NEXT-TO-LEADING ORDER CORRECTIONS TO $e^+e^- \to$ FOUR JETS

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Abstract

I present a general purpose Monte Carlo program for calculating the next-to-leading order corrections to arbitrary four-jet quantities in electron-positron annihilation. In the current version of the program, some subleading in color terms are neglected. As an example, I calculate the four-jet rate in the Durham scheme as well as the Bengtsson-Zerwas angular distribution at $\mathcal{O}(\alpha_s^3)$ and compare the results to existing data.

Four-jet events have been copiously produced at the Z-pole and at smaller energies at electron-positron colliders. These events allow the study of new aspects of QCD. For instance, the three-gluon coupling and the dependence on the number of light flavors, N_f, enter already at tree level. Therefore, this seems to be an ideal place for a measurement of the color factors [1, 2, 3]. Even if there is no doubt about the correctness of QCD as the theory of the strong interactions, these measurements are not purely academic. In fact, the ongoing debate about the existence of light gluinos could be settled immediately by measuring N_f precisely enough (see e.g. ref. [3] and references therein). Indeed, the addition of a massless gluino amounts to a change of N_f from 5 to 8. Also at LEP2 four-jet events will play an important role, since they are the main background for the W pair production. Thus, a next-to-leading order prediction for such quantities is highly desirable.

In this talk I present some next-to-leading order results for the four jet fraction and some angular distributions. These results have been obtained with a program [4] which can compute an arbitrary four-jet quantity at next-to-leading order. However, as will be discussed below, some subleading in color contributions have been neglected so far.

Any next-to-leading order calculation involves basically two major steps. First, the corresponding one-loop amplitudes have to be computed and second, the phase-space integration has to be performed. This second step involves the cancellation of the real and virtual singularities. I used the general version of the subtraction method as proposed in [5] to do the phase space integrals. Thus, no approximation at all has been made in this part of the calculation.

The one-loop amplitudes which are needed for the calculation of four-jet events are $e^+e^- \to q\bar{q}q'\bar{q}'$ and $e^+e^- \to q\bar{q}gg$. Recently, the amplitudes for four quarks in the final state have been computed [6, 7]. However, the situation concerning the one-loop amplitude for the $q\bar{q}gg$ final state is less satisfactory. Unfortunately, only the leading in color terms are known so far [8]. As a result, subleading in color pieces of the cross section can not yet be computed at next-to-leading order. Writing the color decomposition of any four-jet cross section as

$$\sigma_{4-\text{jet}}^{1-\text{loop}} = N_c^2(N_c^2 - 1)\left[\sigma_4^{(a)} + \frac{N_f}{N_c}\sigma_4^{(b)} + \frac{N_f^2}{N_c^2}\sigma_4^{(c)} + \frac{N_f}{N_c^3}\sigma_4^{(d)} + \mathcal{O}\left(\frac{1}{N_c^2}\right)\right] \tag{1}$$

I calculate $\sigma_4^{(a,b,c,d)}$.

Besides the one-loop amplitudes, the tree-level amplitudes for the processes $e^+e^- \to q\bar{q}q'\bar{q}'g$ and $e^+e^- \to q\bar{q}ggg$ are required for the computation of the real contributions. They have been obtained by several groups [9, 10]. In the program the results of ref [9] are used.

In the calculation of these amplitudes all quark and lepton masses have been set to zero. This is usually a very good approximation, although the b-quark mass effects can yield considerable corrections [11]. Unfortunately, the complete inclusion of mass effects at the order $\mathcal{O}(\alpha_s^3)$ is presently out of reach, the main reason being the fact that the one-loop amplitudes are known only for the massless case.

Besides the mass effects and subleading in color terms, three more contributions were neglected, although in principle their inclusion would not pose a problem.

(1) Contributions coming from Pauli exchange. The corresponding $\mathcal{O}(\alpha_s^2)$ terms are known to be numerically tiny and it is expected that the $\mathcal{O}(\alpha_s^3)$ terms are numerically not very significant.
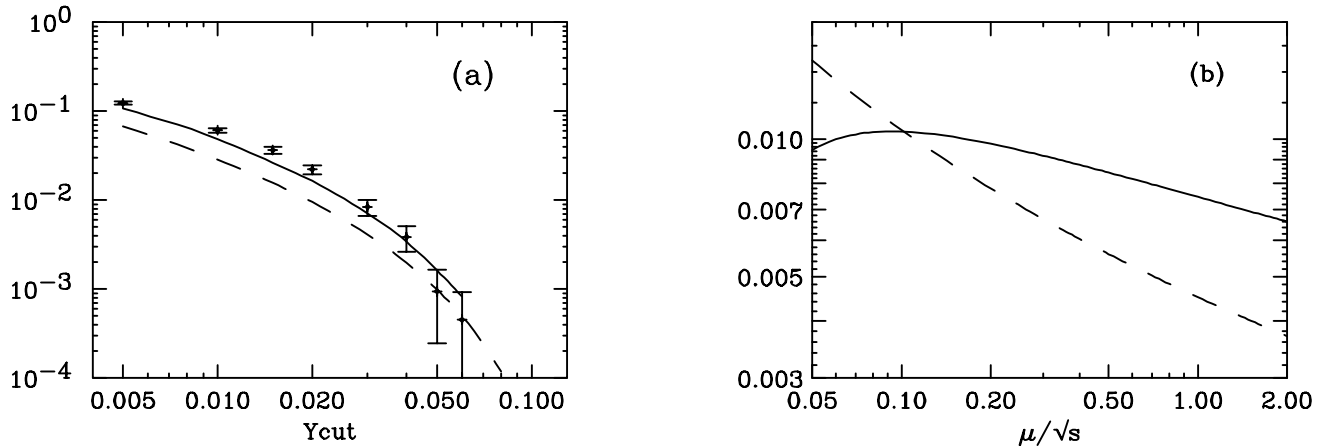
Figure 1: (a) Solid (dashed) lines represent the one-loop (tree-level) predictions for R_4 in the Durham scheme for $\mu = \sqrt{s}$ and $\alpha_s = 0.118$. (b) Solid (dashed) lines show the dependence of R_4 on the renormalization scale μ for the one-loop (tree-level) predictions in the Durham scheme, for $y_{\text{cut}} = 0.03$.

(2) Contributions proportional to the axial coupling a_q of the Z^0 to quarks. Analogous terms have traditionally been omitted from $\mathcal{O}(\alpha_s^2)$ programs, as they cancel precisely between up- and down-type quarks in the final state (for zero quark mass), and their contribution to the three-jet rate is at the percent level [12].

(3) Contributions proportional to $(\sum_q v_q)^2$, where v_q is the quark vector coupling. These "light-by-glue scattering" terms do not appear at $\mathcal{O}(\alpha_s^2)$ at all, have a partial cancellation from the sum over quark flavors, and contribute less than 1% to the $\mathcal{O}(\alpha_s^3)$ term in the total cross-section [13].

I first present the results for the four-jet rate $R_4 \equiv \sigma_{4\text{-jet}}/\sigma_{\text{tot}}$ at next-to-leading order in α_s in the Durham scheme [14]. The y_{cut} dependence is shown in Fig. 1. The solid (dashed) line represents the one-loop (tree-level) prediction. The renormalization scale μ has been chosen to be the center-of-mass energy $\sqrt{s}$, the number of flavors $N_f = 5$ and $\alpha_s = 0.118$ [15]. The data points are preliminary SLD data [16] and are corrected for hadronization.

The truncation of the perturbative expansion for any physical quantity leads to a dependence of the theoretical prediction on the choice of the renormalization scale μ. The tree-level μ dependence is much stronger for the four-jet rate than for the three-jet rate, because the former is proportional to α_s^2 instead of α_s. As expected, this strong renormalization-scale dependence is reduced by the inclusion of the next-to-leading order contribution. Fig. 1 also plots the μ-dependence of R_4 at tree-level and at one-loop for $y_{cut} = 0.03$.

Four-jet angular distributions [17] have been measured by several collaborations [1, 2, 3]. The general procedure is to choose a certain jet definition. Then, in the case of a four-jet event, the jets are ordered according to their energies such that $E_1 > E_2 > E_3 > E_4$. Usually, the most energetic jets are associated with the primary quarks whereas the remaining two jets either origin form a quark or gluon pair (at tree level). This can be exploited to construct angular variables which have a completely different shape for the four-quark and the two-quark-two-gluon final state. Since the two final states are proportional to different color structures one can attempt to measure the various color factors and in particular the number of light flavors N_f. Unfortunately, the four-quark final state is strongly suppressed.
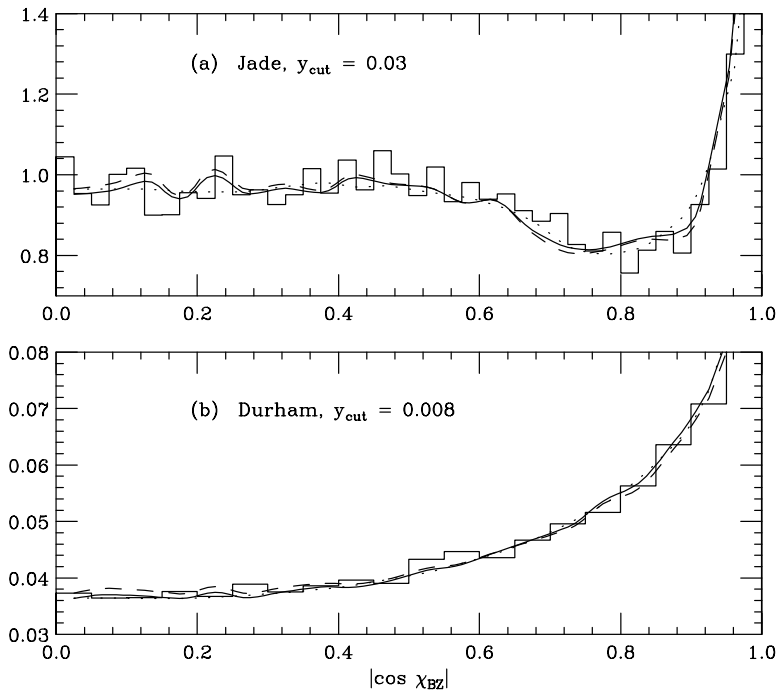
Figure 2: Bengtsson-Zerwas distribution at tree level (dotted), one-loop (solid) and one-loop with $N_f = 8$ (dashed) compared to (a) OPAL [2] and (b) ALEPH [3] data (histograms), which are corrected for detector and hadronization effects.

As a result, the full distributions are not very sensitive to N_f and the error on the measured value of N_f is accordingly large.

An advantage of the angular distributions lies in the fact that one does not need to worry about large logarithms coming from a particular choice of the renormalization scale. The reason for this can easily be understood. At tree level, the strong coupling constant appears only in an overall prefactor. Since these distributions are normalized, the value of α_s and thus the choice of the renormalization scale μ has no influence at all on the result. Only the inclusion of the one-loop corrections introduces a extremely mild μ-dependence.

As an example, I consider the Bengtsson-Zerwas angle, χ_{BZ}, and compare the next-to-leading order prediction with the two most recent analyses of OPAL and ALEPH [2, 3]. In [2] jets were defined according to the JADE scheme with $y_{\text{cut}} = 0.03$, whereas in [3] the Durham jet algorithm with the E0 recombination scheme was used and y_{cut} was chosen to be 0.008. Note also that the two experiments use different normalizations. Fig. 2 compares the tree-level (dotted) and next-to-leading order (solid) predictions to the data which have been corrected for detector and hadronization effects. The dotted line can hardly be seen because it nearly coincides with the solid line. The theoretical curves have been obtained by binning χ_{BZ} into twenty bins. This rather fine binning results in a somewhat larger statistical error, which is of the order of 2% for the shown curves. In order to illustrate the mild dependence on N_f I plotted also the one-loop results for $N_f = 8$ (dashed). Although this dependence and thus the precision on the measurement of N_f may be enhanced by additional cuts or by b-quark tagging [18], Fig. 2 shows that it is very difficult to get a precise measurement of N_f from angular distributions alone.

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