

SLAC-PUB-7473

**OPTIMAL RENORMALIZATION SCALE AND SCHEME FOR  
EXCLUSIVE PROCESSES\***

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hep-ph/9705221 v2 20 May 1997

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\*Research supported in part by the U.S. Department of Energy and National Science Foundation.

## ABSTRACT

We use the BLM method to fix the renormalization scale of the QCD coupling in exclusive hadronic amplitudes such as the pion form factor and the photon-to-pion transition form factor at large momentum transfer. Renormalization-scheme-independent commensurate scale relations are established which connect the hard scattering subprocess amplitudes that control exclusive processes to other QCD observables such as the heavy quark potential and the electron-positron annihilation cross section. The commensurate scale relation connecting the heavy quark potential, as determined from lattice gauge theory, to the photon-to-pion transition form factor is in excellent agreement with  $\gamma e \rightarrow \pi^0 e$  data assuming that the pion distribution amplitude is close to its asymptotic form  $\sqrt{3}f_\pi x(1-x)$ . We also reproduce the scaling and normalization of the  $\gamma\gamma \rightarrow \pi^+\pi^-$  data at large momentum transfer. Because the renormalization scale is small, we argue that the effective coupling is nearly constant, thus accounting for the nominal scaling behavior of the data. However, the normalization of the space-like pion form factor  $F_\pi(Q^2)$  obtained from electroproduction experiments is somewhat higher than that predicted by the corresponding commensurate scale relation. This discrepancy may be due to systematic errors introduced by the extrapolation of the  $\gamma^*p \rightarrow \pi^+n$  electroproduction data to the pion pole.

# 1 Introduction

One of the most critical problems in making reliable predictions in quantum chromodynamics is how to deal with the dependence of the truncated perturbative series on the choice of renormalization scale  $\mu$  and scheme for the QCD coupling  $\alpha_s(\mu)$  [1, 2, 3]. For processes such as jet production in  $e^+e^-$  annihilation and heavy flavor production in hadron collision, where only the leading and next-to-leading predictions are known, the theoretical uncertainties from the choice of renormalization scale and scheme are larger than the experimental uncertainties. The ambiguities due to the renormalization conventions are compounded in processes involving more than one physical scale.

Perturbative QCD has been used to analyze a number of exclusive processes involving large momentum transfers, including the decay of heavy hadrons to specific channels such as  $B \rightarrow \pi\pi$  and  $\Upsilon \rightarrow p\bar{p}$ , baryon form factors at large  $t$ , and fixed  $\theta_{c.m.}$  hadronic scattering amplitudes such as  $\gamma p \rightarrow \pi^+n$  at high energies. As in the case of inclusive reactions, factorization theorems for exclusive processes [4, 5] allow the analytic separation of the perturbatively-calculable short-distance contributions from the long-distance non-perturbative dynamics associated with hadronic binding. For a review of this formalism with many additional references, see [6].

The scale ambiguities for the underlying quark-gluon subprocesses are particularly acute in the case of QCD predictions for exclusive processes, since the running coupling  $\alpha_s$  enters at a high power. Furthermore, since each external momentum entering an exclusive reaction is partitioned among the many propagators of the underlying hard-scattering amplitude, the physical scales that control these processes are inevitably much softer than the overall momentum transfer. Exclusive process phenomenology is further complicated by the fact that the scales of the running couplings in the hard-scattering amplitude depend themselves on the shape of the hadronic wavefunctions.

The renormalization scale ambiguity problem can be resolved if one can optimize the choices of scale and scheme according to some sensible criteria. In the BLM procedure, the renormalization scales are chosen such that all vacuum polarization effects from the QCD  $\beta$  function are re-summed into the running couplings. The coefficients of the perturbative series are thus identical to the perturbative coefficients of the corresponding conformally invariant theory with  $\beta = 0$ . The BLM method has the important advantage of “pre-summing” the large and strongly divergent terms in the PQCD series which grow as  $n!(\alpha_s\beta_0)^n$ , *i.e.*, the infrared renormalons associated with coupling constant renormalization [7, 8]. Furthermore, the renormalization scales

$Q^*$  in the BLM method are physical in the sense that they reflect the mean virtuality of the gluon propagators [3, 8, 9, 10]. In fact, in the  $\alpha_V(Q)$  scheme, where the QCD coupling is defined from the heavy quark potential, the renormalization scale is by definition the momentum transfer caused by the gluon.

In this paper we will use the BLM method to fix the renormalization scale of the QCD coupling in exclusive hadronic amplitudes such as the pion form factor, the photon-to-pion transition form factor and  $\gamma\gamma \rightarrow \pi^+\pi^-$  at large momentum transfer. Renormalization-scheme-independent commensurate scale relations will be established which connect the hard scattering subprocess amplitudes that control these exclusive processes to other QCD observables such as the heavy quark potential and the electron-positron annihilation cross section. Because the renormalization scale is small, we will argue that the effective coupling is nearly constant, thus accounting for the nominal scaling behavior of the data [11, 12].

## 2 Renormalization Scale Fixing In Exclusive Processes

A basic principle of renormalization theory is the requirement that the relations between physical observables must be independent of renormalization scale and scheme conventions to any fixed order of perturbation theory [13]. This property can be explicitly expressed in the form of “commensurate scale relations” [14]. A primary example of a commensurate scale relation is the generalized Crewther relation [14, 15], in which the radiative corrections to the Bjorken sum rule for deep inelastic lepton-proton scattering at a given momentum transfer  $Q$  are predicted from measurements of the  $e^+e^-$  annihilation cross section at a corresponding commensurate energy scale  $\sqrt{s} \propto Q$ .

A scale-fixed relation between any two physical observables  $A$  and  $B$  can be derived by applying BLM scale-fixing to their respective perturbative predictions in, say, the  $\overline{MS}$  scheme and then algebraically eliminating  $\alpha_{\overline{MS}}$ . The choice of the BLM scale ensures that the resulting commensurate scale relation between  $A$  and  $B$  is independent of the choice of the intermediate renormalization scheme [14]. Thus, using this formalism one can relate any perturbatively calculable observable, such as the annihilation ratio  $R_{e^+e^-}$ , the heavy quark potential, and the radiative corrections to structure function sum rules, to each other without any renormalization scale or scheme ambiguity [16].

The heavy-quark potential  $V(Q^2)$  can be identified as the two-particle-irreducible

scattering amplitude of test charges, *i.e.*, the scattering of an infinitely-heavy quark and antiquark at momentum transfer  $t = -Q^2$ . The relation

$$V(Q^2) = -\frac{4\pi C_F \alpha_V(Q^2)}{Q^2}, \quad (1)$$

with  $C_F = (N_C^2 - 1)/2N_C = 4/3$ , then defines the effective charge  $\alpha_V(Q)$ . This coupling provides a physically-based alternative to the usual  $\overline{MS}$  scheme. Recent lattice gauge calculations have provided strong constraints on the normalization and shape of  $\alpha_V(Q^2)$ .

As in the corresponding case of Abelian QED, the scale  $Q$  of the coupling  $\alpha_V(Q)$  is identified with the exchanged momentum. All vacuum polarization corrections due to fermion pairs are incorporated in terms of the usual vacuum polarization kernels defined in terms of physical mass thresholds. The first two terms  $\beta_0 = 11 - 2n_f/3$  and  $\beta_1 = 102 - 38n_f/3$  in the expansion of the  $\beta$  function defined from the logarithmic derivative of  $\alpha_V(Q)$  are universal, *i.e.*, identical for all effective charges at  $Q^2 \gg 4m_f^2$ . The coefficient  $\beta_2$  for  $\alpha_V$  has recently been calculated in the  $\overline{MS}$  scheme [17].

The scale-fixed relation between  $\alpha_V$  and the conventional  $\overline{MS}$  coupling is

$$\alpha_{\overline{MS}}(Q) = \alpha_V(e^{5/6}Q) \left( 1 + \frac{2C_A}{3} \frac{\alpha_V}{\pi} + \dots \right), \quad (2)$$

above or below any quark mass threshold. The factor  $e^{5/6} \simeq 0.4346$  is the ratio of commensurate scales between the two schemes to this order. It arises because of the convention used in defining the modified minimal subtraction scheme. The scale in the  $\overline{MS}$  scheme is thus a factor  $\sim 0.4$  smaller than the physical scale. The coefficient  $2C_A/3$  in the NLO term is a feature of the non-Abelian couplings of QCD; the same coefficient occurs even if the theory had been conformally invariant with  $\beta_0 = 0$ .

As we shall see, the coupling  $\alpha_V$  provides a natural scheme for computing exclusive amplitudes. Once we relate form factors to effective charges based on observables, there are no ambiguities due to scale or scheme conventions.

The use of  $\alpha_V$  as the expansion parameter with BLM scale-fixing has also been found to be valuable in lattice gauge theory, greatly increasing the convergence of perturbative expansions relative to those using the bare lattice coupling [9]. In fact, new lattice calculations of the  $\Upsilon$  spectrum [18] have been used to determine the normalization of the static heavy quark potential and its effective charge:

$$\alpha_V^{(3)}(8.2 \text{ GeV}) = 0.196(3), \quad (3)$$

where the effective number of light flavors is  $n_f = 3$ . The corresponding modified

minimal subtraction coupling evolved to the  $Z$  mass using Eq. (2) is given by

$$\alpha_{\overline{MS}}^{(5)}(M_Z) = 0.115(2). \quad (4)$$

This value is consistent with the world average of 0.117(5), but is significantly more precise. These results are valid up to NLO.

Exclusive processes are particularly challenging to compute in quantum chromodynamics because of their sensitivity to the unknown nonperturbative bound state dynamics of the hadrons. However, in some important cases, the leading power-law behavior of an exclusive amplitude at large momentum transfer can be computed rigorously in the form of a factorization theorem which separates the soft and hard dynamics. For example, the leading  $1/Q^2$  fall-off of the meson form factors can be computed as a perturbative expansion in the QCD coupling [4, 5]:

$$F_M(Q^2) = \int_0^1 dx \int_0^1 dy \phi_M(x, \tilde{Q}) T_H(x, y, Q^2) \phi_M(y, \tilde{Q}), \quad (5)$$

where  $\phi_M(x, \tilde{Q})$  is the process-independent meson distribution amplitude which encodes the nonperturbative dynamics of the bound valence Fock state up to the resolution scale  $\tilde{Q}$  and

$$T_H(x, y, Q^2) = 16\pi C_F \frac{\alpha_s(\mu)}{(1-x)(1-y)Q^2} (1 + O(\alpha_s)) \quad (6)$$

is the leading-twist perturbatively-calculable subprocess amplitude  $\gamma^* q(x) \bar{q}(1-x) \rightarrow q(y) \bar{q}(1-y)$ , obtained by replacing the incident and final mesons by valence quarks collinear up to the resolution scale  $\tilde{Q}$ . The contributions from non-valence Fock states and the correction from neglecting the transverse momentum in the subprocess amplitude from the non-perturbative region are higher twist, *i.e.*, power-law suppressed. The transverse momenta in the perturbative domain lead to the evolution of the distribution amplitude and to next-to-leading-order (NLO) corrections in  $\alpha_s$ . The contribution from the endpoint regions of integration,  $x \sim 1$  and  $y \sim 1$ , are power-law and Sudakov suppressed and thus can only contribute corrections at higher order in  $1/Q$  [4].

The distribution amplitude  $\phi(x, \tilde{Q})$  is boost and gauge invariant and evolves in  $\ln \tilde{Q}$  through an evolution equation [4]. It can be computed from the integral over transverse momenta of the renormalized hadron valence wavefunction in the light-cone gauge at fixed light-cone time [4]:

$$\phi(x, \tilde{Q}) = \int d^2 \vec{k}_\perp \theta \left( \tilde{Q}^2 - \frac{\vec{k}_\perp^2}{x(1-x)} \right) \psi^{(\tilde{Q})}(x, \vec{k}_\perp). \quad (7)$$

The physical pion form factor must be independent of the separation scale  $\tilde{Q}$ . The natural variable to make this separation is the light-cone energy, or equivalently the invariant mass  $\mathcal{M}^2 = k_\perp^2/x(1-x)$ , of the off-shell partonic system [19, 4]. Any residual dependence on the choice of  $\tilde{Q}$  for the distribution amplitude will be compensated by a corresponding dependence of the NLO correction in  $T_H$ . However, the NLO prediction for the pion form factor depends strongly on the form of the pion distribution amplitude as well as the choice of renormalization scale  $\mu$  and scheme.

Another example of an exclusive amplitude which can be computed in perturbative QCD is the transition form factor between a photon and a neutral hadron such as  $F_{\gamma\pi}(Q^2)$ , which has now been measured up to  $Q^2 < 8 \text{ GeV}^2$  in the tagged two-photon collisions  $e\gamma \rightarrow e'\pi^0$  by the CLEO and CELLO collaborations. In this case the amplitude has the factorized form

$$F_{\gamma \rightarrow M}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \phi_M(x, Q^2) T_{\gamma \rightarrow M}^H(x, Q^2), \quad (8)$$

where the hard scattering  $\gamma\gamma^* \rightarrow q\bar{q}$  amplitude gives

$$T_{\gamma \rightarrow M}^H(x, Q^2) = \frac{1}{(1-x)Q^2} (1 + O(\alpha_s)). \quad (9)$$

It is straightforward to obtain commensurate scale relations for these exclusive amplitudes following the procedure outlined above. The CSR relating the pion form factor and the heavy quark potential is

$$\begin{aligned} F_\pi(Q^2) &= \int_0^1 dx \phi_\pi(x) \int_0^1 dy \phi_\pi(y) 4\pi C_F \frac{\alpha_V(Q_V^*)}{(1-x)(1-y)Q^2} \left( 1 + C_V \frac{\alpha_V(Q_V^*)}{\pi} \right) \\ &= - \int_0^1 dx \phi_\pi(x) \int_0^1 dy \phi_\pi(y) V(Q_V^{*2}) \left( 1 + C_V \frac{\alpha_V(Q_V^*)}{\pi} \right), \end{aligned} \quad (10)$$

where  $C_V = -1.91$  is the same coefficient one would obtain in a conformally invariant theory with  $\beta = 0$ , and  $Q_V^{*2} = (1-x)(1-y)Q^2$ . In this analysis we have assumed that the pion distribution amplitude has the asymptotic form  $\phi_\pi = \sqrt{3}f_\pi x(1-x)$ , where the pion decay constant is  $f_\pi \simeq 93 \text{ MeV}$ . In this simplified case the distribution amplitude does not evolve, and there is no dependence on the separation scale  $\tilde{Q}$ . This commensurate scale relation between  $F_\pi(Q^2)$  and  $\langle \alpha_V(Q_V^{*2}) \rangle$  represents a general connection between the form factor of a bound-state system and the irreducible kernel that describes the scattering of its constituents.

If we expand the QCD coupling about a fixed point  $Q_0$  in NLO [9]:  $\alpha_s(Q^*) \approx \alpha_s(Q_0) \left[ 1 - \frac{\beta_0}{2\pi} \ln(Q^*/Q_0) \alpha_s(Q^*) \right]$ , then the integral over the effective charge in the

meson form factor can be performed explicitly. In this approximation  $\langle \ln Q_V^{*2} \rangle = \langle \ln(1-x)(1-y)Q^2 \rangle$ , in agreement with the explicit calculation. Thus, assuming the asymptotic distribution amplitude, the pion form factor at NLO is

$$Q^2 F_\pi(Q^2) = 16\pi f_\pi^2 \alpha_V(e^{-3/2}Q) \left( 1 - 1.91 \frac{\alpha_V}{\pi} \right). \quad (11)$$

A striking feature of this result is that the physical scale controlling the meson form factor in  $\alpha_V$  scheme is very low:  $e^{-3/2}Q \simeq 0.22Q$ , reflecting the characteristic momentum transfer experienced by the spectator valence quark in lepton-meson elastic scattering. We then also have

$$Q^2 F_{\gamma\pi}(Q^2) = 2f_\pi \left( 1 - \frac{5}{3} \frac{\alpha_V(Q_V^*)}{\pi} \right). \quad (12)$$

At this order of approximation, we will take  $Q_V^*$  to be the scale of the coupling that appears in the NLO correction for  $F_\pi$ .

We may also determine the renormalization scale of  $\alpha_V$  for more general forms of the coupling by direct integration over  $x$  and  $y$  in Eq. (10), assuming a specific analytic form for the coupling. Notice, however, that small corrections to the BLM scale will be compensated by a corresponding change in the NLO coefficient.

An important prediction resulting from the factorized form of these results is that the normalization of the ratio

$$R_\pi(Q^2) \equiv \frac{F_\pi(Q^2)}{4\pi Q^2 |F_{\pi\gamma}(Q^2)|^2} \quad (13)$$

$$= \alpha_{\overline{MS}}(e^{-14/6}Q) \left( 1 - 0.56 \frac{\alpha_{\overline{MS}}}{\pi} \right) \quad (14)$$

$$= \alpha_V(e^{-3/2}Q) \left( 1 + 1.43 \frac{\alpha_V}{\pi} \right) \quad (15)$$

$$= \alpha_R(e^{5/12-2\zeta_3}Q) \left( 1 - 0.65 \frac{\alpha_R}{\pi} \right) \quad (16)$$

is formally independent of the form of the pion distribution amplitude. The  $\alpha_{\overline{MS}}$  correction follows from combined references [20, 21, 22]. The next-to-leading correction given here assumes the asymptotic distribution amplitude.

The renormalization scales of the running couplings in Eqs. (14)–(16) have been fixed using BLM commensurate scale relation procedure. The BLM scales are determined from the explicit calculations of the NLO corrections to the pion and transition form factors given by Dittes and Radyushkin [20], Field *et al.* [21], and Braaten [22]. These may be written in the form  $(A(\mu)n_f + B(\mu))\alpha_s/\pi$ , where  $A$  is independent of the separation scale  $\tilde{Q}$ . The  $n_f$  dependence allows one to uniquely identify the



dependence on  $\beta_0$ , which is then absorbed into the running coupling by a shift to the BLM scale  $Q^* = e^{3A(\mu)}\mu$ . An important check of self-consistency is that the resulting value for  $Q^*$  is independent of the choice of the starting scale  $\mu$ .

We emphasize that when we relate  $R_\pi$  to  $\alpha_V$  or  $\alpha_R$  we relate observable to observable and thus there is no scheme ambiguity. The coefficients  $-0.56$ ,  $1.43$  and  $-0.65$  in Eqs. (14)–(16) are identical to those one would have in a theory with  $\beta = 0$ , *i.e.*, conformally invariant theory.

Contrary to the discussion by Chyla [23], the optimized  $Q^*$  is always *scheme* dependent. For example, in the  $\overline{MS}$  scheme one finds  $\mu^2 = (Q_{\overline{MS}}^*)^2 = e^{-5/3}(1-x)(1-y)Q^2$  for  $F_\pi(Q^2)$  [21, 3], whereas in the  $\alpha_V$  scheme the BLM scale is  $(Q_V^*)^2 = (1-x)(1-y)Q^2$ . The final results connecting observables are of course scheme-independent. The result for  $Q_V^{*2}$  is expected since in the  $\alpha_V$  scheme the scale of the coupling is identified with the virtuality of the exchanged gluon propagator, just as in the usual QED scheme, and here, to leading twist, the virtuality of the gluon is  $-(1-x)(1-y)Q^2$ . The resulting relations between the form factors and the heavy quark coupling are independent of the choice of intermediate renormalization scheme, however; they thus have no scale or scheme ambiguities.

Alternatively, we can write the pion form factor in terms of other effective charges such as the coupling  $\alpha_R(\sqrt{s})$  that defines the QCD radiative corrections to the  $e^+e^- \rightarrow X$  cross section:  $R(s) \equiv 3\Sigma e_q^2(1 + \alpha_R(\sqrt{s})/\pi)$ . The commensurate scale relation between  $\alpha_V$  and  $\alpha_R$  is

$$\alpha_V(Q_V) = \alpha_R(Q_R) \left( 1 - \frac{25}{12} \frac{\alpha_R}{\pi} + \dots \right), \quad (17)$$

where the ratio of commensurate scales to this order is  $Q_R/Q_V = e^{23/12 - 2\zeta_3} \simeq 0.614$ .

### 3 The Behavior of the QCD Coupling at Low Momentum

Effective charges such as  $\alpha_V$  and  $\alpha_R$  are defined from physical observables and thus must be finite even at low momenta. The conventional solutions of the renormalization group equation for the QCD couplings which are singular at  $Q \simeq \Lambda_{\text{QCD}}$  are not accurate representations of the effective couplings at low momentum transfer. It is clear that more parameters and information are needed to specify the coupling in the non-perturbative domain.

A number of proposals have been suggested for the form of the QCD coupling in the low-momentum regime. For example, Petronzio and Parisi [24] have argued that

the coupling must freeze at low momentum transfer in order that the perturbative QCD loop integrations are well defined. Mattingly and Stevenson [25] have incorporated such behavior into their parameterizations of  $\alpha_R$  at low scales. Gribov [26] has presented novel dynamical arguments related to the nature of confinement for a fixed coupling at low scales. Zerwas [27] has noted the heavy quark potential must saturate to a Yukawa form since the light-quark production processes will screen the linear confining potential at large distances. Cornwall [28] and others [29, 30] have argued that the gluon propagator will acquire an effective gluon mass  $m_g$  from non-perturbative dynamics, which again will regulate the form of the effective couplings at low momentum. In this work we shall adopt the simple parameterization

$$\alpha_V(Q) = \frac{4\pi}{\beta_0 \ln \left( \frac{Q^2 + 4m_g^2}{\Lambda_V^2} \right)}, \quad (18)$$

which effectively freezes the  $\alpha_V$  effective charge to a finite value for  $Q^2 \leq 4m_g^2$ .

We can use the non-relativistic heavy quark lattice results [18, 31] to fix the parameters. A fit to the lattice data of the above parameterization gives  $\Lambda_V = 0.16$  GeV if we use the well-known momentum-dependent  $n_f$  [32]. Furthermore, the value  $m_g^2 = 0.2$  GeV<sup>2</sup> gives consistency with the frozen value of  $\alpha_R$  advocated by Mattingly and Stevenson [25]. Their parameterization implies the approximate constraint  $\alpha_R(Q)/\pi \simeq 0.27$  for  $Q = \sqrt{s} < 0.3$  GeV, which leads to  $\alpha_V(0.5 \text{ GeV}) \simeq 0.37$  using the NLO commensurate scale relation between  $\alpha_V$  and  $\alpha_R$ . The resulting form for  $\alpha_V$  is shown in Fig. 1. The corresponding predictions for  $\alpha_R$  and  $\alpha_{\overline{MS}}$  using the commensurate scale relations at NLO are also shown. Note that for low  $Q^2$  the couplings, although frozen, are large. Thus the NLO and higher-order terms in the CSRs are large, and inverting them perturbatively to NLO does not give accurate results at low scales. In addition, higher-twist contributions to  $\alpha_V$  and  $\alpha_R$ , which are not reflected in the CSR relating them, may be expected to be important for low  $Q^2$  [33].

It is clear that exclusive processes such as the pion and photon to pion transition form factors can provide a valuable window for determining the magnitude and the shape of the effective charges at quite low momentum transfers. In particular, we can check consistency with the  $\alpha_V$  prediction from lattice gauge theory. A complimentary method for determining  $\alpha_V$  at low momentum is to use the angular anisotropy of  $e^+e^- \rightarrow Q\overline{Q}$  at the heavy quark thresholds [34]. It should be emphasized that this parameterization (Eq. (18)) is just an approximate form. The actual behavior of  $\alpha_V(Q^2)$  at low  $Q^2$  is one of the key uncertainties in QCD phenomenology. In this paper we shall use exclusive observables to deduce information on this quantity.

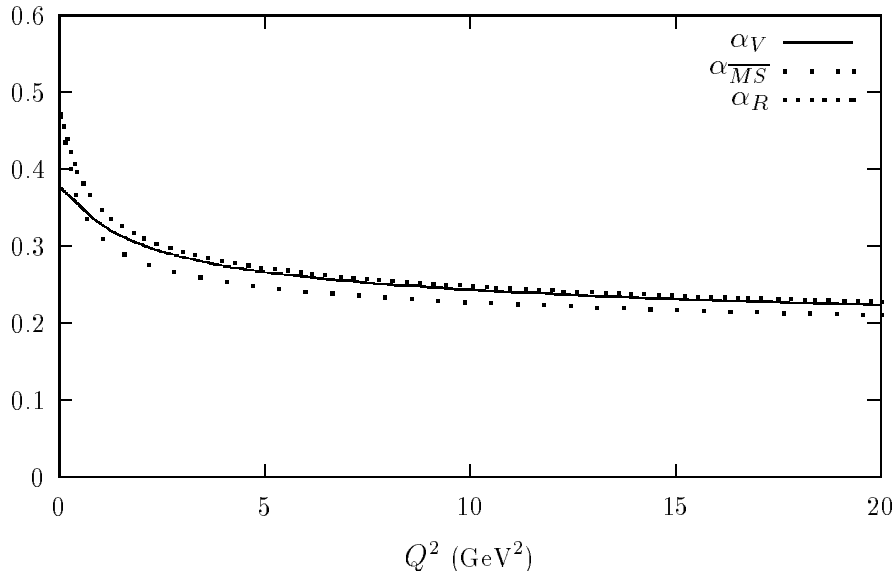


Figure 1: The coupling function  $\alpha_V(Q^2)$  as given in Eq. (18). Also shown are the corresponding predictions for  $\alpha_{\overline{MS}}$  and  $\alpha_R$  following from the NLO commensurate scale relations [Eqs. (2) and (17)].

## 4 Applications

As we have emphasized, exclusive processes are sensitive to the magnitude and shape of the QCD couplings at quite low momentum transfer:  $Q_V^{*2} \simeq e^{-3}Q^2 \simeq Q^2/20$  and  $Q_R^{*2} \simeq Q^2/50$  [35]. The fact that the data for exclusive processes such as form factors, two photon processes such as  $\gamma\gamma \rightarrow \pi^+\pi^-$ , and photoproduction at fixed  $\theta_{c.m.}$  are consistent with the nominal scaling of the leading twist QCD predictions (dimensional counting) at momentum transfers  $Q$  up to the order of a few GeV can be immediately understood if the effective charges  $\alpha_V$  and  $\alpha_R$  are slowly varying at low momentum. The scaling of the exclusive amplitude then follows that of the subprocess amplitude  $T_H$  with effectively fixed coupling. Note also that the Sudakov effect of the end point region is the exponential of a double log series if the coupling is constant, and thus is strong.

In Fig. 2, we compare the recent CLEO data [36] for the photon to pion transition form factor with the prediction

$$Q^2 F_{\gamma\pi}(Q^2) = 2f_\pi \left( 1 - \frac{5}{3} \frac{\alpha_V(e^{-3/2}Q)}{\pi} \right). \quad (19)$$

The flat scaling of the  $Q^2 F_{\gamma\pi}(Q^2)$  data from  $Q^2 = 2$  to  $Q^2 = 8$  GeV<sup>2</sup> provides an

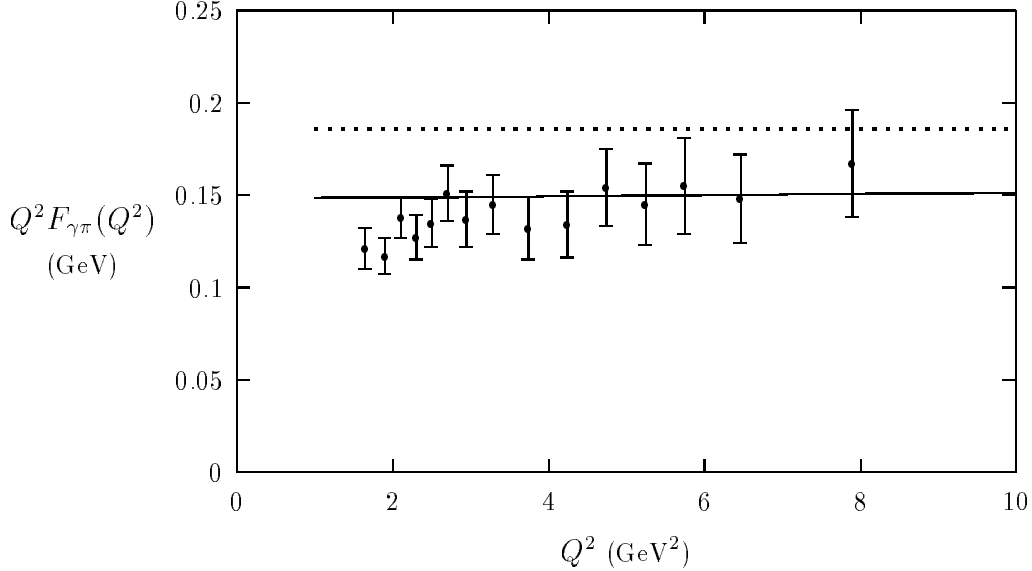


Figure 2: The  $\gamma \rightarrow \pi^0$  transition form factor. The solid line is the full prediction including the QCD correction [Eq. (19)]; the dotted line is the LO prediction  $Q^2 F_{\gamma\pi}(Q^2) = 2f_\pi$ .

important confirmation of the applicability of leading twist QCD to this process. The magnitude of  $Q^2 F_{\gamma\pi}(Q^2)$  is remarkably consistent with the predicted form assuming the asymptotic distribution amplitude and including the LO QCD radiative correction with  $\alpha_V(e^{-3/2}Q)/\pi \simeq 0.12$ . Radyushkin [37], Ong [38] and Kroll [39] have also noted that the scaling and normalization of the photon-to-pion transition form factor tends to favor the asymptotic form for the pion distribution amplitude and rules out broader distributions such as the two-humped form suggested by QCD sum rules [40]. One cannot obtain a unique solution for the non-perturbative wavefunction from the  $Q^2 F_{\pi\gamma}$  data alone. However, we have the constraint that

$$\frac{1}{3} \left\langle \frac{1}{1-x} \right\rangle \left[ 1 - \frac{5}{3} \frac{\alpha_V(Q^*)}{\pi} \right] \simeq 0.8 \quad (20)$$

(assuming the renormalization scale we have chosen in Eq. (12) is approximately correct). Thus one could allow for some broadening of the distribution amplitude with a corresponding increase in the value of  $\alpha_V$  at low scales.

In Fig. 3 we compare the existing measurements of the space-like pion form factor  $F_\pi(Q^2)$  [41, 42] (obtained from the extrapolation of  $\gamma^* p \rightarrow \pi^+ n$  data to the pion pole) with the QCD prediction 11, again assuming the asymptotic form of the

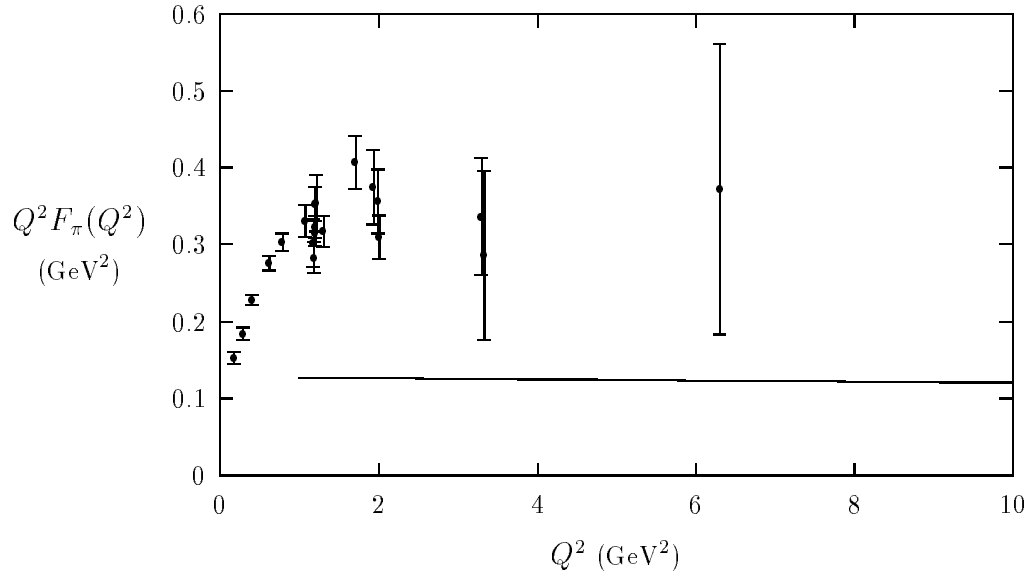


Figure 3: The space-like pion form factor.

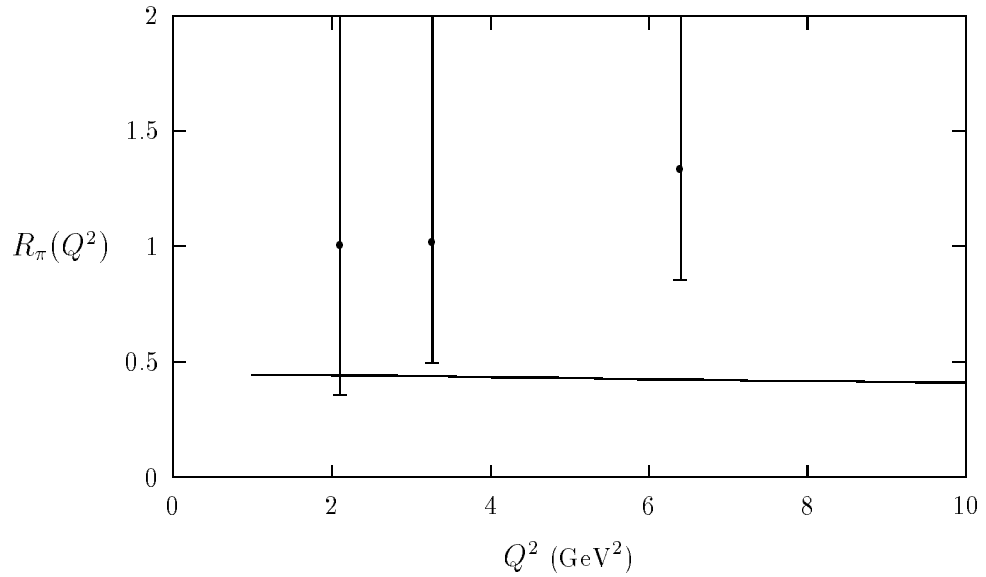


Figure 4: The ratio  $R_\pi(Q^2) \equiv \frac{F_\pi(Q^2)}{4\pi Q^2 |F_{\pi\gamma}(Q^2)|^2}$ .

pion distribution amplitude and  $\alpha_V(e^{-3/2}Q)/\pi \simeq 0.12$ . The scaling of the pion form factor data is again important evidence for the nominal scaling of the leading twist prediction. However, the prediction is lower than the data by approximately a factor of 2. The same feature can be seen in the ratio  $R_\pi(Q^2)$  (Fig. 4), in which the uncertainties due to the unknown form of the pion distribution amplitude tend to cancel out.

We have also analyzed the  $\gamma\gamma \rightarrow \pi^+\pi^-$  data. These data exhibit true leading-twist scaling, so that one would expect this process to be a good test of theory. One can show [43] that to LO

$$\frac{\frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+\pi^-)}{\frac{d\sigma}{dt}(\gamma\gamma \rightarrow \mu^+\mu^-)} = \frac{4|F_\pi(s)|^2}{1 - \cos^4 \theta_{c.m.}} \quad (21)$$

in the CMS, where  $dt = (s/2)d(\cos \theta_{c.m.})$  and here  $F_\pi(s)$  is the *time-like* pion form factor. The ratio of the time-like to space-like pion form factor for the asymptotic distribution amplitude is given by

$$\frac{|F_\pi^{(timelike)}(-Q^2)|}{F_\pi^{(spacelike)}(Q^2)} = \frac{|\alpha_V(-Q^{*2})|}{\alpha_V(Q^{*2})}. \quad (22)$$

If we simply continue Eq. (18) to negative values of  $Q^2$  (Fig. 5), then for  $1 < Q^2 < 10$  GeV<sup>2</sup>, and hence  $0.05 < Q^{*2} < 0.5$  GeV<sup>2</sup>, the ratio of couplings in Eq. (22) is of order 1.5. Of course this assumes the analytic application of Eq. (18). Thus if we assume the asymptotic form for the distribution amplitude, then we predict  $F_\pi^{(timelike)}(-Q^2) \simeq (0.3 \text{ GeV}^2)/Q^2$  and hence

$$\frac{\frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+\pi^-)}{\frac{d\sigma}{dt}(\gamma\gamma \rightarrow \mu^+\mu^-)} \simeq \frac{.36}{s^2} \frac{1}{1 - \cos^4 \theta_{c.m.}}. \quad (23)$$

The resulting prediction for  $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-)$  is shown in Fig. 6, along with the data of Ref. [44]. Considering the possible contribution of the resonance  $f_2(1270)$ , the agreement is reasonable.

It should be noted that the leading-twist prediction  $Q^2 F_\pi^{(timelike)}(-Q^2) = 0.3$  GeV<sup>2</sup> is a factor of two below the measurement of the pion form factor obtained from the  $J/\psi \rightarrow \pi^+\pi^-$  branching ratio. The  $J/\psi$  analysis assumes that the  $\pi^+\pi^-$  is created only through virtual photons. However, if the  $J/\psi \rightarrow \pi^+\pi^-$  amplitude proceeds through channels such as  $\gamma gg$ , then the branching ratio is not a precise method for obtaining  $F_\pi^{(timelike)}$ . It is thus important to have direct measurement of the  $e^+e^- \rightarrow \pi^+\pi^-$  amplitude off-resonance. We also show the prediction for the pion

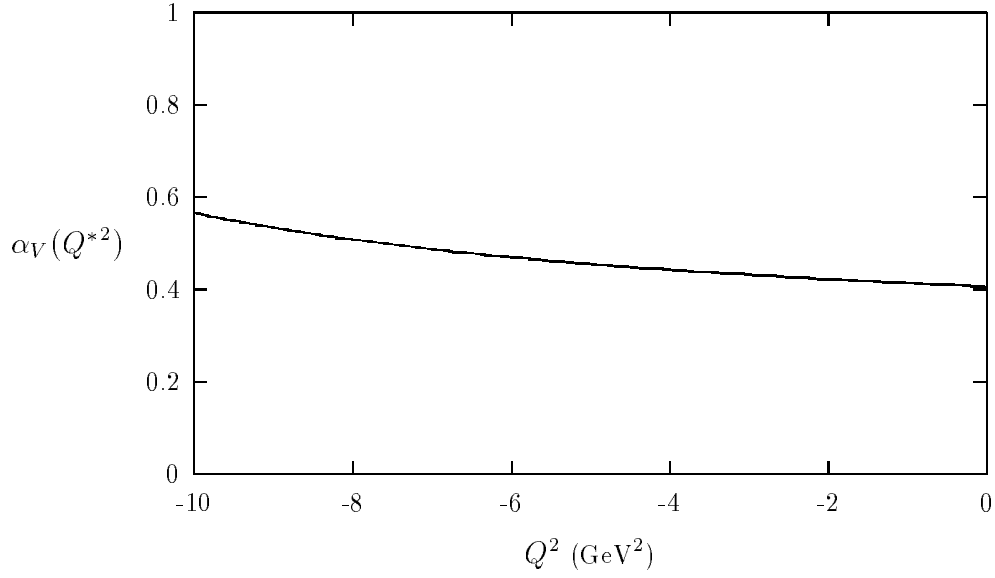


Figure 5: Continuation of Eq. (18) to negative  $Q^2$ . Note that  $Q^{*2} \equiv e^{-3}Q^2$ .

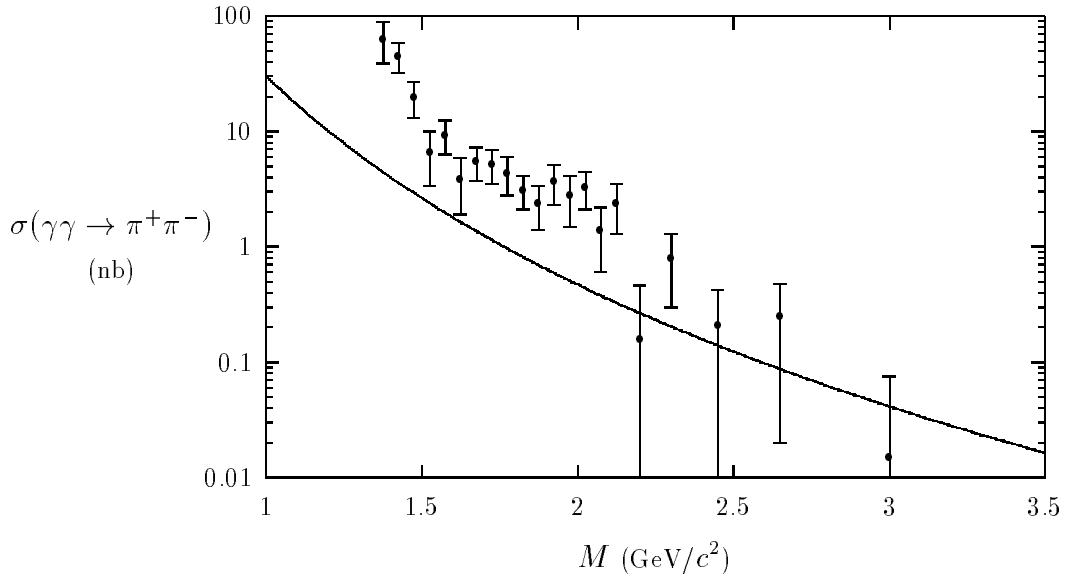


Figure 6: Two-photon annihilation cross section as a function of CMS energy.

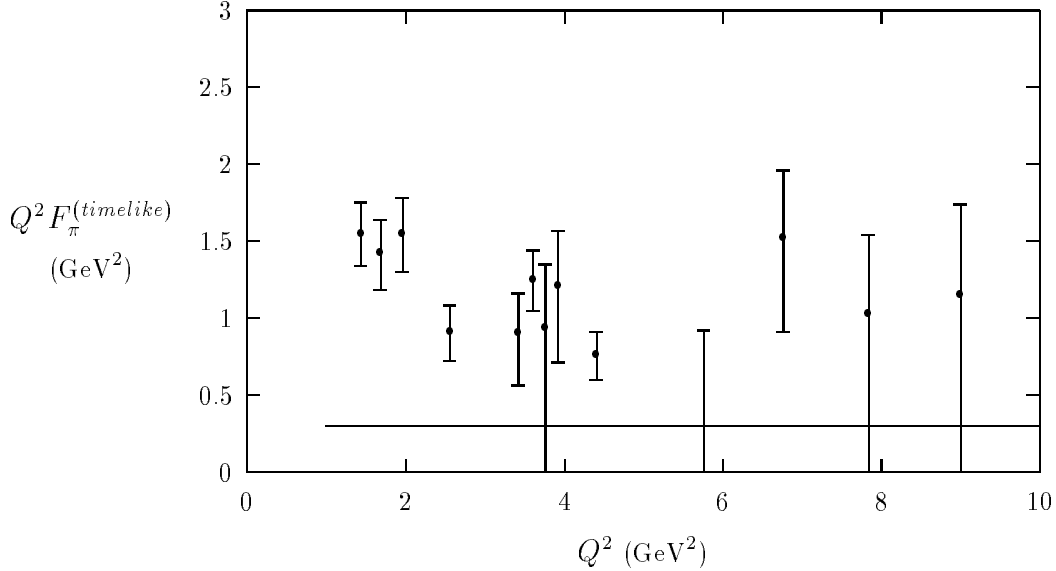


Figure 7: Pion electromagnetic form factor in the time-like region.

form factor in the time-like region compared with the data of Bollini, *et al.* [45] in Fig. 7. We emphasize that the normalization of the prediction

$$\begin{aligned}
 F_{\pi}^{(timelike)}(-Q^2) &= \frac{16\pi f_{\pi}^2}{Q^2} \alpha_V(-Q^{*2}) \left(1 - 1.9 \frac{\alpha_V}{\pi}\right) \\
 &\approx \frac{0.3 \text{ GeV}^2}{Q^2}
 \end{aligned}
 \tag{24}$$

assumes the asymptotic form for the pion distribution amplitude and the form of  $\alpha_V$  given in Eq. (18), with the parameters  $m_g^2 = 0.2 \text{ GeV}^2$  and  $\Lambda_V = 0.16 \text{ GeV}$ . There is clearly some room to readjust these parameters. However, even at the initial stage of approximation done in this paper, which includes NLO corrections at the BLM scale, there is no significant discrepancy with the relevant experiments.

The values for the space-like pion form factor  $F_{\pi}(Q^2)$  obtained from the extrapolation of  $\gamma^* p \rightarrow \pi^+ n$  data to the pion pole thus appear to be systematically higher in normalization than predicted by commensurate scale relations; however, it should be emphasized that this discrepancy may be due to systematic errors introduced by the extrapolation procedure [46]. What is at best measured in electroproduction is the transition amplitude between a mesonic state with an effective space-like mass  $m^2 = t < 0$  and the physical pion. It is theoretically possible that the off-shell form factor  $F_{\pi}(Q^2, t)$  is significantly larger than the physical form factor because of its bias



towards more point-like  $q\bar{q}$  valence configurations in its Fock state structure. The extrapolation to the pole at  $t = m_\pi^2$  also requires knowing the analytic dependence of  $F_\pi(Q^2, t)$  on  $t$ . These considerations are discussed further in Ref. [47]. If we assume that there are no significant errors induced by the electroproduction extrapolation, then one must look for other sources for the discrepancy in normalization. Note that the NLO corrections in Eqs. (11) and (15) are of order 20–30%. Thus there may be large contributions from NNLO and higher corrections which need to be re-summed. There are also possible corrections from pion rescattering in the final state of the electroproduction process. It thus would be very interesting to have unambiguous data on the pion form factors from electron-pion collisions, say, by scattering electrons on a secondary pion beam at the SLAC Linear Collider.

We also note that the normalization of  $\alpha_V$  could be larger at low momentum than our estimate. This would also imply a broadening of the pion distribution amplitude compared to its asymptotic form since one needs to raise the expectation value of  $1/(1-x)$  in order to maintain consistency with the magnitude of the  $Q^2 F_{\gamma\pi}(Q^2)$  data.<sup>†</sup> A full analysis will then also require consideration of the breaking of scaling from the evolution of the distribution amplitude.

In any case, we find no compelling argument for significant higher-twist contributions in the few GeV regime from the hard scattering amplitude or the endpoint regions, since such corrections violate the observed scaling behavior of the data.

The time-like pion form factor data obtained from  $e^+e^- \rightarrow \pi^+\pi^-$  annihilation does not have complications from off-shell extrapolations or rescattering, but it is also more sensitive to nearby vector meson poles in the  $t$  channel. If we analytically continue the leading twist prediction and the effective form of  $\alpha_V$  to the time-like regime, we obtain the prediction shown in Fig. 7, again assuming the asymptotic form of the pion distribution amplitude.

The analysis we have presented here suggests a systematic program for estimating exclusive amplitudes in QCD. The central input is  $\alpha_V(0)$ , or

$$\frac{1}{Q_0^2} \int_0^{Q_0^2} dQ'^2 \alpha_V(Q'^2), \quad Q_0^2 \leq 1 \text{ GeV}^2, \quad (25)$$

which largely controls the magnitude of the underlying quark-gluon subprocesses for hard processes in the few-GeV region. In this work, the mean coupling value for  $0 < Q^2 < Q_0^2 \simeq 1 \text{ GeV}^2$  corresponding to Eq. (25) is  $\overline{\alpha_V} \simeq 0.37$ . The main focus will then be to determine the shapes and normalization of the process-independent meson and baryon distribution amplitudes.

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<sup>†</sup>Again, this assumes that the scale in Eq. (12) has been set correctly.

## 5 Conclusions

In this paper we have shown that dimensional counting rules emerge if the effective coupling  $\alpha_V(Q^*)$  is approximately constant in the domain of  $Q^*$  relevant to the hard scattering amplitudes of exclusive processes. In the low- $Q^*$  domain, evolution of the quark distribution amplitudes is also minimal. Furthermore, Sudakov suppression of the long-distance contributions is strengthened if the coupling is frozen because of the exponentiation of a double log series. The Ansatz of a frozen coupling at small momentum transfer has not been demonstrated from first principles. However, the behavior of exclusive amplitudes point strongly to scaling behavior in the kinematic regions we discussed. We have also found that the commensurate scale relation connecting the heavy quark potential, as determined from lattice gauge theory, to the photon-to-pion transition form factor is in excellent agreement with  $\gamma e \rightarrow \pi^0 e$  data assuming that the pion distribution amplitude is close to its asymptotic form  $\sqrt{3}f_\pi x(1-x)$ . We also reproduce the scaling and normalization of the  $\gamma\gamma \rightarrow \pi^+\pi^-$  data at large momentum transfer. However, the normalization of the space-like pion form factor  $F_\pi(Q^2)$  obtained from electroproduction experiments is somewhat higher than that predicted by the corresponding commensurate scale relation. This discrepancy may be due to systematic errors introduced by the extrapolation of the  $\gamma^*p \rightarrow \pi^+n$  electroproduction data to the pion pole.

### Acknowledgements

It is a pleasure to thank Hung Jung Lu for many valuable discussions during the early stages of this work. We also thank Martin Beneke and Volodya Braun for helpful conversations. S.J.B. is supported in part by the U.S. Department of Energy under contract no. DE-AC03-76SF00515. C.-R.J. is supported in part by the U.S. Department of Energy under contract no. DE-FG02-96ER40947. The North Carolina Supercomputer Center is also acknowledged for the grant of supercomputing time allocation. A.P. is supported in part by the National Science Foundation under research contract NSF-PHY94-08843. D.G.R. is supported in part by the U.S. Department of Energy under contract no. DE-FG02-91ER40690.

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