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[1]Light-Cone Wavefunctions at Small x

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Abstract

There exist an infinite number of exact small momentum fraction-x boundary conditions on light-cone wavefunctions of bound states in gauge theory. They are necessary for finite expectation values of the invariant mass operator and relate components of the wavefunction from different Fock sectors. We illustrate their consequences by analyzing the small-x quark Regge behavior of a heavy large-N meson, finding power-law rise of unpolarized distributions. The polarized distribution changes sign and then vanishes with minus the unpolarized Regge intercept.

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1 Light-Cone Dynamics

Light-cone quantization of QCD is a promising tool to describe the wealth of experimental information about hadronic structure in terms of quark and gluon degrees of freedom [1]. It has the advantage of dealing explicitly with the hadronic wavefunction in a general Lorentz frame, and it is particularly convenient for analyzing matrix elements of currents such as form factors and light-cone dominated inclusive processes. In practice the calculation of a hadronic wavefunction presents a formidable many-body problem since arbitrary numbers of gluons and sea quarks can play a significant role.

Large numbers of partons necessarily have large free energy in the light-cone formalism. In this paper we shall point out some rather simple but important exact restrictions on the light-cone wavefunctions which follow from high light-cone energy boundary conditions alone. These 'ladder relations' relate Fock space sectors containing different numbers of partons. Thus the different Fock components of a hadronic wavefunction in QCD are not analytically independent. We give a simple application to the quark distribution function of a heavy meson as an illustration of the physical consequences.

Each hadronic bound state in the light-cone Hamiltonian formalism of QCD is an eigenstate $|\Psi(P^+, \mathbf{P}^\perp)>$ of the invariant mass operator $\hat{M}^2 = 2P^+P^- - |\mathbf{P}^\perp|^2$ where $P^- = (P^0 - P^3)/\sqrt{2}$ is the light-cone energy, which is the displacement operator in light-cone time $x^+ = (x^0 + x^3)/\sqrt{2}$, while $P^+ = (P^0 + P^3)/\sqrt{2}$ and $\mathbf{P}^\perp$ are the conserved total momenta. The operator P^- contains both the kinetic energy and interaction parts of the light-cone Hamiltonian. The eigenfunction of the bound state can be expanded on the Fock basis of free quark and gluons. The light-cone energy k^- of each such constituent of mass m carrying light-cone longitudinal momentum k^+ and transverse momentum $\mathbf{k}^\perp = (k^1, k^2)$ is

$$k^- = \frac{m^2 + |\mathbf{k}^\perp|^2}{2k^+} \ . \tag{1}$$

The light-cone wavefunction for the $n-$parton state can be labelled by its constituents' momenta $\mathbf{k}_i = (k_i^+, \mathbf{k}_i^\perp)$ and helicities α_i : $f_{\alpha_1 \cdots \alpha_n}(\mathbf{k}_1, \cdots, \mathbf{k}_n)$ where $\sum_i k_i^+ = P^+$ and $\sum \mathbf{k}_i^\perp = \mathbf{0}^\perp$. At first sight, one would expect that the finiteness of the kinetic part of the operator P^- would always force the light-cone wavefunction for each Fock component to vanish at $x_i = k_i^+/P^+ = 0$ for each fermion constituent since $m^2 > 0$. In fact, we shall show that in theories with Yukawa-like (i.e. fermion-boson-fermion) interactions such as gauge theories, that this is not the case.

For example, consider the light-cone $SU(N)$ gauge theory quantized on the surface $x^+ = 0$ in the light-cone gauge $A^+ = A_- = 0$ with one quark flavor. It is well-known that A_+ and half the components of the quark spinor, the left-moving half (v_+, v_-) of a chiral representation say, with $\pm$ chiralities, are constrained fields. They may be eliminated by their equations of motion

$$\mathrm{i}\partial_- v_\pm \quad = \quad F_\mp \tag{2}$$

$$\partial_-^2 A_+ \quad = \quad J \tag{3}$$

where

$$F_+ \quad = \quad \mathrm{i}(\partial_z + \mathrm{i}gA_z)u_- + \frac{m}{\sqrt{2}}u_+ \tag{4}$$

$$F_- \quad = \quad -\mathrm{i}(\partial_{\bar{z}} + \mathrm{i}gA_{\bar{z}})u_+ + \frac{m}{\sqrt{2}}u_- \tag{5}$$

$$\begin{aligned} J \quad = \quad & \partial_-(\partial_z A_{\bar{z}} + \partial_{\bar{z}} A_z) + g(\mathrm{i}[A_z, \partial_- A_{\bar{z}}] + \mathrm{i}[A_{\bar{z}}, \partial_- A_z]) \\ & + g(u_+ u_+^\dagger + u_- u_-^\dagger) \ . \end{aligned} \tag{6}$$

In the above expressions

$$A_z \equiv \frac{1}{\sqrt{2}}(A_1 - \mathrm{i}A_2) \qquad , \qquad \partial_z \equiv \frac{1}{\sqrt{2}}(\partial_1 - \mathrm{i}\partial_2) \ , \tag{7}$$

$$A_{\bar{z}} \equiv \frac{1}{\sqrt{2}}(A_1 + \mathrm{i}A_2) \qquad , \qquad \partial_{\bar{z}} \equiv \frac{1}{\sqrt{2}}(\partial_1 + \mathrm{i}\partial_2) \ , \tag{8}$$

and $u_\pm$ form respectively the positive and negative chiralities of the remaining right-moving components of the quark. The exchange of non-propagating particles associated with the constrained fields results in non-local interactions in the light-cone Hamiltonian

$$P^- = \int dx^- d\mathbf{x}^\perp \left\{ F_+^\dagger \frac{1}{\mathrm{i}\partial_-} F_+ + F_-^\dagger \frac{1}{\mathrm{i}\partial_-} F_- + \frac{1}{2}\mathrm{Tr}\left[-J\frac{1}{\partial_-^2} J + (\mathcal{F}_{12})^2 \right] \right\} \tag{9}$$

where $\mathcal{F}_{12} = \partial_1 A_2 - \partial_2 A_1 + \mathrm{i}g[A_1, A_2]$. We see that the free-fermion kinetic term is replaced by a gauge-field dependent expression,

$$\frac{m^2}{2} u^\dagger \frac{1}{\mathrm{i}\partial_-} u \to F^\dagger \frac{1}{\mathrm{i}\partial_-} F \ , \tag{10}$$

in analogy with the replacement

$$\frac{p^2}{2m} \to \frac{(p - eA)^2}{2m} \tag{11}$$

of non-relativistic electrodynamics. Thus the combination F plays a special role as a fermionic 'mechanical velocity' in the gauge extension of the free kinetic energy of the gauge theory. The zero momentum limit of the constraint equation (2) now forces an interaction-dependent condition on the quark-gluon combined system

$$\int_{-\infty}^{+\infty} dx^- F_\pm = 0 \ , \tag{12}$$

implying that the field v vanishes at $x^- = \pm\infty$. This condition is necessary for finiteness of the interactions non-local in x^- involving F in (9), at fixed transverse co-ordinates and simply translates into a condition on the fields at vanishing longitudinal momentum $k^+ = 0$. However, the finiteness condition does not imply the individual fixed particle number light-cone wavefunctions have to vanish at $x_i = 0$, but rather that combinations of the wavefunctions involving one more and one less gluon quanta are related at the small x boundary.

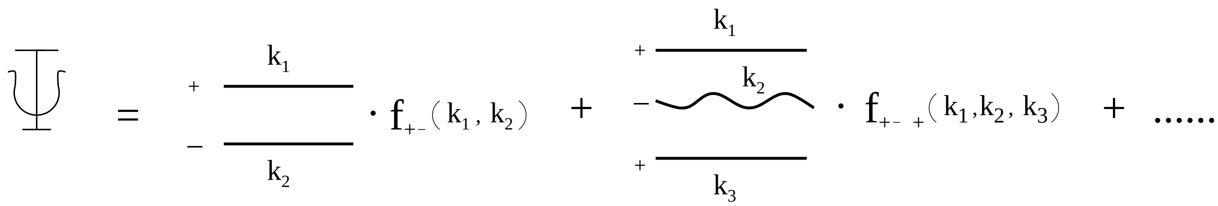

Figure 1: Expansion of the large-N meson light-cone wavefunction (for a total helicity zero example) in terms of parton Fock components, represented by propagators for quark and anti-quark (solid) and gluons (wavy).

2 Ladder Relations.

In order to elucidate the consequences of (12) in the quantum theory, we can consider its effect on specific light-cone wavefunctions. Introducing the harmonic oscillator modes of the physical fields[3]

$$
\begin{aligned}
A_{z\,ij}(x^-,\mathbf{x}^\perp) &= \frac{1}{(2\pi)^{3/2}}\int_0^\infty \frac{dk^+}{\sqrt{2k^+}}\int d\mathbf{k}^\perp \times \\
&\left[a_{+ij}(k^+,\mathbf{k}^\perp)e^{-\mathrm{i}(k^+x^- - \mathbf{k}^\perp\cdot\mathbf{x}^\perp)} + a^\dagger_{-ji}(k^+,\mathbf{k}^\perp)e^{+\mathrm{i}(k^+x^- - \mathbf{k}^\perp\cdot\mathbf{x}^\perp)}\right] \qquad (13) \\
u_{\pm\,i}(x^-,\mathbf{x}^\perp) &= \frac{1}{(2\pi)^{3/2}}\int_0^\infty dk^+\int d\mathbf{k}^\perp \times \\
&\left[b_{\pm i}(k^+,\mathbf{k}^\perp)e^{-\mathrm{i}(k^+x^- - \mathbf{k}^\perp\cdot\mathbf{x}^\perp)} + d^\dagger_{\mp i}(k^+,\mathbf{k}^\perp)e^{+\mathrm{i}(k^+x^- - \mathbf{k}^\perp\cdot\mathbf{x}^\perp)}\right]\ , \qquad (14)
\end{aligned}
$$

we can expand any hadron state $|\Psi(P^+,\mathbf{P}^\perp)>$ in terms of a Fock basis. The operators $a^\dagger_\pm$ create gluons with helicity ± 1, while $b^\dagger_\pm$ and $d^\dagger_\pm$ correspond to quarks and antiquarks (respectively) with helicities $\pm\frac{1}{2}$. For concreteness we will consider a meson in the frame $\mathbf{P}^\perp = 0$ in the large N limit, using a basis of Fock states singlet under residual global gauge transformations. At large N there is gluon but not quark pair production, so a meson is the superposition of $\bar{q}q$, $\bar{q}gq$, $\bar{q}ggq$, $\bar{q}gggq$, and so on. Explicitly,

$$
\begin{aligned}
|\Psi(P^+)> = \sum_{n=2}^{\infty}\int_0^{P^+} & dk_1^+\ldots dk_n^+ \sum_{\alpha_i=\pm}\ \delta(k_1^+ + \cdots + k_n^+ - P^+)\times \\
\int & d\mathbf{k}_1^\perp\ldots d\mathbf{k}_n^\perp\ \delta(\mathbf{k}_1^\perp+\cdots+\mathbf{k}_n^\perp)\ f_{\alpha_1\ldots\alpha_n}(\mathbf{k}_1,\mathbf{k}_2,\ldots,\mathbf{k}_n)\times \\
& \frac{1}{\sqrt{N^{n-1}}} d^\dagger_{\alpha_1 i}(\mathbf{k}_1)a^\dagger_{\alpha_2 ij}(\mathbf{k}_2)a^\dagger_{\alpha_3 jk}(\mathbf{k}_3)\ldots a^\dagger_{\alpha_{n-1}lm}(\mathbf{k}_{n-1})b^\dagger_{\alpha_n m}(\mathbf{k}_n)|0> \qquad (15)
\end{aligned}
$$

where repeated indices are summed over and the coefficients f, depending on the momenta and helicities, diagonalize $\hat{M}^2$. If one writes the Fock wavefunctions $f_{\alpha_1\ldots\alpha_n}$ in terms of the light-cone momentum fractions $x_i = k_i^+/P^+$ and relative transverse momentum $\mathbf{k}_i^\perp - x_i\mathbf{P}^\perp$ then the Fock representation is independent of the total momentum P^+ and $\mathbf{P}^\perp$.

The quantum version of the statements in the preceding section turns out to be a little delicate due to operator ordering ambiguities. We have chosen a prescription which is at least consistent at large N.

[3] $i,j = 1,\ldots,N$ are gauge indices and $\dagger$ is here understood as the quantum complex conjugate, so it does not transpose them.

Introducing

$$\tilde{F}_{\pm}(k^+, \mathbf{k}^\perp) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} dx^- d\mathbf{x}^\perp \; F_{\pm}(x^-, \mathbf{x}^\perp) e^{-\mathrm{i}(k^+ x^- - \mathbf{k}^\perp \cdot \mathbf{x}^\perp)} \tag{16}$$

we find that (12) can be meaningfully applied as an annihilator of physical states for the cases

$$\lim_{k^+ \to 0^-} \tilde{F}_{\pm i}(k^+, \mathbf{k}^\perp) \cdot |\Psi(P^+, \mathbf{P}^\perp) > \;=\; 0 \tag{17}$$

$$\lim_{k^+ \to 0^+} \tilde{F}^\dagger_{\pm i}(k^+, \mathbf{k}^\perp) \cdot |\Psi(P^+, \mathbf{P}^\perp) > \;=\; 0 \; . \tag{18}$$

The first relation yields a condition on the Fock space wavefunctions f involving vanishing quark longitudinal momentum, the second on vanishing anti-quark longitudinal momentum:

$$\begin{aligned} & m f_{\mp\pm\alpha_1\cdots\alpha_n}(\mathbf{k}, \mathbf{k}_1, \ldots, \mathbf{k}_{n+1}) \\ & \pm (k^1 \pm \mathrm{i}k^2) f_{\pm\pm\alpha_1\cdots\alpha_n}(\mathbf{k}, \mathbf{k}_1, \ldots, \mathbf{k}_{n+1}) \\ & \quad = \; \pm \frac{g\sqrt{N}}{(2\pi)^{3/2}} \left[\frac{f_{\pm\alpha_1\cdots\alpha_n}(\mathbf{k}+\mathbf{k}_1, \mathbf{k}_2, \ldots, \mathbf{k}_{n+1})}{\sqrt{k_1^+}} \right. \\ & \qquad + \int_0^\infty \frac{dp^+ dq^+}{\sqrt{q^+}} \, \delta(p^+ + q^+ - k^+) \int d\mathbf{p}^\perp d\mathbf{q}^\perp \; \delta(\mathbf{p}^\perp + \mathbf{q}^\perp - \mathbf{k}^\perp) \times \\ & \qquad\qquad \left. f_{\pm\mp\pm\alpha_1\cdots\alpha_n}(\mathbf{p}, \mathbf{q}, \mathbf{k}_1, \ldots, \mathbf{k}_{n+1}) \right] \end{aligned} \tag{19}$$

and

$$\begin{aligned} & m f_{\mp\mp\alpha_1\cdots\alpha_n}(\mathbf{k}, \mathbf{k}_1, \ldots, \mathbf{k}_{n+1}) \\ & \pm (k^1 \pm \mathrm{i}k^2) f_{\pm\mp\alpha_1\cdots\alpha_n}(\mathbf{k}, \mathbf{k}_1, \ldots, \mathbf{k}_{n+1}) \\ & \quad = \; \pm \frac{g\sqrt{N}}{(2\pi)^{3/2}} \int_0^\infty \frac{dp^+ dq^+}{\sqrt{q^+}} \, \delta(p^+ + q^+ - k^+) \int d\mathbf{p}^\perp d\mathbf{q}^\perp \; \delta(\mathbf{p}^\perp + \mathbf{q}^\perp - \mathbf{k}^\perp) \times \\ & \qquad\qquad f_{\pm\mp\mp\alpha_1\cdots\alpha_n}(\mathbf{p}, \mathbf{q}, \mathbf{k}_1, \ldots, \mathbf{k}_{n+1}) \; , \end{aligned} \tag{20}$$

with a similar set of relations for quarks; in (19) and (20) $\mathbf{k} = (k^+, \mathbf{k}^\perp)$ and the limit $k^+ \to 0^+$ is understood. We can interpret the above relations as the leading result for small but finite k^+, with corrections at higher order in k^+/k_1^+ etc. Similar 'ladder relations' were first considered in the context of a 'collinear' one-space, one-time approximation to light-cone QCD [2, 3] If we adopt the following momentum-space operator ordering in P^- (9)

$$\int d\mathbf{k}_\perp \left\{ - \int_{-\infty}^0 \frac{dk^+}{k^+} \left(\tilde{F}^\dagger_{+i} \tilde{F}_{+i} + \tilde{F}^\dagger_{-i} \tilde{F}_{-i} \right) + \int_0^\infty \frac{dk^+}{k^+} \left(\tilde{F}_{+i} \tilde{F}^\dagger_{+i} + \tilde{F}_{-i} \tilde{F}^\dagger_{-i} \right) \right\} \; , \tag{21}$$

this manifestly ensures finiteness at the $k^+ = 0$ pole. Normal ordering the oscillator modes in P^- would spoil finiteness. Since we do not use normal ordering of the form (21), infinite quark self energies (self-inertias) are generated but no vacuum energies are generated. One sees explicitly from (19) and (20) that the wavefunction components with at least one gluon do not in general vanish for small quark or antiquark k^+. This is an intrinsic property of the bound state, i.e. no reference has been made to perturbation theory or special $\mathbf{k}_\perp$ kinematic regimes. It is a boundary condition on wavefunctions necessary for finite expectation value of the invariant mass operator.

The wavefunction components f are the solutions to the bound state problem as represented by a light-cone relativistic Schrödinger many-body matrix equation: if $\mathcal{M}$ is the bound state mass eigenvalue, projecting $\hat{M}^2|\Psi>=\mathcal{M}^2|\Psi>$ onto a specific n-parton Fock state one derives

$$\left(\mathcal{M}^2 - \sum_{i=1}^{n}\left[\frac{(\mathbf{k}_i^\perp)^2 + m_i^2}{x_i}\right]\right) f_{\alpha_1\ldots\alpha_n}(\mathbf{k}_1,\mathbf{k}_2,\ldots,\mathbf{k}_n) = \hat{V}\left[f_{\alpha_1\ldots\alpha_n}(\mathbf{k}_1,\mathbf{k}_2,\ldots,\mathbf{k}_n)\right] \tag{22}$$

with interaction kernel $\hat{V}$ (including self-inertias). In eq.(22) a small momentum limit $x_i = k_i^+/P^+ \to 0$ for given i also has the potential to yield a boundary condition on the wavefunction independent of the mass $\mathcal{M}$. However this is equivalent to imposing that $\sim \tilde{F}^\dagger \tilde{F}$ at zero momentum should annihilate physical states, evidently a more complicated condition than eq.(17). Eq.(22) does have the advantage that one can consider more than one parton having small momentum and corrections to the small momentum limit.

So far, the parameters in $\hat{V}$ have been interpreted as the bare ones of an ultraviolet-regulated Hamiltonian. When the quark small-x boundary approaches another high-energy corner of phase space, such as large transverse momentum or gluon small x, further counterterms must be added to obtain finite answers. The wavefunctions which are integrated in the relations (19) and (20) involve *two* vanishingly small longitudinal momenta as $k^+ \to 0^+$. Because of this, although the integration domain has vanishingly small measure, the integrands are sufficiently singular to give a non-zero result. To determine the singular behavior of the wavefunction components appearing in the integrand requires us in general to study the renormalization of the light-cone Schrödinger equation (22), taking account of all the high energy cut-offs which must be applied, which is beyond the scope of this paper. However the net effect will be virtual corrections to non-integral terms in (19) and (20). One uses (22) to express the integrands in terms of different Fock sector components. This generates corrections to the non-integral terms in (19) and (20) plus further integrals involving more partons and/or higher powers of the coupling. The above procedure may then be repeated iteratively for the latter, so generating renormalizations of the non-integral terms in (19)(20) in powers of the coupling constant; additional non-integral terms are also generated with more exotic helicity structure. This procedure is demonstrated with examples in the context of the collinear approximation to QCD in ref.[3].

For the remainder of this paper we shall consider a 'tree-level' approximation to the ladder relations, dropping the integral terms. This should be a good approximation for heavy quarks, when the expansion parameter for loop corrections is g/m. In this regime we may also neglect helicity non-conserving pieces, i.e. we consider transverse momenta bounded by $\Lambda^\perp$ where $\Lambda^\perp/m << 1$. The new ladder relations in the heavy quark regime may then be written

$$f_{\pm\pm\alpha_1\cdots\alpha_n}(\mathbf{k},\mathbf{k}_1,\ldots,\mathbf{k}_{n+1}) = 0 \tag{23}$$

$$f_{\mp\pm\alpha_1\cdots\alpha_n}(\mathbf{k},\mathbf{k}_1,\ldots,\mathbf{k}_{n+1}) = \pm\frac{g\sqrt{N}}{m(2\pi)^{3/2}}\left[\frac{f_{\pm\alpha_1\cdots\alpha_n}(\mathbf{k}+\mathbf{k}_1,\mathbf{k}_2,\ldots,\mathbf{k}_{n+1})}{\sqrt{k_1^+}}\right] \tag{24}$$

where $\mathbf{k} = (k^+ \to 0^+, \mathbf{k}^\perp)$ as before.

3 Small-x Distribution Functions.

We now shall show that the ladder relations reproduce results for the leading $\log 1/x$ approximation of quark and anti-quark distribution functions, which are usually obtained by summing ladder diagrams [4]. For illustration, we shall calculate these distributions for heavy-quarkonium (15) in the large-N limit. Following ref.[5] we define the probability of finding an anti-quark with longitudinal momentum fraction $x = k^+/P^+$ and collinear with the hadron up to scale $\Lambda^\perp$ as

$$\begin{aligned} Q(x,\Lambda^\perp) = & \sum_{n=2}^{\infty}\sum_{\alpha_i}\int_0^{P^+} dk_1^+\ldots dk_n^+\ \delta(k_1^+ + \cdots + k_n^+ - P^+)\times \\ & \int_0^{\Lambda^\perp} d\mathbf{k}_1^\perp\ldots d\mathbf{k}_n^\perp\ \delta(\mathbf{k}_1^\perp + \cdots + \mathbf{k}_n^\perp)\times \\ & \delta(k_1^+ - k^+)\ |f_{\alpha_1\ldots\alpha_n}(\mathbf{k}_1,\mathbf{k}_2,\ldots,\mathbf{k}_n)|^2 \end{aligned} \tag{25}$$

(the analysis may be repeated trivially for quarks). In this *intrinsic* contribution to the distribution function, $\Lambda^\perp$ is not necessarily large, but it could be used as the input for an *extrinsic* evolution to large momentum scales.

Consider the contribution to $Q(x \to 0)$ in a helicity $+1$ polarized meson from partons with alternating helicity f_{++}, f_{-++}, f_{+-++}, f_{-+-++}, ... For heavy quarks only these components of the wavefunction will contribute as a result of eqs.(23)(24) and the dominance of f_{++} in the valence part if we assume zero orbital angular momentum $L = 0$. If $k^+ << k_2^+$ in (25) we may use the ladder relation (24) to re-express the n-parton wavefunction in terms of that for $n-1$ partons. Evidently, the new integrand in (25) gives its dominant contribution in the region of small k_2^+, so we may apply the ladder relation again. This process may be iterated until one arrives at the $\bar{q}q$ valence wavefunction. Consider therefore the contribution from the integration region

$$\int_{\mathcal{C}^+} = \int_{k_1^+} dk_2^+ \int_{k_2^+} dk_3^+ \cdots \int_{k_{n-3}^+} dk_{n-2}^+\ . \tag{26}$$

Then the n-parton contribution to (25) is approximately

$$\left(\frac{g^2N}{8\pi^3}\right)^{n-2}\int_{\mathcal{C}^+}\int_{\mathcal{C}^\perp}\frac{I_{n-2}}{k_2^+k_3^+\cdots k_{n-2}^+}\int_0^{P^+} dk_n^+\int d\mathbf{k}_n^\perp\,\frac{|f_{++}((P^+-k_n^+,-\mathbf{k}_n^\perp),\mathbf{k}_n)|^2}{P^+-k_n^+} \tag{27}$$

where

$$\int_{\mathcal{C}^\perp} I_{n-2} = \int \frac{d\mathbf{k}_1^\perp}{m^2} \cdot \int \frac{d\mathbf{k}_2^\perp}{m^2} \cdot \cdots \cdot \int \frac{d\mathbf{k}_{n-2}^\perp}{m^2} \tag{28}$$

For simplicity we have used the same (global) transverse cut-off $|\mathbf{k}^\perp| \leq \Lambda^\perp$ for each parton. The heavy quark limit is a choice rigorously compatible with the leading $\log 1/x$ approximation, namely $(g^2)^{n-1} \int I_{n-1} << (g^2)^n \int I_n$. [4]

Thus

$$Q(x \to 0) \approx \frac{g^2 N}{8\pi^3} \cdot < P^+/k_n^+ >_{\rm val} \sum_{n=3}^{\infty} \frac{1}{(n-3)!} \left[\frac{g^2 N}{8\pi^3} \ln \frac{1}{x} \right]^{n-3} \int_{\mathcal{C}^\perp} I_{n-2} \tag{29}$$

$$= < P^+/k_n^+ >_{\rm val} \frac{g^2 N (\Lambda^\perp)^2}{8 m^2 \pi^2} x^{-g^2 N (\Lambda^\perp)^2 / 8 m^2 \pi^2} \tag{30}$$

where $< P^+/k_n^+ >_{\rm val}$ is the expectation value of the inverse anti-quark momentum fraction in the $\bar{q}q$ valence sector. Note that this result ignores the contribution to Q from the valence wavefunction itself, which is expected to vanish at small x. The answer (30) is similar to that of Reggeon exchange between virtual photon and hadron which one could have obtained by summing appropriate ladder graphs under the same assumptions. We would have to generalize our ladder relation to finite N and m, thus allowing sea quark pair production, in order to observe pomeron-like contributions in the quark structure function of the meson; alternatively such contributions could be seen in the gluon structure function of a heavy large-N meson, as has been shown by Mueller [6].

The corresponding polarized (anti-)quark distribution function ΔQ has an extra $\mathrm{sgn}(\alpha_1)$ factor in the definition (25), leading to an extra $(-1)^n$ in the sum (29). Thus the polarization asymmetry for a helicity $+1$ meson in the leading $\log 1/x$ heavy quark approximation behaves as

$$\frac{\Delta Q}{Q}(x \to 0) \approx -x^{g^2 N (\Lambda^\perp)^2 / 4 m^2 \pi^2} \tag{31}$$

and is negative because the dominant process at small x for heavy quarks is the helicity-flip emission of one gluon. Note that this implies that ΔQ has convergent Regge behaviour with minus the Regge intercept of Q. This result again neglects the direct contribution of the $\bar{q}q$ sector, which is also expected to vanish at small x, only more quickly than (31). The convergent behavior (31), although a correct consequence of our approximation, may be unrealistic for physical applications given that our approximation (large N, heavy quarks) neglects quark pair production. That, and sub-leading log contributions, could upset the delicate cancelations which lead to vanishing ΔQ above. We shall assume nevertheless that the ladder relations give the correct sign of $\Delta Q(x)$ at small x. Since the helicity of any parton as $x \to 1$ tends to align with that of the hadron[7], the polarization asymmetry should then change sign at

[4] If $\Lambda^\perp >> m$, then it is the wavefunctions with totally aligned helicities which dominate in the distribution functions — we would need the most general form of the ladder relations now — but we cannot be sure that taking leading $\log 1/x$'s is accurate in this case because the momentum dependence of the corresponding transverse integrals I_n would tend to cancel the asymptotic freedom of the running coupling.

sufficiently small x due to (31). This sign change has been explicitly observed in the non-perturbative solution of the collinear model for QCD[3].

These considerations should also be relevant to the sign of the heavy sea quark polarization in the nucleon. For example, strange quarks are predicted to be helicity-aligned with the nucleon polarization at $x \to 1$[7]. However, most of the strange quark distribution occurs at small x, where according to the ladder relations and Eq. (31), $\Delta s(x)$ will have a negative sign. We also note that explicit models of the intrinsic strangeness distribution of the nucleon[8, 9] predict a negative Δs for the strange quarks[9]. Thus we also expect a change in sign of the strange quark helicity distribution of the nucleon at an intermediate value of x.

4 Conclusions.

The methods of this paper allow the derivation of generalized ladder relations and their corrections for any number of partons by considering small-x expansions of the renormalized light-cone bound-state equation (22). As we have seen, the leading orders of this expansion can yield interesting relations and information on the Reggeon structure of quark structure functions which are formally independent of the details of the bound state, such as its mass $\mathcal{M}$. The analysis given in this paper of heavy quarkonium ladder relations demonstrates the origin of the Regge behavior of QCD for both the polarized and unpolarized structure functions, although further work will be required to show that this gives the rigorous behavior at $x \to 0$. We have also applied the ladder relations to derive constraints on the Regge behavior and sign of the polarized distributions at $x \to 0$. These results also have interesting phenomenological predictions for the polarization correlations of non-valence quark and anti-quark distributions.

It is also possible to develop further relations between Fock components of the light-cone wavefunctions which follow from the zero momentum limit of matrix elements of the current J in eq.(3). Inspection of Eq.(9) shows that this is again a finite energy condition; i.e., it is a boundary condition for small gluon k^+ [6].

We also note that the behavior of light-cone wavefunctions at very small x are generally difficult to obtain by numerical solution of the discretized bound-state problem without enormous computational cost. However, the analytic ladder relations derived here may be usefully employed to extend the coarse-structured numerical results into otherwise inaccessible regions of phase space.

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