SLAC-PUB-7452
PUPT-1696
Imperial/TP/96-97/40
April 1997

Intermediate Scalars and the Effective String Model of Black Holes

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Abstract

We consider five-dimensional black holes modeled by D-strings bound to D5-branes, with momentum along the D-strings. We study the greybody factors for the non-minimally coupled scalars which originate from the gravitons and R-R antisymmetric tensor particles polarized along the 5-brane, with one index along the string and the other transverse to the string. These scalars, which we call intermediate, couple to the black holes differently from the minimal and the fixed scalars which were studied previously. Analysis of their fluctuations around the black hole reveals a surprising mixing between these NS-NS and R-R scalars. We disentangle this mixing and obtain two decoupled scalar equations. These equations have some new features, and we are able to calculate the greybody factors only in certain limits. The results agree with corresponding calculations in the effective string model provided one of the intermediate scalars couples to an operator of dimension (1,2), while the other to an operator of dimension (2,1). Thus, the intermediate scalars are sensitive probes of the chiral operators in the effective string action.

Submitted to Nuclear Physics B

* Supported in part by the Department of Energy under contract no. DE-AC03-76SF00515.

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1. Introduction

There has been much progress recently in describing the microstates of black holes through D-brane physics. The Bekenstein-Hawking entropy of certain extremal and near-extremal black holes can be understood through the counting of D-brane microstates [1,2,3,4,5]. Furthermore, the Hawking radiation and the semi-classical absorption were shown in many cases to agree with the calculation of the corresponding process in the D-brane picture. This was demonstrated for the charged black holes in four and five dimensions that are described by effective string models [6,7,8,9,10,11,12,13,14,15,16], as well as for the extremal threebranes that admit a direct D-brane description [17,18].

The results mentioned so far refer to minimally coupled scalar fields. Not all scalars, however, are minimally coupled. There are other scalars which couple to the non-trivial vector backgrounds. Examples of these are the 'fixed' scalars considered in [19,20], which have different cross-sections from the minimally coupled scalars. In the $D = 5$ black hole background there are two specific fixed scalars, which mix with each other and with the gravitational perturbations [20]. Recently, the complexities of this mixing were disentangled in [21]. The greybody factors calculated from the diagonalized equations of motion were found to be of the form obtained earlier in [20]: in the effective string model such greybody factors are reproduced by operators of dimension $(2, 2)$. This poses a puzzle, since the effective string action derived in [20] also contains couplings to dimension $(1, 3)$ and $(3, 1)$ operators which produce greybody factors of a different form. Thus, it is of special interest to study other situations in which chiral operators appear in the effective string couplings. This will be the subject of the present paper.

We will be concerned with yet a third type of scalars, which we call intermediate, first considered in [20]. This type is different from both the minimally coupled and the fixed scalars. The intermediate scalars originate from the fields A_5^i (denoted by h_{5i} in [20]) and B_{5i}, i.e. the gravitons and the R-R 2-form particles polarized along the 5-brane, with one index pointing along the string and the other transversely to the string. In this paper, we will calculate the semi-classical absorption cross-sections of the intermediate scalars and compare them with the effective string model predictions.

In Section 2, after presenting the setup, we derive the classical equations of motion for the intermediate scalars in the $D = 5$ black hole background (an alternative derivation based on the 6-dimensional theory will be presented in the Appendix). This turns out to be quite nontrivial due to a mixing between A_5^i and B_{5i}. In Section 3 we propose a coupling for these scalars in the effective string model of the black hole. Part of this coupling term is not present in the standard Nambu-type D-string action. It turns out that requiring the scalars to couple to operators of a given dimension on the world sheet is a very restrictive guiding principle. We find that the necessary operators are of dimensions (1,2) and (2,1) and then calculate the resulting cross-sections as predicted by the effective string model.

Finally, in Section 4 we compare the absorption cross-sections derived by semi-classical considerations to the cross-sections predicted by the string. The classical equations of motion are complicated and we are only able to solve for the cross-sections in various limits. In every case that we can treat analytically, there is exact agreement between the semi-classical gravity and the effective string. This is evidence that the effective string model reproduces the dynamics of the intermediate scalars. However, our inability to solve for the general semi-classical greybody factor leaves the question of the complete agreement open.

2. Derivation of the Equations of Motion

As in [20] we start with the action of the $D = 10$ type IIB supergravity reduced to 5 dimensions. The relevant part of it is

$$S_5 = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{g}\Big[R - \frac{4}{3}(\partial_\mu\phi_5)^2 - \frac{1}{4}G^{pl}G^{qn}(\partial_\mu G_{pq}\partial^\mu G_{ln} + \sqrt{G}\partial_\mu B_{pq}\partial^\mu B_{ln}) \tag{2.1}$$

$$- \frac{1}{4}e^{-\frac{4}{3}\phi_5}G_{pq}F^p_{\mu\nu}F^q_{\mu\nu} - \frac{1}{4}e^{\frac{2}{3}\phi_5}\sqrt{G}G^{pq}H_{\mu\nu p}H_{\mu\nu q} - \frac{1}{12}e^{\frac{4}{3}\phi_5}\sqrt{G}H^2_{\mu\nu\lambda} + \ldots\Big] ,$$

where $\mu,\nu,\ldots = 0,1,2,3,4$; $p,q,\ldots = 5,6,7,8,9$. ϕ_5 is the 5-d dilaton and G_{pq} is the metric of internal 5-torus,

$$\phi_5 \equiv \phi_{10} - \frac{1}{4}G = \phi_6 - \frac{1}{2}\lambda , \qquad G = \det G_{pq} ,$$

and B_{pq} are the internal components of the R-R 2-tensor. $F^p_{\mu\nu}$ is the field strength of the Kaluza-Klein vectors A^p_μ. It will be crucial in what follows that $H_{\mu\nu p}$ and $H_{\mu\nu\lambda}$ are given explicitly by (see, e.g., [22])

$$H_{\mu\nu p} = F_{\mu\nu p} - B_{pq}F^q_{\mu\nu} , \qquad F_p = dB_p , \qquad F^p = dA^p , \tag{2.2}$$

$$H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} - \frac{1}{2}A^p_\mu F_{\nu\lambda p} - \frac{1}{2}B_{\mu p}F^p_{\nu\lambda} + \text{cyclic permutations} ,$$

where $B_{\mu p}$ and $B_{\mu\nu}$ differ from the $D = 10$ components of the R-R 2-form field by terms proportional to A^p_μ. The 'shifts' in these field strengths vanish for the $D = 5$ black hole backgrounds which correspond to bound states of RR-charged 5-branes and strings with momentum flow. For such black holes, $B_{pq} = 0$, the vector fields A^p and B_p have electric backgrounds, while $H_{\mu\nu\lambda}$ has a magnetic one (we shall assume that the electric charges Q_{Kp} and Q^p corresponding to the vectors A^p and B_p have only the $p = 5$ component). However, in general the field strength shifts in (2.2) are important for the discussion of perturbations. We will argue, in fact, that while the shift in $H_{\mu\nu\lambda}$ does not contribute in the present case, the shift in $H_{\mu\nu p}$ will lead to a mixing between perturbations of G_{pq} and B_{pq} for $p=5$ and $q=i$ (5 is the direction of the string and $i=6,7,8,9$ label the directions of T^5 orthogonal to the string).

The 5-dimensional charged black hole metric is [23,24,4]

$$ds_5^2 = g_{mn}dx^m dx^n + ds_3^2 = -h\mathcal{H}^{-2}dt^2 + h^{-1}\mathcal{H}dr^2 + r^2\mathcal{H}d\Omega_3^2 \ , \tag{2.3}$$

$$h = 1 - \frac{r_0^2}{r^2} \ , \quad \mathcal{H} \equiv (H_n H_1 H_5)^{1/3} \ , \quad \sqrt{g} = r^3 (H_n H_1 H_5)^{1/3} \ ,$$

$$H_1 = 1 + \frac{\hat{Q}}{r^2} \ , \quad H_5 = 1 + \frac{\hat{P}}{r^2} \ , \quad H_n = 1 + \frac{\hat{Q}_K}{r^2} \ ,$$

where $\hat{Q} = \sqrt{Q^2 + \frac{1}{4}r_0^4} - \frac{1}{2}r_0^2$, etc. The background values of the internal metric and the dilaton are (see [20] for more details)

$$(ds_{10}^2)_{T^5} = G_{pq}dx^p dx^q = e^{2\nu_5}dx_5^2 + e^{2\nu}(dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2) \ , \tag{2.4}$$

$$\nu_5 = -2\phi_5 \equiv \lambda \ , \qquad e^{2\lambda} = \frac{H_n}{(H_1 H_5)^{1/2}} \ .$$

It is useful to choose the following parametrization for the full (background plus perturbation) internal metric

$$G_{pq} = e^{2\nu}\begin{pmatrix} e^{2\lambda-2\nu} + e^{2\nu}A_5^i A_5^i & A_5^i \\ A_5^j & \delta_{ij} \end{pmatrix} \ , \qquad \sqrt{G} = e^{\lambda+4\nu} \ , \tag{2.5}$$

$$G^{pq} = e^{-2\lambda}\begin{pmatrix} 1 & -A_5^i \\ -A_5^j & e^{2\lambda-2\nu}\delta^{ij} + A_5^i A_5^j \end{pmatrix} \ ,$$

For the present discussion of the 'off-diagonal' perturbations the fluctuations of ϕ_5, as well as those of $\sqrt{G}$, can be ignored. Therefore, we concentrate on the dependence on A_5^i and B_{5i} and do not keep track of other scalar perturbations which were already discussed in [20].

The $D=5$ scalars A_5^i and B_{5i} originate from the $M=5$ components of the KK vector A_M^i and the vector component B_{Mi} of the R-R 2-tensor in type IIB supergravity reduced to 6 dimensions. An alternative derivation of the equations for the A_5^i and B_{5i} perturbations, which directly uses the $D=6$ theory, will be presented in the Appendix.

The relevant terms in the $D=5$ action are[1]

$$S_5 = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{g}\Big[-\frac{1}{2}e^{-2\lambda+2\nu}\partial_\mu A_5^i \partial^\mu A_5^i - \frac{1}{2}e^{-2\lambda+2\nu}\partial_\mu B_{5i}\partial^\mu B_{5i} \tag{2.6}$$

[1] The μ,ν indices are always contracted using the curved 5-dimensional metric and assuming that $F_{\mu\nu}F_{\mu\nu} \equiv F_{\mu\nu}F^{\mu\nu}$, etc. The repeated i,j-indices are summed with δ_{ij} with no extra factors (all factors in $5,i$ directions are given explicitly).

$$- \frac{1}{4} e^{\frac{2}{3}\lambda+2\nu} [F^i_{\mu\nu} F^i_{\mu\nu} + 2F^5_{\mu\nu} F^i_{\mu\nu} A^i_5 + F^5_{\mu\nu} F^5_{\mu\nu} (e^{2\lambda-2\nu} + A^i_5 A^i_5)]$$

$$- \frac{1}{4} e^{-\frac{4}{3}\lambda+4\nu} \Big[H_{\mu\nu 5} H_{\mu\nu 5} - 2H_{\mu\nu i} H_{\mu\nu 5} A^i_5 + H_{\mu\nu i} H_{\mu\nu j} (e^{2\lambda-2\nu}\delta^{ij} + A^i_5 A^j_5)\Big]\Big] \ ,$$

where

$$H_{\mu\nu 5} = F_{\mu\nu 5} - B_{5i} F^i_{\mu\nu} \ , \qquad H_{\mu\nu i} = F_{\mu\nu i} + B_{5i} F^5_{\mu\nu} \ .$$

Here only $F_{\mu\nu 5}$ and $F^5_{\mu\nu}$ have background values, which we denote by $\tilde{F}$,

$$e^{\frac{8}{3}\lambda} \sqrt{g} (\tilde{F}^5)^{0r} = 2Q_K \ , \qquad e^{-\frac{4}{3}\lambda+4\nu} \sqrt{g} (\tilde{F}_5)^{0r} = 2Q \ ,$$

so that

$$\tilde{F}^5_{\mu\nu} = \frac{Q_K}{Q} e^{-4\lambda+4\nu} \tilde{F}_{\mu\nu 5} \ . \tag{2.7}$$

As a result, we may integrate out $F_{\mu\nu i}$ or all of $H_{\mu\nu i}$ easily. This gives[2]

$$-\frac{1}{4} e^{-\frac{4}{3}\lambda+4\nu} \big(- e^{-2\lambda+2\nu} \tilde{F}_{\mu\nu 5} \tilde{F}_{\mu\nu 5} A^i_5 A^i_5 \big) \ . \tag{2.8}$$

To show this it is crucial that $\tilde{F}^5_{\mu\nu}$ has only the electric component and depends only on r, and that the scalar perturbations depend only on r and t. This is similar to what happens in the fixed scalar case [19,20].

The mixing that contributes a new term is $\tilde{F}_{\mu\nu 5} B_{5i} F^i_{\mu\nu}$ which comes from the $H^2_{\mu\nu 5}$ term.[3] The relevant vector-scalar terms are

$$- \frac{1}{4} e^{\frac{2}{3}\lambda} e^{2\nu} \big[F^i_{\mu\nu} F^i_{\mu\nu} + 2\tilde{F}^5_{\mu\nu} F^i_{\mu\nu} A^i_5 + \tilde{F}^5_{\mu\nu} \tilde{F}^5_{\mu\nu} A^i_5 A^i_5 - 2e^{-2\lambda+2\nu} \tilde{F}_{\mu\nu 5} B_{5i} F^i_{\mu\nu}\big] \ .$$

It remains to integrate out $F^i_{\mu\nu}$. One should actually integrate over the corresponding gauge potential, but since the background is electric and static, and the scalars depend only on r and t, this is equivalent to just solving for the field strength.

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[2] The $H_i H_i hh$ term is of subleading order being quartic in the fluctuations.

[3] One way to see why the mixing terms inside $H_{\mu\nu\lambda}$ in (2.1) and (2.2) do not contribute is to dualize $B_{\mu\nu}$ into a vector, V_μ. The resulting terms in the action will have the following structure: $\int d^5x [-\frac{1}{4}\sqrt{g} e^{-\frac{4}{3}\phi_5} G^{-1/2} F^2_{\mu\nu}(V) + \epsilon^{\mu\nu\lambda\sigma\kappa} V_\mu F_{\nu\lambda p} F^p_{\sigma\kappa}]$. The three vectors, V_μ, $A_{\mu 5}$, A^5_μ, have electric backgrounds with charges P, Q, Q_K respectively. The trilinear Chern-Simons-type term produces a non-zero contribution in the gaussian approximation only if the two fluctuation fields have indices different from 0 and r, which are the directions of the electric background of the third field in the product. This means that the Chern-Simons-type term does not mix the 'electric' perturbations of the fields, but it is the 'electric' perturbations of the vectors that couple to the off-diagonal scalars we discuss.

Adding the $A_5^i A_5^i$ term already obtained in (2.8), we get

$$-\frac{1}{4}e^{-\frac{2}{3}\lambda+2\nu}\Big(- \big[\tilde{F}^5_{\mu\nu}A^i_5 - e^{-2\lambda+2\nu}\tilde{F}_{\mu\nu 5}B_{5i}\big]^2$$

$$+ \tilde{F}^5_{\mu\nu}\tilde{F}^5_{\mu\nu}A^i_5A^i_5 - e^{-4\lambda+4\nu}\tilde{F}_{\mu\nu 5}\tilde{F}_{\mu\nu 5}A^i_5A^i_5\Big).$$

We can simplify this expression using (2.7):

$$\frac{1}{4}e^{-\frac{2}{3}\lambda+2\nu}\Big(e^{-4\lambda+4\nu}\tilde{F}_{\mu\nu 5}\tilde{F}_{\mu\nu 5}(A^i_5A^i_5 + B_{5i}B_{5i}) - 2e^{-2\lambda+2\nu}\tilde{F}^i_{\mu\nu}\tilde{F}_{\mu\nu 5}A^i_5B_{5i}\Big)$$

$$= \frac{1}{4}e^{-\frac{14}{3}\lambda+6\nu}\tilde{F}_{\mu\nu 5}\tilde{F}_{\mu\nu 5}\Big(A^i_5A^i_5 + B_{5i}B_{5i} - 2\frac{Q_K}{Q}e^{-2\lambda+2\nu}A^i_5B_{5i}\Big) . \tag{2.9}$$

The novelty is the mixing term in the brackets

$$-2\frac{Q_K}{Q}e^{-2\lambda+2\nu}A^i_5B_{5i} = -2\frac{Q_KH_1}{QH_n}A^i_5B_{5i} ,$$

which is thus present for arbitrary non-vanishing values of P, Q and Q_K.

Remarkably, the full A^i_5, B_{5i} scalar action with the kinetic terms included can be diagonalized in terms of the fields ξ_i and η_i defined by

$$\eta_i = \frac{1}{\sqrt{2}}(A^i_5 + B_{5i}) , \qquad \xi_i = \frac{1}{\sqrt{2}}(A^i_5 - B_{5i}) . \tag{2.10}$$

With these definitions,

$$S_5 = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{g}\Big(- \frac{1}{2}e^{-2\lambda+2\nu}[(\partial_\mu\xi_i)^2 + (\partial_\mu\eta_i)^2] \tag{2.11}$$

$$+ \frac{1}{4}e^{-\frac{14}{3}\lambda+6\nu}\tilde{F}_{\mu\nu 5}\tilde{F}_{\mu\nu 5}\big[(1 + \frac{Q_K}{Q}e^{-2\lambda+2\nu})\xi_i^2 + (1 - \frac{Q_K}{Q}e^{-2\lambda+2\nu})\eta_i^2\big]\Big) .$$

Rescaling the fields to eliminate the background-dependent factors $e^{-2\lambda+2\nu}$ in the kinetic parts, we arrive at the following decoupled equations (we shall use the same notation, ξ_i and η_i, for the redefined fields, $e^{-\lambda+\nu}\xi_i$ and $e^{-\lambda+\nu}\eta_i$)

$$\begin{aligned}\Big[hr^{-3}\frac{d}{dr}(hr^3\frac{d}{dr}) + \omega^2H_1H_5H_n - M_\xi\Big]\,\xi_i &= 0 ,\\ \Big[hr^{-3}\frac{d}{dr}(hr^3\frac{d}{dr}) + \omega^2H_1H_5H_n - M_\eta\Big]\,\eta_i &= 0 ,\end{aligned} \tag{2.12}$$

where

$$M_\xi = M_{\lambda-\nu} + M_+ , \qquad M_\eta = M_{\lambda-\nu} + M_- , \tag{2.13}$$

$$M_{\lambda-\nu} = h e^{\lambda-\nu} r^{-3} \frac{d}{dr}(r^3 h \frac{d}{dr} e^{-\lambda+\nu}) ,$$

$$M_\pm = \frac{4Q^2}{r^6 H_1^2}(1 \pm \frac{Q_K H_1}{Q H_n}) h .$$

Somewhat surprisingly, all the dependence on P disappears from the "mass" terms since

$$e^{-\lambda+\nu} = (H_1/H_n)^{1/2} ,$$

so that

$$M_{\lambda-\nu} = \frac{(\hat{Q}_K - \hat{Q})(r^2 - r_0^2)}{r^4 (r^2 + \hat{Q}_K)^2 (r^2 + \hat{Q})^2} \Big[(\hat{Q} + 3\hat{Q}_K + 2r_0^2) r^4$$

$$+ (4\hat{Q}\hat{Q}_K + \hat{Q} r_0^2 - \hat{Q}_K r_0^2) r^2 - 2\hat{Q}\hat{Q}_K r_0^2 \Big] .$$

In the extremal limit, $r_0 = 0$, $\hat{Q} = Q$, $\hat{P} = P$, $\hat{Q}_K = Q_K$, the resulting "mass" terms are found to be

$$M_\xi = \frac{8Q^2 Q_K^2 + 8QQ_K(Q + Q_K) r^2 + (3Q^2 + 2QQ_K + 3Q_K^2) r^4}{r^2 (r^2 + Q_K)^2 (r^2 + Q)^2} ,$$

$$M_\eta = \frac{3(Q - Q_K)^2 r^2}{(r^2 + Q_K)^2 (r^2 + Q)^2} . \tag{2.14}$$

They have the following asymptotics

$$r \to 0: \qquad M_\xi = \frac{8}{r^2} , \qquad M_\eta = 0 ,$$

$$r \to \infty: \qquad M_\xi = \frac{3Q^2 + 2QQ_K + 3Q_K^2}{r^6} , \qquad M_\eta = \frac{3(Q - Q_K)^2}{r^6} .$$

At the horizon η_i behaves as the $l = 0$ partial wave of a minimally coupled scalar. ξ_i, on the other hand, behaves as the $l = 2$ partial wave, which is the behavior previously encountered for the fixed scalars [20,21]. The expressions (2.14) can be simplified if $Q \gg Q_K$,

$$M_\xi = \frac{Q^2 (8Q_K^2 + 8Q_K r^2 + 3r^4)}{r^2 (r^2 + Q_K)^2 (r^2 + Q)^2} , \tag{2.15}$$

$$M_\eta = \frac{3Q^2 r^2}{(r^2 + Q_K)^2 (r^2 + Q)^2} .$$

Note that, for $Q_K = 0$,

$$M_\eta = M_\xi = \frac{3Q^2}{r^2 (r^2 + Q)^2} . \tag{2.16}$$

Thus, as one switches on Q_K, there is a remarkable jump from the $l = 1$ to the $l = 0$ or $l = 2$ behaviors near the horizon.

3. Absorption in the Effective String Model

In the previous section we found a surprising mixing between the off-diagonal components of the Kaluza-Klein scalars A^i_5 and the internal components B_{5i} of the R-R 2-tensor. In this section we discuss this mixing from the effective string point of view, and show what it implies about the greybody factors.

First, we have to write down the lowest-dimension couplings to the effective string for the fields in question. In [20] the scalar fields A^i_5 were included, but the components B_{5i} of the R-R field were omitted. In fact, as the discussion of the gravitational perturbations shows, these two field mix and one should keep both of them. The simplest assumption that one usually makes is that the effective string action is the same as the D-string action with a rescaled tension. The necessary terms in the action are then

$$S = -T_{\text{eff}} \int d^2\sigma \, \left(\sqrt{-\gamma} - \hat{B}_{05}\right) ,$$

where

$$\gamma_{ab} = G_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu \ , \qquad \hat{B}_{ab} = B_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu \ .$$

The leading order couplings are found to be

$$-\frac{T_{\text{eff}}}{2}[A^i_5(\partial_+ + \partial_-)X^i + B_{5i}(\partial_+ - \partial_-)X^i] = -\frac{T_{\text{eff}}}{\sqrt{2}}[\xi_i \partial_- X^i + \eta_i \partial_+ X^i] \ ,$$

where the same mixtures of the fields naturally emerge as the ones needed in the effective field theory (GR) calculation, (2.10). We see that these mixtures couple to operators of dimension $(0,1)$ and $(1,0)$ respectively. Clearly, these couplings do not contribute to absorption. Expanding further we find the term

$$-\frac{T_{\text{eff}}}{4} A^i_5 \ [\partial_- X^i (\partial_+ X)^2 + \partial_+ X^i (\partial_- X)^2] \ ,$$

whose natural supersymmetric completion is

$$-\frac{T_{\text{eff}}}{4} \ A^i_5 \ [\ \partial_- X^i \ T^{tot}_{++} \ + \ \partial_+ X^i \ T^{tot}_{--} \ ] \ ,$$

with T^{tot} including the fermionic contribution as well. It is interesting that, using this coupling in the case $Q_K = 0$, we find the greybody factor which *exactly* agrees with the GR result. So, for $Q_K = 0$ (the non-chiral case) we may just use the coupling stated in [20] and arrive at complete agreement with the semi-classical calculation.

The structure of the action is less clear for $Q_K > 0$. While we do not readily see a cubic coupling for B_{5i}, we will add it by hand to enforce the principle that ξ_i and η_i

couple to operators of a given dimension. With this assumption, the terms that arise in the effective string action are

$$\delta S = -\frac{T_{\text{eff}}\sqrt{2}}{4}\int d^2\sigma\ [\ \eta_i\ \partial_- X^i\ T^{tot}_{++}\ +\ \xi_i\ \partial_+ X^i\ T^{tot}_{--}\ ]\ . \tag{3.1}$$

Using the action (3.1), let us now derive the effective string absorption cross-section for η_i. The absorption cross-section is due to processes $\eta_i \to L+L+R$ and $\eta_i + L \to L+R$ (L and R stand for the left-moving and right-moving modes on the string). The matrix element between properly normalized states, including the kinetic term normalization factor $\kappa_5\sqrt{2}$ for η_i (see (2.11)), is found to be

$$\frac{2\kappa_5}{\sqrt{T_{\text{eff}}}}\sqrt{\frac{q_1 p_1 p_2}{\omega}}\ . \tag{3.2}$$

Adding up the absorption rates for the two processes gives (see [20] for details of analogous cross-section computations)

$$\begin{aligned}&\frac{3\kappa_5^2 L_{\text{eff}}}{2\pi T_{\text{eff}}}\frac{1}{1-e^{-\frac{\omega}{2T_R}}}\int_{-\infty}^{\infty} dp_1 dp_2\, \delta\left(p_1+p_2-\frac{\omega}{2}\right)\frac{p_1}{1-e^{-\frac{p_1}{T_L}}}\frac{p_2}{1-e^{-\frac{p_2}{T_L}}}\\ =&\frac{\kappa_5^2 L_{\text{eff}}}{32\pi T_{\text{eff}}}\frac{\omega}{\left(1-e^{-\frac{\omega}{2T_L}}\right)\left(1-e^{-\frac{\omega}{2T_R}}\right)}\left(\omega^2+16\pi^2 T_L^2\right)\ .\end{aligned} \tag{3.3}$$

The values of the parameters in the effective string model have been fixed in [25,10,20],

$$\kappa_5^2 L_{\text{eff}} = 4\pi^3 r_1^2 r_5^2\ , \qquad T_{\text{eff}} = \frac{1}{2\pi r_5^2}\ , \tag{3.4}$$

where

$$r_1^2 \equiv \hat{Q}\ , \qquad r_5^2 \equiv \hat{P}\ .$$

Note that this effective string tension is the tension of the D-string divided by n_5, the number of 5-branes. This value of the tension is necessary for agreement with the entropy of near-extremal 5-branes [25], as well as for the agreement of the fixed-scalar cross-section for $r_1 = r_5$ [20]. In this paper we will show that it also leads to agreement of the absorption cross-sections for the scalars η_i and ξ_i.

Using (3.3), (3.4), and the detailed balance, we find that the absorption cross-section for η_i is

$$\sigma_\eta(\omega) = \frac{\pi^3 r_1^2 r_5^4}{4}\frac{\omega\left(e^{\frac{\omega}{T_H}}-1\right)}{\left(e^{\frac{\omega}{2T_L}}-1\right)\left(e^{\frac{\omega}{2T_R}}-1\right)}(\omega^2+16\pi^2 T_L^2)\ . \tag{3.5}$$

After analogous steps, the absorption cross-section for ξ_i is found to be

$$\sigma_\xi(\omega) = \frac{\pi^3 r_1^2 r_5^4}{4} \frac{\omega\left(e^{\frac{\omega}{T_H}} - 1\right)}{\left(e^{\frac{\omega}{2T_L}} - 1\right)\left(e^{\frac{\omega}{2T_R}} - 1\right)} (\omega^2 + 16\pi^2 T_R^2) \ . \tag{3.6}$$

In the next section we will check these greybody factors against semi-classical effective field theory calculations. We will need the following expressions for the temperatures [10],

$$T_L = \frac{r_0 e^\sigma}{2\pi r_1 r_5} \ , \qquad T_R = \frac{r_0 e^{-\sigma}}{2\pi r_1 r_5} \ , \qquad \frac{2}{T_H} = \frac{1}{T_L} + \frac{1}{T_R} \ , \tag{3.7}$$

where σ is defined by

$$r_n^2 = r_0^2 \sinh^2 \sigma \ , \qquad r_n^2 \equiv \hat{Q}_K \ .$$

This may be solved with the result,

$$e^{\pm 2\sigma} = 1 + \frac{2}{r_0^2}(r_n^2 \pm Q) \ .$$

Under $Q_K \to -Q_K$, we therefore find that $\sigma \to -\sigma$, which implies that T_L and T_R are interchanged. This transformation reverses the momentum flow along the string, so that the operators of dimension $(1,2)$ and $(2,1)$, and therefore ξ_i and η_i, are interchanged. The classical equations for ξ_i and η_i, (2.12), (2.13), are also interchanged under $Q_K \to -Q_K$. This is the first, and very important, consistency check between the effective string and the semi-classical descriptions of the intermediate scalars.

4. Comparison with Semiclassical Greybody Factors

In this section we carry out a number of calculations which indicate agreement, at least in various limits, between the semi-classical cross-sections and those in the effective string model. First we discuss the case $Q_K = 0$ where the classical calculation is the easiest. Then we address various limits of the $Q_K > 0$ case.

4.1. $Q_K = 0$

Here we consider the case $r_n^2 = 0$ (i.e. $Q_K = 0$), where η_i and ξ_i satisfy identical equations (2.12),(2.16). Since here $T_L = T_R$, the two effective string greybody factors are also the same, and they will turn out to be identical to the semi-classical ones.

The non-extremal equation satisfied by both η_i and ξ_i is (for $r_0 \ll r_1, r_5$)

$$\left[hr^{-3}\partial_r(hr^3\partial_r) + f(r)\omega^2 - 3h\frac{r_1^4}{r^2(r_1^2+r^2)^2} + h\frac{r_0^2 r_1^2(r_1^2+2r^2)}{r^4(r_1^2+r^2)^2}\right] R = 0 \ , \tag{4.1}$$

where we set $\eta_i, \xi_i = R(r)e^{i\omega t}$, and

$$h(r) = 1 - \frac{r_0^2}{r^2} , \qquad f(r) = \left(1 + \frac{r_1^2}{r^2}\right)\left(1 + \frac{r_5^2}{r^2}\right) .$$

In the near region ($r \ll r_1, r_5$) we find, in terms of the variable $z = h(r)$,

$$\left[z\partial_z(z\partial_z) + D + \frac{C}{(1-z)} + \frac{E}{(1-z)^2} \right] R = 0 , \tag{4.2}$$

where

$$D = -\frac{1}{4} , \qquad C = \frac{\omega^2 r_1^2 r_5^2}{4r_0^2} + 1 , \qquad E = -\frac{3}{4} .$$

This may be reduced to a hypergeometric equation by a substitution of the form

$$R = z^\alpha (1-z)^\beta F(z) . \tag{4.3}$$

After some algebra we find that, if α and β satisfy

$$E + \beta(\beta - 1) = 0 , \qquad \alpha^2 + D + C + E = 0 ,$$

then the equation for $F(z)$ becomes

$$z(1-z)\frac{d^2F}{dz^2} + [(2\alpha+1)(1-z) - 2\beta z]\frac{dF}{dz} - [(\alpha+\beta)^2 + D]F = 0 , \tag{4.4}$$

which is the hypergeometric equation. In general, the solution to

$$z(1-z)\frac{d^2F}{dz^2} + [C - (1 + A + B)z]\frac{dF}{dz} - ABF = 0 , \tag{4.5}$$

which satisfies $F(0) = 1$, is the hypergeometric function $F(A, B; C; z)$. Thus, the solution in the inner region is

$$R_I = z^\alpha (1-z)^\beta F(\alpha + \beta + i\sqrt{D}, \alpha + \beta - i\sqrt{D}; 1 + 2\alpha; z) , \tag{4.6}$$

where

$$\beta = -\frac{1}{2} , \qquad \alpha = -i\frac{\omega r_1 r_5}{2r_0} = -i\frac{\omega}{4\pi T} .$$

In the last equation we used the fact that, for $r_n = 0$,

$$T = T_L = T_R = T_H = \frac{r_0}{2\pi r_1 r_5} .$$

Using the asymptotics of the hypergeometric functions for $z \to 1$, we find that, for large r,

$$R_I \to \frac{r}{r_0} E ,$$

where

$$E = \frac{\Gamma\left(1 - i\frac{\omega}{2\pi T}\right)}{\Gamma\left(2 - i\frac{\omega}{4\pi T}\right)\Gamma\left(1 - i\frac{\omega}{4\pi T}\right)} \ .$$

In the middle region ($r_0 \ll r \ll 1/\omega$) the approximate solution is

$$R_{II} \approx E\ \frac{r_1}{r_0}\left(1 + \frac{r_1^2}{r^2}\right)^{-1/2} .$$

In the outer region, the dominant solution, which matches to the asymptotic form in region II, is

$$R_{III} = 2A\rho^{-1}J_1(\rho) \ , \qquad \rho = \omega r \ .$$

By matching we find that

$$A = E\ \frac{r_1}{r_0} \ .$$

The absorption cross-section may now be obtained using the method of fluxes (see, e.g., [10,20] and references therein). The flux per unit solid angle is

$$\mathcal{F} = \frac{1}{2i}(R^* h r^3 \partial_r R - \text{c.c.}) \ . \tag{4.7}$$

The absorption probability is the ratio of the incoming flux at the horizon to the incoming flux at infinity,

$$P = \frac{\mathcal{F}_{\text{horizon}}}{\mathcal{F}_\infty^{\text{incoming}}} = \frac{\pi\omega^3}{2} r_1 r_5 r_0\ |A|^{-2} \ .$$

The absorption cross-section is related to the s-wave absorption probability by

$$\sigma_{\text{abs}} = \frac{4\pi}{\omega^3}P = \frac{2\pi^2 r_0^3 r_5}{r_1}|E|^{-2} \ .$$

Thus,

$$\sigma_{\text{abs}} = \frac{2\pi^2 r_0^3 r_5}{r_1} x(1 + x^2)\frac{e^{2\pi x} + 1}{e^{2\pi x} - 1} \ ,$$

where

$$x = \frac{\omega}{4\pi T} = \frac{\omega r_1 r_5}{2r_0} \ .$$

It follows that

$$\sigma_{\text{abs}} = \frac{\pi^3}{4} r_1^2 r_5^4 \frac{e^{\frac{\omega}{2T}} + 1}{e^{\frac{\omega}{2T}} - 1}\omega(\omega^2 + 16\pi^2 T^2) \ . \tag{4.8}$$

This is in exact agreement with the cross-sections (3.5) and (3.6) derived in the effective string model! In particular, the agreement of the overall normalization provides new evidence in favor of the effective string tension (3.4) being given by the D-string tension divided by n_5.

4.2. The ξ_i cross-section for $Q_K > 0$

The scalar ξ_i has the fluctuation equation (2.12) with the effective mass term $M_\xi = M_{\lambda-\nu} + M_+$. We will try to solve for the cross-section exactly in the regime where $r_0 \ll r_n \ll r_1, r_5$, so that $T_R \ll T_L$. We will take ω/T_R to be of order 1. Hence we should be able to find the dependence of the cross-section on ω/T_R, which is a test of the greybody factor dependence.

We will match the approximate solutions in several regions. First, consider the inner region, $r \ll r_n$. Here the effective mass is approximately $\frac{8h}{r^2}$, and the equation becomes

$$\left[hr^{-3}\partial_r(hr^3\partial_r) + \frac{\omega^2 r_1^2 r_5^2 r_n^2}{r^6} - \frac{8(r^2 - r_0^2)}{r^4}\right] R = 0 \ .$$

In terms of the variable $z = 1 - \frac{r_0^2}{r^2}$,

$$\left[z\partial_z(z\partial_z) + \frac{\omega^2 r_1^2 r_5^2 r_n^2}{4r_0^4} - \frac{2z}{(1-z)^2}\right] R = 0 \ .$$

This equation has the same form as (4.2) with

$$D = \frac{\omega^2 r_1^4 r_n^2}{4r_0^4} \ , \qquad C = 2 \ , \qquad E = -2 \ .$$

We will again use the substitution (4.3), where now

$$\begin{aligned} E + \beta(\beta - 1) = 0 \ &\rightarrow \ (\beta - 2)(\beta + 1) = 0 \ , \\ \alpha^2 + D + C + E = 0 \ &\rightarrow \ \alpha = -i\frac{\omega r_1 r_5 r_n}{2r_0^2} \ . \end{aligned}$$

For $r_0 \ll r_n$, we have

$$T_R \approx \frac{r_0^2}{4\pi r_1 r_5 r_n} \ . \tag{4.9}$$

Thus,

$$\alpha = -i\frac{\omega}{8\pi T_R} \ . \tag{4.10}$$

We also choose $\beta = -1$. Hence, the solution is

$$R_I = z^\alpha (1-z)^{-1} F(-1 + \alpha + i\sqrt{D}, -1 + \alpha - i\sqrt{D}; 1 + 2\alpha; z) \ .$$

Away from the horizon, i.e. as $z \rightarrow 1$,

$$R_I \rightarrow \frac{r^2}{r_0^2}\frac{2}{1 - i\frac{\omega}{4\pi T_R}} \equiv K_1 \frac{r^2}{r_0^2} \ .$$