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Removing Discrete Ambiguities in CP Asymmetry Measurements*

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Abstract

We discuss methods to resolve the ambiguities in CP violating phase angles ϕ that are left when a measurement of $\sin 2\phi$ is made. We show what knowledge of hadronic quantities will be needed to fully resolve all such ambiguities.

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I. INTRODUCTION

If we assume Standard Model unitarity there are two independent angles in the "unitarity triangle", both of which are related to the underlying non-zero phases of CKM matrix elements. We use the definition $\gamma = \pi - \beta - \alpha$, where

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \qquad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \tag{1.1}$$

have simple interpretations as phases of particular combinations of CKM matrix elements.

In B factory experiments we seek to measure quantities that, in the absence of physics from beyond the Standard Model, are simply related to these angles. Ignoring for the moment the effects of subleading amplitudes, CP violating asymmetries are proportional to $\sin 2\phi$ where ϕ is one of the angles of the triangle. In particular, the first two CP asymmetries to be measured are likely to be in $B \to \psi K_S$ which measures $\sin 2\beta$, and in $B \to \pi^+\pi^-$ which measures $\sin 2\alpha$. However, measurement of $\sin 2\phi$ can only determine the angle ϕ up to a four fold ambiguity: $\{\phi, \pi/2 - \phi, \pi + \phi, 3\pi/2 - \phi\}$ with the angles defined by convention to lie between 0 and 2π. Thus, with two independent angles, there can be a priori a total 16 fold ambiguity in their values as determined from CP asymmetry measurements. These ambiguities can limit our ability to test the consistency between the measured value of these angles and the range allowed by other measurements interpreted in terms of the Standard Model CKM matrix elements [1].

In any model where the angles measured by the asymmetries in $B \to \psi K_S$ and $B \to \pi^+\pi^-$ are two angles of a triangle only 4 of the 16 choices are allowed, since the other combinations are incompatible with this geometry [2]. Within the Standard Model, the present data on the CKM matrix elements further reduce the allowed range, implying that 2β is in the first quadrant ($0 < \beta < \pi/4$), that $0 < \alpha < \pi$, and that there is a correlation between the values of α and β [3]. Thus, among the 16 possible solutions at most two, and probably only one, will be found to be consistent with Standard Model results.

In the presence of physics beyond the Standard Model the values of the "would be" α and β extracted from asymmetry measurements may not fall within their Standard Model allowed range. Such new physics cannot be detected if the values of the asymmetry angles happen to be related via the ambiguities to values that do overlap the Standard Model range. Clearly, the fewer ambiguous pairings that remain, the better our chance of recognizing non-Standard Model physics should it occur.

One way to resolve these ambiguities is to measure asymmetries that depend on very small angles [2, 4]: $\arg[-V_{cs}V_{cb}^*/V_{ts}V_{tb}^*]$ or $\arg[-V_{cd}V_{cs}^*/V_{ud}V_{us}^*]$. In this work we discuss other ways to resolve the ambiguities by measuring asymmetries that relate to large angles only. That is not to say we discuss only easy measurements. We will later briefly discuss the experimental difficulties, but first we review the issue from a theoretical perspective. In addition to the *values* of $\sin 2\phi$, only the *signs* of $\cos 2\phi$ and $\sin\phi$ for both $\phi = \alpha$ and $\phi = \beta$ need to be determined. These four signs resolve the ambiguities completely:

- $\text{sign}(\cos 2\phi)$ is used to resolve the $\phi \to \pi/2 - \phi$ ambiguity.
- $\text{sign}(\sin\phi)$ is used to resolve the $\phi \to \pi + \phi$ ambiguity.

Several measurements which can determine $\text{sign}(\cos 2\phi)$ have been proposed [2, 5, 1, 6]. Uncertainties in calculation of hadronic effects do not affect the interpretations of these measurements, although they do depend on the known value of hadronic quantities such as the width and the mass of the ρ. The determination of $\text{sign}(\sin\phi)$, however, cannot be achieved without some theoretical input on hadronic physics. Quantities that are independent of hadronic effects always appear as the ratio of a product of CKM matrix elements to the complex conjugate of the same product. Such pure phases are thus always twice the difference of phases of the CKM elements. Any observable that directly involves a weak phase difference of two CKM elements, ϕ, (rather than 2ϕ) also involves hadronic quantities such as the ratio of magnitudes of matrix elements and the difference of their strong phases. Thus, in order to determine the sign of $\sin\alpha$ or $\sin\beta$ some knowledge about hadronic physics is required.

We note that this is true even for our current knowledge of the Standard Model CP violating phase, $\sin\delta > 0$ (where δ is the single independent phase in the standard parametrization of the CKM matrix [7]). In order to determine sign($\sin\delta$) input on the sign of B_K is used [2]. The quantity B_K is a ratio of hadronic matrix elements. Its value is totally determined by the strong interactions and thus, a-priori, is not reliably calculable. However, by now many methods of determining B_K, including lattice calculations, all find that $B_K > 0$, though the range of allowed values is still quite large. As a result, it is now widely accepted that the sign of B_K is reliable and thus that, in the Standard Model, $\sin\delta > 0$.

Many weak decay amplitudes include two terms with different weak phases. In this work we show how the presence of a second term can be used to determine the sign of $\sin\alpha$ and $\sin\beta$. The needed theoretical input is the sign of the real part of the ratio of the two amplitude terms (excluding CKM elements). The focus of this paper is to examine what input assumptions are needed to determine this sign, and discuss the status of these assumptions. Our aim is to clarify what is the minimum understanding of strong interaction effects that will be needed to resolve the angle ambiguities. Our current arguments alone cannot stand as a convincing reason to exclude an angle consistent with the Standard Model range in favor of a choice that is not consistent. However, were such a choice favored by this argument, it would at least pose a serious challenge to theorists to understand better the strong interaction effects involved. Eventually it may be that we have to piece together many such puzzles to get a view of non-Standard Model physics from the low energy frontier of B decays.

In section 2 we review the general formalism of CP asymmetries in B decays. In section 3 we review methods to determine sign($\cos 2\phi$). In section 4 we explain how to determine sign($\sin\phi$), and what is the theoretical input that has to be supplied. Finally, section 5 contains discussions and conclusions.

II. GENERAL FORMALISM

In this section we present the general formalism of CP asymmetries in B decays. We start by explaining how we group penguin and tree diagrams and then present the needed formalism.

A. Two-term weak decay amplitudes.

The terms "penguin" and "tree" amplitudes are standard in the field for weak decay amplitudes, but are actually only meaningful at the short-distance, quark-diagram level. Our argument here is quite general and is not in any way affected by the ambiguity inherent in these short distance labels. We group amplitude terms together by weak phase, rather than by individual diagrams. Then there is no need to attempt the unphysical distinction between rescattering of a tree diagram and a long-distance cut of a penguin diagram. Further we use CKM unitarity to eliminate one out of the up, charm and top penguin diagrams terms. In this way any B decay amplitude, including all tree and penguin diagrams, can be written as a sum of two terms, each with a definite weak phase related to particular CKM-matrix elements. The most convenient choice of how to group terms depends on the final state quarks.

For $b \to q\bar{q}s$ decays, for any final state f, it is convenient to choose the two terms as

$$A_f^s = V_{cb}V_{cs}^* A_f^{ccs} + V_{ub}V_{us}^* A_f^{uus}. \tag{2.1}$$

The second term here is Cabbibo suppressed compared to the first and is negligible in most cases. For $b \to c\bar{c}s$ decays (e.g., $B \to \psi K_S$) the second term gets further suppression since the dominant term includes a tree level diagram while the CKM-suppressed term contains only one loop (penguin) diagrams, namely, $A_f^{ccs} \gg A_f^{uus}$. In $b \to u\bar{u}s$ decays the tree diagram contributes to the second term while the first term has only penguin contributions and hence $A_f^{ccs} \ll A_f^{uus}$, thus in this case there is no clear hierarchy among the two terms.

For $b \to q\bar{q}d$ decays all the CKM coefficients are of the same order of magnitude. It is then convenient to express the amplitude as

$$A_f^d = V_{qb}V_{qd}^* A_f^{qqd} + V_{tb}V_{td}^* A_f^{ttd}, \tag{2.2}$$

where $q = u$ or c is chosen so that the first term includes any tree diagram contribution for the channel in question. (When there is no tree diagram the choice is arbitrary.) The second term here has a weak phase predicted in the Standard Model to be half the weak phase of the mixing amplitude. Thus, only one unknown weak phase difference enters the analysis when the amplitude is written in this way.

For any given channel at most one of these two terms has a tree diagram contribution. The tree diagram is generally expected to be the dominant contribution to any $A_f^{qqq'}$ for which it is non-zero, so we will call this the "tree-dominated" term to remind the reader that it also contains a difference of loop (or penguin) contributions with the same weak phase. We then refer to the other term, which has no tree diagram contribution, as the "penguin-only" term.

We note, as an aside, that the two-term structure of decay amplitudes can also accommodate any beyond-Standard-Model physics contribution, since any additional term in a decay amplitude, whatever its phase, can always be written as a sum of two terms of definite phase with (possibly negative) real magnitudes. The difference between Standard Model physics and non-Standard-Model physics then comes down to the expected relative sizes of the two terms. These expected sizes are, in general, dependent on our understanding of hadronic matrix elements. This just shows once again how difficult it could be to recognize the presence of non-Standard Model physics. The only reliable way to find new effect in decay amplitudes is to examine cases in which a single term significantly dominates the weak decay amplitude in the Standard Model [8].

B. General formalism

Here we recall the general formalism of CP asymmetries in B decays. We use the standard notations [9]. We assume the Standard Model all the way.

The time dependent CP asymmetry in B decays into a final CP eigenstate state f is defined as [9]

$$a_f(t) \equiv \frac{\Gamma[B^0(t)\to f] - \Gamma[\bar{B}^0(t)\to f]}{\Gamma[B^0(t)\to f] + \Gamma[\bar{B}^0(t)\to f]}, \tag{2.3}$$

and is given by

$$a_f(t) = a_f^{\cos}\cos(\Delta M t) + a_f^{\sin}\sin(\Delta M t), \tag{2.4}$$

with

$$a_f^{\cos} \equiv \frac{1-|\lambda|^2}{1+|\lambda|^2}, \qquad a_f^{\sin} \equiv \frac{-2\,\mathrm{Im}\lambda}{1+|\lambda|^2}, \qquad \lambda \equiv \frac{q}{p}\frac{\bar{A}}{A}, \tag{2.5}$$

where p and q are the components of the interaction eigenstates in the mass eigenstates, $|B_{L,H}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle$, and $A(\bar{A})$ is the $B_d(\bar{B}_d) \to f$ transition amplitude [9]. The time-dependent measurement can separately determine $a_f^{\cos}$ and $a_f^{\sin}$. We always consider decays with a leading tree diagram amplitude. Then, we write the amplitude as

$$A = A_T e^{i\phi'_T} e^{i\delta_T} + A_P e^{i\phi'_P} e^{i\delta_P}, \qquad \bar{A} = A_T e^{-i\phi'_T} e^{i\delta_T} + A_P e^{-i\phi'_P} e^{i\delta_P} \tag{2.6}$$

where T and P stand for the tree-dominated and penguin-only terms respectively. The weak phases of the decay amplitudes, ϕ'_T and ϕ'_P are convention dependent, as is $\arg(q/p)$ but the differences $\phi_T = \phi'_T - \arg(q/p)/2$ and $\phi_P = \phi'_P - \arg(q/p)/2$ are convention independent quantities that we seek to determine. Similarly, the strong phases are all subject to arbitrary redefinitions, only the relative strong phase of the two terms $\delta \equiv \delta_P - \delta_T$ is a physically meaningful quantity. We have introduced strong phases for each term so that we can always fix both A_T and A_P to be real quantities, independent of any phase convention choice. We then define the real quantity

$$r \equiv \frac{A_P}{A_T}. \tag{2.7}$$

Note that we allow $r < 0$. The CP violation sensitive quantity λ is then

$$\lambda = \eta_f \frac{e^{-i\phi_T} + r e^{-i\phi_P} e^{i\delta}}{e^{i\phi_T} + r e^{i\phi_P} e^{i\delta}}. \tag{2.8}$$

Here η_f is the CP parity of the final state. In particular, $\eta_{\psi K_s} = -1$ and $\eta_{\pi^+\pi^-} = \eta_{D^+D^-} = 1$.

For $b \to c\bar{c}s$ decays, leading for example to the final state ψK_S, the penguin-only term is Cabbibo suppressed and can be safely neglected. Thus $r = 0$ should be an excellent approximation and we get the well known result [9]

$$a_f^{\cos} = 0, \qquad a_f^{\sin} = \eta_f \sin 2\phi_T. \tag{2.9}$$

We next consider $b \to q\bar{q}d$ decays, leading for example to the final states $B \to \pi^+\pi^-$ or $B \to D^+D^-$. Here, by definition, $\phi_P = 0$ since the penguin contributions with a different weak phase are subsumed in A_T. Then

$$a_f^{\cos} = \frac{2r \sin\phi_T \sin\delta}{1 + r^2 + 2r\cos\phi_T\cos\delta}, \qquad a_f^{\sin} = \eta_f \frac{\sin 2\phi_T + 2r\sin\phi_T\cos\delta}{1 + r^2 + 2r\cos\phi_T\cos\delta}. \tag{2.10}$$

III. DETERMINING $\mathrm{sign}(\cos 2\phi)$

In this section we review measurements that can be used to extract $\mathrm{sign}(\cos 2\alpha)$ and $\mathrm{sign}(\cos 2\beta)$. These signs resolve the $\phi \to \pi/2 - \phi$ ambiguities.

A. $B \to \rho\pi$

All the three decays $B \to \rho^+\pi^-$, $B \to \rho^-\pi^+$ and $B \to \rho^0\pi^0$ can lead to a $\pi^+\pi^-\pi^0$ final state. Due to interferences between these channels sufficient information is encoded in the $B \to \rho\pi$ decays to distinguish between the α and $\pi/2 - \alpha$ choices. This was shown in Ref. [5], where it was explained how both $\sin 2\alpha$ and $\cos 2\alpha$ can be measured using a full Dalitz plot distribution analysis. To resolve the ambiguity one needs only to fix the sign of $\cos 2\alpha$, which should be relatively easy to achieve.

We do not repeat here the detailed explanations of Ref. [5]. In that work it was shown that there are several observables that, in the absence of penguins, directly measure $\cos 2\alpha$. (These observables all involve the imaginary part of an overlap between two different Breit-Wigner functions describing two different charges of ρ meson.) The presence of penguins spoils the simple relationship between these quantities and $\cos 2\alpha$. However, even when penguin terms are present, there is enough information in the interference regions to determine the sign of $\cos 2\alpha$. A multiparameter fit can obtain a preferred choice between α and $\pi/2 - \alpha$, even allowing for arbitrarily large penguin contributions.

Here, and throughout this paper, we neglect the effects of electroweak penguins. These give a correction to isospin-based treatments for isolating certain CKM factors. The isospin structure of the amplitudes contributing to $\rho\pi$ decays is used to isolate terms with isospin two, because they receive no contribution from QCD penguin graphs, and hence show pure $\sin 2\alpha$ and/or $\cos 2\alpha$ dependence. Electroweak penguin graphs can give isospin two parts but the relevant contributions here are expected to be quite small and hence unlikely to confuse the extraction of the sign of $\cos 2\alpha$.

Experimentally, the $\cos 2\alpha$ determination involves fitting parameters to the contributions of a broad resonance. Under these resonances there are non-resonant B decay contributions which must also be fit in order to extract the relevant resonant effects. The question of how best to parameterize these non-resonant contributions is under study [10]. It will have to be resolved to extract useful results from these channels.

B. $B \to DD^{**}$

The idea of using overlapping decays to add information on $\cos 2\phi$ can be in principle applied to B decays to higher D resonances [6]. In that case, a full Dalitz plot distribution of $D^{(*)}D^{(*)}\pi$ final states can be used to determine the sign of $\cos 2\beta$. Since the D^* are rather narrow the interference effects are probably too small to be detected in $B \to DD^*$ since there is essentially no overlap kinematic region between different D^*'s. The $B \to D^{(*)}D^{**}$ decays are better candidates. The D^{**} widths are larger and the effect may be measurable. More details are expected to be given in Ref. [6]. Once again, it may be a problem to parameterize non-resonant $D^{(*)}D^{(*)}\pi$ that contribute in the same region as the resonances and could potentially destroy the analysis.

C. $B^\pm \to DK^\pm$

The angle γ satisfies the condition

$$\alpha + \beta + \gamma = \pi \text{ (mod } 2\pi). \tag{3.1}$$

Since γ is defined modulo 2π, the 16 possibilities for α and β result in an eightfold ambiguity in γ. These eight values give two different values for $\cos 2\gamma$ and four different values for $\sin 2\gamma$. Thus, by measuring $\cos 2\gamma$ or $\sin 2\gamma$ some of the ambiguities can be resolved. Here we focus on $\cos 2\gamma$ and in the next subsection we discuss $\sin 2\gamma$.

The value of $\cos 2\gamma$ can be used to resolve some combination of the $\phi \to \pi/2 - \phi$ ambiguities. The trigonometric identity

$$\cos 2\gamma = \cos 2\beta \cos 2\alpha - \sin 2\alpha \sin 2\beta, \tag{3.2}$$

implies that the transformations $\beta \to \pi/2 - \beta$ or $\alpha \to \pi/2 - \alpha$ (but not both) change the value of $\cos 2\gamma$. As we assume that $\sin 2\beta$ and $\sin 2\alpha$ are known, $\cos 2\gamma$ can distinguish between the two cases $\{\alpha, \beta\}, \{\pi/2 - \alpha, \pi/2 - \beta\}$ or $\{\pi/2 - \alpha, \beta\}, \{\alpha, \pi/2 - \beta\}$. Thus, for example, if $\cos 2\alpha$ in known from the $B \to \rho\pi$ analysis, the sign of $\cos 2\beta$ can be determined from the measurement of $\cos 2\gamma$.

Several methods to extract $\sin^2\gamma$ (or equivalently $\cos 2\gamma$) using $B^\pm \to DK^\pm$ decays [11, 12] or B_s decays [13] have been proposed [14]. For the purpose of illustration, below we concentrate on the method of [11]. This method uses measurements of six $B^\pm \to DK^\pm$ decay rates to extract $\cos 2\gamma$ up to a two fold ambiguity. This two-fold ambiguity is due to an unknown strong phase. In general, this ambiguity can be removed by applying the same analysis for several final states [11] with the same flavor quantum numbers as $DK^\pm$. All these modes have the same weak phase but, in general, different strong phases. Thus, only one solution of $\cos 2\gamma$ is consistent in all the modes while the second (incorrect) one should be different in the different modes, since strong phases differ from one mode to another.

We note that even if we have a two-fold ambiguity in $\cos 2\gamma$ because we have studied only a single final state system, the incorrect value of $\cos 2\gamma$ should not be the same as that obtained using the incorrect value of β or α. In that case there are going to be two possible solutions for $\cos 2\gamma$ from the $B^{\pm} \to DK^{\pm}$ measurement, and two predictions arising from the measurements of $\sin 2\beta$ and $\sin 2\alpha$. In general, only one of the solutions will coincide and the other not. Choosing the one that coincides is sufficient to resolve the ambiguity in the $\cos 2\gamma$ measurement and at the same time to fix the relative sign of $\cos 2\alpha$ and $\cos 2\beta$.

D. $B_s \to \rho K_S$

The time dependent CP asymmetry in certain B_s decays (e.g., $B_s \to \rho K_S$) directly measures $\sin 2\gamma$ if the penguin-only term in the decay amplitude is neglected. A measurement of $\sin 2\gamma$ would determine the signs of $\cos 2\beta$ and $\cos 2\alpha$ [2], assuming their magnitudes are known. The trigonometric identity

$$\sin 2\gamma = -(\cos 2\beta \sin 2\alpha + \cos 2\alpha \sin 2\beta), \tag{3.3}$$

implies that either or both of the transformations $\beta \to \pi/2 - \beta$ and $\alpha \to \pi/2 - \alpha$, change the value of $\sin 2\gamma$. Thus, the signs of both $\cos 2\alpha$ and $\cos 2\beta$ can be determined, once $\sin 2\gamma$ is known.

Experimentally, it will be very hard, if at all possible, to measure this asymmetry. In addition, the penguin-only term is expected to be significant in $b \to u\bar{u}d$ decays, making the relationship between the asymmetry and the angle γ more complicated [14]. These problems imply that the methods we mentioned before are better than the time dependent CP asymmetry in $B_s \to \rho K_S$ decay for determining γ [14]. However, all these other methods determine $\cos 2\gamma$. The justification to study the time dependent CP asymmetry in $B_s \to \rho K_S$ is that it probes a different functional dependence of γ, namely, $\sin 2\gamma$.

As we need only to choose between few discrete choices of γ the problems mentioned before may not be so severe in our case. By the time measurement of the CP asymmetry in $B_s \to \rho K_S$ is feasible we will probably already know the rough value of the penguin contribution, from its relationship to similar effects in $B \to \pi\pi$, extracted via isospin analysis, and those determined from fits to $B_d \to \rho\pi$. If $\cos 2\gamma$ is already measured as discussed above, then we need this measurement only to distinguish between the two values of the sign of $\sin 2\gamma$. In general only one sign will be consistent with the allowed range for the ratio of penguin-only to tree-dominated terms, so the ambiguity will be resolved even though an a-priori measurement of $\sin 2\gamma$ cannot be achieved.

IV. DETERMINING $\text{sign}(\sin\phi)$

In this section we discuss how $\text{sign}(\sin\alpha)$ and $\text{sign}(\sin\beta)$ can be determined. These signs resolve the $\phi \to \pi + \phi$ ambiguity. As we already explained, this ambiguity cannot be resolved in any theoretically clean way. Some knowledge of hadronic physics is always needed. In the following we describe several methods that can be used to resolve the ambiguity, and explain what is the needed theoretical input.

In order to get sensitivity to $\text{sign}(\sin\phi)$ we focus on cases where two terms with different weak phases are involved in the decay amplitude. Then, in principle, the relative phase between these two terms can be determined. However, there is also a relative strong phase between these two terms. Therefore, theoretical input is required in order to disentangle the strong and the weak phases. The relevant hadronic quantity is found to be the sign of $r\cos\delta$, that is the sign of the real part of the ratio of the two amplitude terms (excluding weak phases).

A. $B \to \psi K_S$ vs $B \to D^+D^-$

In the case of the angle β we have one class of measurements, from $b \to c\bar{c}s$ processes such as $B \to \psi K_S$, that have very small r. For these channels Eq. (2.9) with $\phi_T = \beta$ is valid and the asymmetry measurement determines β up to the usual four-fold ambiguity [9]

$$a^{\sin}_{\psi K_S} = -\sin 2\beta. \tag{4.1}$$

The other class of measurements is from $b \to c\bar{c}d$ decays such as $B \to D^+D^-$. In this case we expect r to be significantly larger and Eq. (2.10) with $\phi_T = \beta$ is valid. For simplicity we will here give results valid only to leading order in r, however we have checked that the full expression contains enough information to avoid this approximation if needed. We get

$$a^{\sin}_{D^+D^-} = \sin 2\beta - 2r_{DD}\cos 2\beta \sin\beta \cos\delta_{DD}. \tag{4.2}$$

where δ_{DD} is the strong phase difference between the tree-dominated and penguin-only $B \to D^+D^-$ amplitudes, and r_{DD} is the signed ratio of their magnitudes. Comparing Eqs. (4.2) and (4.1) we find

$$a^{\sin}_{\psi K_S} + a^{\sin}_{D^+D^-} = -2r_{DD}\cos\delta_{DD}(\cos 2\beta \sin\beta). \tag{4.3}$$

It is clear from this expression that we can fix the sign of $\sin\beta$ only if we know the sign of $\cos 2\beta$ and, in addition, the sign of $r_{DD}\cos\delta_{DD}$. We assume the first of these is given by the methods discussed in the previous section.

Currently, there is no reliable way to determine the sign of the real part of the ratio of hadronic matrix elements ($r_{DD}\cos\delta_{DD}$). In order to proceed, we assume factorization. (We will discuss the reliability of this and subsequent assumptions later.) Assuming factorization and that the top penguin is dominant, we can infer from the results of Ref. [15], $r_{DD} < 0$. Within the factorization approximation the relevant strong phases (almost) vanish, so that $\delta_{DD} \simeq 0$, and hence the sign of $r_{DD}\cos\delta_{DD}$ is given by the sign of r_{DD}.

Assuming $r_{DD}\cos\delta_{DD} < 0$ as given by the factorization calculation we get

$$\mathrm{sign}(a^{\sin}_{\psi K_S} + a^{\sin}_{D^+D^-}) = \mathrm{sign}(\cos 2\beta \sin\beta). \tag{4.4}$$

Note, in particular, that the Standard Model predicts $\cos 2\beta \sin\beta > 0$, and therefore also that the asymmetry in D^+D^- is smaller in magnitude than the asymmetry in ψK_S (and opposite in sign).

We need only measure the sign of the sum of the two asymmetries to resolve the ambiguity. Even this may not be an easy task if r_{DD} is small, however a recent estimate found that in the Standard Model $3\% \lesssim r_{DD} \lesssim 30\%$ [16], and certainly in the upper end of this range the required sign should be measurable.

B. $B \to \rho\pi$ vs $B \to \pi^+\pi^-$

We first explain how to get $\sin 2\alpha$ uniquely out of the $B \to \rho\pi$ decays without uncertainties due to penguin only terms. Then, the comparison with the asymmetry in $B \to \pi^+\pi^-$ can be used to determined sign($\sin\alpha$) using a similar approach to that discussed for β above.

While the experiment may well proceed to determine all the various amplitudes and phases simultaneously by a maximum likelihood fit, it is instructive to inspect the expressions analytically to see what combination of terms actually enters into the measurement of $\sin 2\alpha$. We follow the treatment of [5] and write

$$\begin{aligned} A_3 &= A(B^0 \to \rho^+\pi^-) = T_3 + P_1 + P_0\,, & \bar{A}_3 &= A(\bar{B}^0 \to \rho^-\pi^+) = \bar{T}_3 + \bar{P}_1 + \bar{P}_0\,, \\ A_4 &= A(B^0 \to \rho^-\pi^+) = T_4 - P_1 + P_0\,, & \bar{A}_4 &= A(\bar{B}^0 \to \rho^+\pi^-) = \bar{T}_4 - \bar{P}_1 + \bar{P}_0\,, \\ A_5 &= A(B^0 \to \rho^0\pi^0) = T_5 - P_0\,, & \bar{A}_5 &= A(\bar{B}^0 \to \rho^0\pi^0) = \bar{T}_5 - \bar{P}_0\,, \end{aligned} \tag{4.5}$$

where T_i is the tree-dominated amplitude and P_1 and P_0 are the (suitably rescaled) penguin-only contribution for isospin one and isospin zero respectively. The CP conjugate amplitudes $\bar{A}_i$, $\bar{T}_i$ and $\bar{P}_i$ differ from the original amplitudes, A_i, T_i and P_i only in the sign of the weak phase of each term. We further define

$$\begin{aligned} A_{sum} &\equiv A_3 + A_4 + 2A_5 = \left(|T_3|e^{i\delta_3} + |T_4|e^{i\delta_4} + 2|T_5|e^{i\delta_5}\right) e^{i\phi'_T}, \\ \bar{A}_{sum} &\equiv \bar{A}_3 + \bar{A}_4 + 2\bar{A}_5 = \left(|T_3|e^{i\delta_3} + |T_4|e^{i\delta_4} + 2|T_5|e^{i\delta_5}\right) e^{-i\phi'_T}. \end{aligned} \tag{4.6}$$

Here, δ_i is the strong phase of T_i, and ϕ'_T is the common weak phase of the tree-dominated terms. We see that $\bar{A}_{sum} = A_{sum}e^{-2i\phi'_T}$. From Table I of Ref. [5] we see that both $A_{sum}A^*_{sum}$ and $\mathrm{Im}\left(q\bar{A}_{sum}\, p^* A^*_{sum}\right)$ are observables. (Note that q as defined in Ref. [5] is equal to $\sqrt{2}qp^*$ in our standard notation.) In particular, we see that from the data we can extract

$$a_{\rho\pi}^{Dalitz} \equiv -\mathrm{Im}\left(\frac{q}{p}\frac{\bar{A}_{sum}}{A_{sum}}\right) = -\sin 2\alpha, \tag{4.7}$$

where for the last equation we used $|q/p| = 1$ and $\phi_T = \pi - \alpha$. Eq. (4.7) shows that $\sin 2\alpha$ can be extracted using $B \to \rho\pi$ decays without penguin pollution. We emphasize that in order to obtain this result we did not have to assume that the top penguin is dominant. All penguin terms are included, either as a subdominant part in the tree-dominated amplitudes, or in the penguin-only term.

Alternately, $B \to \pi\pi$ decay modes can also be used to extract $\sin 2\alpha$ without hadronic uncertainties using isospin analysis. The needed measurements are the time-dependent rate for $B \to \pi^+\pi^-$ together with the time-integrated rates of $B^0 \to \pi^0\pi^0$, $B^+ \to \pi^+\pi^-$ and their conjugate decays [17], and a geometrical construction then allows extraction of $\sin 2\alpha$. However, discrete ambiguities in this construction imply that $\sin 2\alpha$ can only be extracted up to certain discrete choices, which correspond also to differences in the relative phase and the ratio of magnitudes of certain tree-dominated and penguin-only terms (but not the same combinations as we identify below). The determination from $\rho\pi$ does not suffer from this problem. (These ambiguities could in principle be removed by a precise measurement of the time dependent asymmetry in $B \to \pi^0\pi^0$ [17], but this measurement is unlikely.)

Now, assuming we have determined $\sin 2\alpha$, we look again at the $B \to \pi^+\pi^-$ decay, here using the interference of the two terms in the amplitude to determine the sign of $\sin\alpha$, just as we did in the D^+D^- case for β. Here, $\phi_T = \pi - \alpha$ and $\phi_P = 0$, and Eq. (2.10) gives the asymmetry. Once again, for simplicity, we work to leading order in r, but this approximation can be avoided if needed. We get

$$a^{\sin}_{\pi^+\pi^-} = -\sin 2\alpha - 2r_{\pi\pi}\cos 2\alpha \sin\alpha \cos\delta_{\pi\pi}, \tag{4.8}$$

where $\delta_{\pi\pi}$ is the strong phase difference between the tree-dominated and penguin-only $B \to \pi^+\pi^-$ amplitudes, and $r_{\pi\pi}$ is the signed ratio of their magnitudes. Comparing Eqs. (4.7) and (4.8) we get

$$a^{\sin}_{\pi^+\pi^-} - a^{Dalitz}_{\rho\pi} = -2r_{\pi\pi}\cos\delta_{\pi\pi}(\cos 2\alpha \sin\alpha) \tag{4.9}$$

Thus, once $\mathrm{sign}(r_{\pi\pi}\cos\delta_{\pi\pi})$ is known, the measurements will determine $\mathrm{sign}(\cos 2\alpha \sin\alpha)$. If the $\mathrm{sign}(\cos 2\alpha)$ is known from the treatments discussed above, $\mathrm{sign}(\sin\alpha)$ is then determined; if not, at least the fourfold ambiguity of $\{\mathrm{sign}(\cos 2\alpha), \mathrm{sign}(\sin\alpha)\}$ is reduced to a two-fold ambiguity.

Again, there is as yet no reliable way to calculate the sign of $r_{\pi\pi} \cos\delta_{\pi\pi}$. Therefore, we turn to the short-distance calculation with factorization to determine [15] that $r_{\pi\pi} < 0$ and that $\delta_{\pi\pi}$ is very small. This then gives

$$\mathrm{sign}(a^{\sin}_{\pi^+\pi^-} - a^{Dalitz}_{\rho\pi}) = \mathrm{sign}(\cos 2\alpha \sin\alpha). \tag{4.10}$$

With the knowledge of $\cos 2\alpha$ this difference can be used to fix the sign of $\sin\alpha$.

C. CP asymmetries in inclusive decays

In the above, the main obstacle in getting theoretically clean predictions is that we do not have a reliable way to calculate the ratio of the relevant hadronic matrix elements. An alternative way, which does not suffer from this problem, is to measure asymmetries in semi-inclusive decays, e.g. to all states with a given flavor content [18]. Here matrix elements are not needed. However a crucial assumption in this case is that the semi-inclusive measurements are described by the quark level calculations, which are needed to determine ξ: the fraction of CP-odd final states. The quantity $1-2\xi$ is referred to as the "dilution factor". The assumption, called local quark-hadron duality, that the quark-diagram kinematics are unaltered by hadronization, is essential to this calculation and is not well justified. In addition, we are convinced that full semi-inclusive measurements are not experimentally feasible, some data cuts will be needed. The effect of such cuts on the ratio of CP-even to CP-odd contributions is difficult to calculate and likely to be even more sensitively dependent on the local quark-hadron duality assumption.

However, our game here is to determine signs, so we can possibly use these methods despite large uncertainties in the calculation of the relevant dilution factors, as long as the sign of $(1 - 2\xi)$ is reliably determined. The hope is that by the time the inclusive measurements will be carried out, we will have consistency checks that will either support or rule out local duality. For example, the inclusive asymmetry calculations are similar to that of the B_s width difference [19]. If future measurements of the B_s width difference agree with this calculation, it would support the local duality assumption.

A potentially useful measurement is the asymmetry in the $B_d \to DX$ where X is multi pion state with no K meson contributions. Such decays are governed by the $b \to c\bar{u}d$ and $b \to u\bar{c}d$ transitions. The inclusive calculation gives [18]

$$a^{\sin}_{c\bar{u}d\bar{d}} = -(1-2\xi)\left|\frac{V_{cd}V_{ub}}{V_{ud}V_{cb}}\right| \sin(\alpha-\beta). \tag{4.11}$$

On the practical side, we note that the large inclusive rate may help compensate the CKM suppression of the asymmetry. We see that the $\alpha \to \pi+\alpha$ or $\beta \to \pi+\beta$ transformations (but not both) will change the sign of the result. The quantity $(1-2\xi)$ is calculated to be about 0.21 [18], but the range of uncertainty on this quantity, and its dependence on the necessary experimental cuts remains to be explored. If we can convince ourselves that we know the sign of this quantity, as calculated for the specific data sample used to determine the asymmetry, we can use such a measurement to reduce the set of ambiguous choices for the two angles. Perhaps one way to proceed will be to explore, both in the theory and in the data, the sensitivity of the signs to changes in the selected sample.

Another measurement that can be useful is that of B_s decays governed by the $b \to c\bar{u}s$ and $b \to u\bar{c}s$ transitions. For this case Ref. [18] found

$$a^{\sin}_{c\bar{u}s\bar{s}} \approx (1-2\xi)\left|\frac{V_{cs}V_{ub}}{V_{us}V_{cb}}\right| \sin(\alpha+\beta), \tag{4.12}$$

where here $1-2\xi \approx 0.28$ [18]. Again, the $\alpha \to \pi+\alpha$ or $\beta \to \pi+\beta$ transformations (but not both) will change the sign of the result. Note that unlike the previous case, here the CKM suppression is not very small. However, asymmetries in B_s decays are expected to be harder to measure. Once again the dilution factor calculation needs to be further explored to determine whether the sign of this quantity can reliably be calculated.

D. Remarks about the theoretical assumptions

We here examine the points at which it is important to clarify our theoretical understanding if we are to use the results of B factory experiments to look for indications of non-Standard Model physics. Our arguments can be strengthened by a combination of improved calculational methods (such as lattice calculations of matrix elements) and by testing the implications of similar arguments in a variety of channels, in addition to those studied for the CP studies. It is to be hoped that, by the time we have sufficient data to perform the measurements described above, both of these avenues will have been explored and our arguments, e.g. on the sign of the $r\cos\delta$ terms, either discredited or more firmly established. The point of this paper is that we need to pursue this further understanding to resolve the ambiguous choices.

We will discuss here the exclusive final states. There, we use factorization to calculate the sign of $r\cos\delta$. Here we discuss why it is plausible that this sign is correctly predicted by the factorization calculation. Our calculation uses the operator product expansion approach, which is rigorous, but adds to it the less rigorous ingredients of a model to calculate matrix elements. We apply this model only in color-allowed decays where the outcome is insensitive to the variation of the parameter governing the relative contribution of color-suppressed terms.

The factorization approximation treats each quark-antiquark combination separately, the only strong phase, in this approximation, is a small effect that arises from cuts of the short-distance penguin diagrams involving u or c quarks. Thus, $\delta \approx 0$. To go beyond the factorization approximation we consider a two step picture in which the decay and hadronization occurs as calculated in the factorization approximation but (elastic and inelastic) final state rescattering are allowed. While here we present only the D^+D^- final state, similar treatment apply also to the $\pi^+\pi^-$ final state with similar conclusions. The way to proceed is to work in the isospin basis. Each of the terms $A_T e^{i\delta_P}$ and $A_P e^{i\delta_P}$ has two isospin contributions (labeled by the final state isospin $I_f = 0, 1$). These terms acquire strong phases through rescattering effects. We emphasize that the rescattering phases for the same isospin channel can be different in the penguin-only and tree-dominated terms. These amplitudes have different overlap between the D^+D^- state and the other hadronic states with the same charm-quark content and isospin. Because the light quark content in

D^+D^- is $d\bar{d}$ we know that in both the tree-dominated and penguin-only terms separately the two isospin contributions are equal in magnitude. Thus, the effect of rescattering can be taken into account by writing the tree-dominated and penguin-only amplitudes as

$$A_T e^{i\delta_T} \cos\delta_T^{01}, \qquad A_P e^{i\delta_P} \cos\delta_P^{01}. \tag{4.13}$$

Here the phases are given by

$$\delta_X = (\delta_X^0 + \delta_X^1)/2, \qquad \delta_X^{01} = (\delta_X^0 - \delta_X^1)/2, \tag{4.14}$$

where δ_X^i is the phase shift of the isospin i term in the $X = T, P$ amplitude. Thus, after rescattering, we find

$$r_{DD} = r_{DD}^{fact} \frac{\cos\delta_P^{01}}{\cos\delta_T^{01}}, \qquad \cos\delta_{DD} = \cos(\delta_T - \delta_P), \tag{4.15}$$

where r_{DD}^{fact} is r_{DD} as calculated using factorization. Thus, the sign of $r_{DD}\cos\delta_{DD}$ is unchanged by rescattering if the relevant phase shifts are all sufficiently small that the cosines in Eqs. (4.15) are all positive.

It seems to be a reasonable assumption that all the relevant strong phases are small. There are no known nearby resonances with isospin 0 or 1 in the spin zero partial wave in the D^+D^- system at the B mass. Furthermore, some cross checks on this argument are available. The rates of D^+D^- and $D^0\bar{D}^0$ productions are given by

$$\begin{aligned} \Gamma(B \to D^+D^-) &= \left| A_T \cos\delta_T^{01} e^{i\delta_T} e^{i\phi_T'} + A_P \cos\delta_P^{01} e^{i\delta_P} e^{i\phi_P'} \right|^2 . \\ \Gamma(B \to D^0\bar{D}^0) &= \left| A_T \sin\delta_T^{01} e^{i\delta_T} e^{i\phi_T'} + A_P \sin\delta_P^{01} e^{i\delta_P} e^{i\phi_P'} \right|^2 . \end{aligned} \tag{4.16}$$

If the $D^0\bar{D}^0$ rate is small compared to the D^+D^- rate it provides some confirmation that the rescattering phases δ_T^{01} and δ_P^{01} are small.

Direct CP violation effects in these channels depend on the same rescattering phases and can be predicted in terms of the same parameterization. Such effects are proportional to $\sin\delta$ and so are small if all rescattering effects are small. Large direct CP violations in the D^+D^- or $\pi^+\pi^-$ channels would be a reason to mistrust our argument for the sign of $r\cos\delta$.

However, small direct CP violations are consistent with, but not a convincing argument for small δ. An interesting example would be if $\sin\delta$ is found to be small in several channels with the same quark content (e.g. DD, DD^* and D^*D^*). Then, we would have to conclude that either $\delta \sim 0$ or $\delta \sim \pi$ in each of these channels. There is no reason to believe that any rescattering strong phases should be close to π and it is even less likely that several at once have this value. However, due to the arguments for factorization, it is quite plausible that all of them are small at the same time.

To conclude: the needed theoretical input is the sign of $r\cos\delta$. Here, we argue that it is plausible that the correct sign can be predicted by factorization in color-allowed channels. Moreover, some cross-check can be done. However, we emphasize again that we believe that there is currently no reliable way to determine this sign.

V. FINAL REMARKS AND CONCLUSIONS

Our goal is to find physics beyond the Standard Model. While in this paper we present our results as a way to resolve the discrete ambiguities in the values of α and β, it should be remembered that in the context of the Standard Model, because of constraints from other measurements, there is only two fold ambiguity in α and no ambiguity in β. The importance of resolving the ambiguities is to expose a possible inconsistency with the Standard Model values. This will then indicate new physics.

When looking for new physics, one should try to assume as little as possible about its nature. Here, we allowed any kind of new physics. This new physics can be (any combination of) new contribution to $B-\bar{B}$, $B_s-\bar{B}_s$ or $K-\bar{K}$ mixing, violation of the three generation CKM unitarity, or a new contribution to decay amplitudes. Once some inconsistency within the Standard Model is found, then the pattern it exhibits can perhaps be used to get some insight about the kind of the new physics responsible for it.

The ideas presented here should be, of course, additional to other methods of looking for new physics [20]. New physics can be found in several other ways: if the values of α and β

are outside the Standard Model allowed range; if the asymmetry in B_s decay mediated by $b \to c\bar{c}s$ it significant; or, if asymmetries that should be the same in the Standard Model are found to be different [8]. Because any discrepancy can be an indication of physics beyond the Standard Model, it is important to try to have as many independent tests as possible.

If some of the above hints for new physics were found, the ideas we presented have to be modified. For example, if $a_{CP}(B \to \phi K_S) \neq a_{CP}(B \to \psi K_S)$ which would indicate a new contribution to the $b \to s$ transition [8, 21], we will not be able to determine sign$(\sin\beta)$ by comparing $a_{CP}(B \to \psi K_S)$ to $a_{CP}(B \to D^+ D^-)$. The underlying assumption in this analysis is that the former measures $\sin 2\beta$ to very high accuracy. A new significant contribution to the $b \to s$ transition would invalidate this assumption.

However, in some situations of new physics, the methods we discuss can still be useful. For example, in models where the only significant new physics effects are significant contribution to the $B - \bar{B}$ or $K - \bar{K}$ mixing amplitude the unitarity triangle can, in principle, be reconstructed. However, the combination of discrete ambiguities and hadronic uncertainties make it impractical [22]. Reduction of the ambiguities, in a manner discussed here, may help in making this program feasible [22].

In our analysis we always care only about a sign of a specific quantity. Usually, the sign of a specific quantity can be determined more easily than its magnitude. For example, the determination of $\cos 2\gamma$ from $B^\pm \to DK^\pm$ decays is experimentally very challenging. However, even a measurement with large errors may be sufficient for our purpose. Of course, if no choice is found to be consistent across the set of measurements we have an immediate indication for non-Standard Model physics.

While the methods we describe work in generic points of the parameter space, there are some values of the angles where they will not work. This is the case where some of the quantities we need to determine are (very close to) zero. For example, when $\alpha = \pi/4$ we have $\cos 2\alpha = 0$. Then, the ambiguity in the value of α is only two fold, but it cannot be removed by the methods we presented. We used the ratio $\cos 2\alpha \sin\alpha / \cos 2\alpha$ to determine sign$(\sin\alpha)$. However, when $\cos 2\alpha \approx 0$ we will not be able to measure this ratio.

From the experimental point of view, since many of the channels we have discussed have yet to be reliably observed it is not clear how feasible the comparisons we discuss will be. All these studies are certainly at least second generation B factory work, not feasible until large data samples have been accumulated. For example, the determinations of sign$(\sin\phi)$ using exclusive decays involve comparisons of measured asymmetries in two different channels. Determining the sign of a difference of two measured quantities, each of which will have significant errors, is certainly not going to be easy, and will be harder if the actual values of the asymmetries are small (e.g. if $|\alpha|$ is close to $\pi/2$).

To conclude: we explain how the determination of sign$(\cos 2\phi)$ and sign$(\sin\phi)$ (for $\phi = \alpha, \beta$) fully resolve the 16 fold ambiguity in the values of α and β as can be extracted from CP asymmetries in B decays. The determinations of sign$(\cos 2\alpha)$ and sign$(\cos 2\beta)$ are theoretically clean. The determination of sign$(\sin\alpha)$ and sign$(\sin\beta)$, however, are plagued with some theoretical input, which, at present, is not reliable. The hope is that by the time the measurements will be carried out, our theoretical toolkit will be improved and we will be able to calculate more reliably the sign of the relevant hadronic effects. From the experimental side, none of the methods we described is easy to carry out. Hopefully, some of them will turn out to be useful.

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