, a self-consistent calculation of the impedance, including both the geometric and resistive effects, is needed.

4 APPLICATION TO THE NLC AND THE SLC

For the case of the NLC collimators the surface is copper, for which $\sigma = 5.8 \times 10^{17}$ s^{-1}, and the linac bunch length $\sigma_z = 100$ μm; therefore, the effective skin depth $\delta = .1$ μm. If we assume $f\alpha = 10$, then for the kick due to wall roughness, Eq. 7, to be small compared to the resistive wall kick, Eq. 8, implies that the granularity at the surface of the collimators would need to be small compared to 50 nm.

For the case of the SLC linac collimators the surface is made of vanadium, for which $\sigma = 4 \times 10^{16}$ s^{-1}, or titanium nitride, for which $\sigma = 2 - 4 \times 10^{16}$ s^{-1}. The granularity is expected to be on the order of $a \sim 1$ μm[9]. We expect $f\alpha \sim 5 - 15$. The typical bunch length in the SLC linac is $\sigma_z = 1$ mm, and the corresponding skin depth $\delta = 1.1$ μm (for vanadium). With these parameters we find that the kick due to the bumps is 1–2.5 times as large as that due to the wall resistance. Although this calculation is rough the results suggest that surface roughness may conceivably explain the factor 3–4 discrepancy in the SLC collimator wakefield measurements. For a more quantative result, however, more study, particularly concerning the microstructure at the surface of the collimators, is needed.

5 ACKNOWLEDGEMENTS

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