

Supersymmetric Baryogenesis at the Electroweak Phase Transition

Mihir Worah

*Stanford Linear Accelerator Center,
Stanford University, Stanford, CA 94309*

Submitted to *Physical Review D*

SUPERSYMMETRIC BARYOGENESIS AT THE ELECTROWEAK PHASE TRANSITION

Mihir P. Worah

*Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309*

Abstract

We study the possibility of baryogenesis in the case of supersymmetry breaking with large mixing between the \tilde{c}_R and \tilde{t}_R or \tilde{u}_R and \tilde{t}_R squarks resulting in one light right-handed up-type squark mass eigenstate. We argue that in this case the electroweak phase transition will be first order, and that large phases already present in the quark mass matrices can generate a baryon asymmetry of the correct magnitude without introducing any new phases specifically for this purpose. We study in detail a particular ansatz for supersymmetry breaking and CP violation where there is only one CP violating phase in the theory: in the up-type quark mass matrix. We study the constraints placed on this model by baryogenesis and flavor physics. This scenario has robust implications for low energy flavor physics including $D^0 - \bar{D}^0$ mixing and an electric dipole moment for the neutron that are close to the experimental bounds, and CP violation in the $B - \bar{B}$ system that is different from that in the Standard Model.

1 Introduction

The three physical requirements needed for baryogenesis [1]: baryon number violation, CP violation, and a departure from thermal equilibrium all exist at the electroweak phase transition (EWPT). The existence of baryon number violating standard model field configurations at high temperatures was demonstrated in [2]. CP violating interactions are known to exist in nature, and a sufficiently first order phase transition would provide the departure from equilibrium needed in order to generate the baryon asymmetry and then shut off the baryon number violation fast enough to avoid washing it out. The possibility of baryogenesis occurring at the electroweak phase transition is exciting because the physics occurs at $\simeq 100$ GeV, and is plausibly testable in terrestrial experiments.

An interesting attempt to generate the asymmetry using only the fields and CP violation in the Standard Model was made in [3], but it was later shown in [4] that decoherence effects prevent the generation of a sufficiently large asymmetry in this case. In addition, the phase transition in the Standard Model isn't quite strong enough to preserve any asymmetry that might have been generated. Thus it seems that one needs to go beyond the Standard Model in order to generate the baryon number asymmetry. Although many models have been proposed that successfully accomplish this [5], there is no one commonly accepted or most plausible paradigm.

A well motivated extension of the Standard Model is the Minimal Supersymmetric Standard Model (MSSM). Introducing the extra sparticle content of the MSSM brings with it a plethora of new masses, mixing angles and CP violating phases associated with the soft breaking of supersymmetry. These are constrained by the assumptions of no new CP violation beyond that of the Cabibbo Kobayashi Maskawa (CKM) matrix, universal supersymmetry breaking squark and gaugino masses, and universal and proportional trilinear scalar couplings. The phenomenology of this constrained model has been extensively studied [6]. While rich in implications for high energy collider experiments, the constrained MSSM by construction has negligible effects on low energy flavor physics and baryogenesis.

In a recent series of papers [7, 8] it has been shown that if the assumption of universal squark masses was relaxed to allow a sufficiently light *stop*, the strength of the electroweak phase transition would be enhanced over that in the Standard Model, allowing any baryon asymmetry that might have been generated at the phase transition to persist until today. This light *stop* would have to be predominantly the \tilde{t}_R because a light \tilde{t}_L would require a degenerate \tilde{b}_L to avoid conflicts with electroweak observables like the ρ parameter, and generating a light *stop* due to large $\tilde{t}_L - \tilde{t}_R$ mixing weakens the strength of the EWPT.

One could then introduce a small phase in the left-right *stop* mixing term (A_t), which would provide the CP violation required to generate a baryon asymmetry [9] (models that have a large phase in the diagonal trilinear term A_t have the possibility of generating too large a nucleon EDM due to an RGE induced phase in A_u [10]). Possible aesthetic objections to this scenario are the presence of an extremely small or negative supersymmetry breaking mass squared for \tilde{t}_R , and the introduction of a new small CP violating phase.

In this paper we suggest a model that generates a baryon asymmetry by a generalization of the mechanism first proposed in [9], which does not suffer from the two unnatural features mentioned above. The limits on neutral meson mixing constrain the mixing between the first two squark generations of either charge, but do not limit the mixing between the first and third or second and third generation up-type squarks [11]. Thus, there could be large $\tilde{c}_R - \tilde{t}_R$ or $\tilde{u}_R - \tilde{t}_R$ mixing in the soft susy breaking terms with universal diagonal terms [12]. In such a scenario one of the mass eigenstates is light, while still having a large Yukawa coupling (proportional to the top quark mass) to the Higgs boson, thus enhancing the electroweak phase transition as in [7]. In addition, due to this texture of the (RR) up squark mass matrix, the supersymmetry breaking (LR) piece of the squark mass matrix (A term) is in general no longer proportional to the quark mass matrix, while the supersymmetry conserving part (μ term) is [13, 14]. As a result of this non-proportionality between the A and μ terms the complex parameters already present in the quark mass matrix give rise to a physical phase in the space and time dependent squark mass matrix at the electroweak phase transition, which can generate a baryon number asymmetry of the correct magnitude.

The outline of our paper is as follows: in Sec. II we present the model. Sec. III which is broken up into 4 sub-sections discusses the details of the baryogenesis. Sec. IV studies the implications for low energy flavor physics and Sec. V concludes.

2 The Model

Consider the MSSM [15] with the most general soft supersymmetry breaking terms [14]

$$\mathcal{L}_{MSSM} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft} \quad (1)$$

where

$$\mathcal{L}_{SUSY} = \int d^4\theta \Phi^\dagger e^{2gV} \Phi + \int d^2\theta W(\Phi) + h.c. \quad (2)$$

and

$$\mathcal{L}_{soft} = \int d^4\theta \Phi^\dagger [\bar{\eta}\Gamma^* + \eta\Gamma - \bar{\eta}\eta Z] e^{2gV} \Phi - \int d^4\theta \Phi^T \frac{\Lambda}{2} \Phi + \int d^2\theta W'(\Phi) + h.c. \quad (3)$$

In the above equations Φ is a column vector of chiral superfields, V are the vector superfields, and $W(\Phi)$ is the superpotential containing bilinear and trilinear terms M_{ij} and Y_{ijk} . In Eq. (3), η is the spurion whose $vev = m_0\theta^2$ breaks supersymmetry, Γ , Z and Λ are matrices of supersymmetry breaking parameters, and $W'(\Phi)$ is a holomorphic function of the superfields with bilinear couplings M'_{ij} and trilinear couplings Y'_{ijk} . This leads in component notation to

$$-\mathcal{L}_{soft} = m_{ij}^2 \phi_i^* \phi_j + \frac{1}{6} A_{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} B_{ij} \phi_i \phi_j + h.c. \quad (4)$$

where ϕ_i are the scalar fields and

$$\begin{aligned} A_{ijk} &= m_0 [Y'_{ijk} + Y_{ljk} \Gamma_{li} + Y_{ilk} \Gamma_{lj} + Y_{ijl} \Gamma_{lk}] \\ B_{ij} &= m_0 [M'_{ij} + M_{lj} \Gamma_{li} + M_{il} \Gamma_{lj} + m_0 \Lambda_{ij}], \\ m_{ij}^2 &= m_0^2 [Z_{ij} + \Gamma_{il}^* \Gamma_{lj}], \end{aligned} \quad (5)$$

and we have ignored the gaugino mass terms.

Since the new effects we consider are a result of the non-diagonal texture of the right-handed up-type squark mass matrix squared ($M_{\tilde{u}RR}^2$) and the corresponding form of the trilinear supersymmetry breaking term ($A_{\tilde{u}LR}$) we concentrate primarily on these. Consider

$$M_{\tilde{u}RR}^2 = m_0^2 A_{U_{RR}} \quad (6)$$

and

$$A_{\tilde{u}LR} = m_0 \lambda_U A_{U_{LR}} \quad (7)$$

where from Eq. (5) $A_{U_{RR}} = Z_{U_{RR}} + \Gamma_{U_{RR}}^\dagger \Gamma_{U_{RR}}$, λ_U is the matrix of Yukawa couplings for the up-type quarks, and $A_{U_{LR}} = \lambda_U^{-1} Y'_U + \Gamma_{U_{RR}}$. With the rest of the squark mass matrices diagonal and universal, the CP violating invariant responsible for baryogenesis in this model is given by

$$J_{CP} = Im Tr [A_{U_{LR}}^\dagger \lambda_U^\dagger \lambda_U A_{U_{RR}}] \quad (8)$$

Since $\lambda_U^\dagger \lambda_U$ and $A_{U_{RR}}$ in Eq. (8) are Hermitian, a necessary condition for baryogenesis is that $A_{U_{LR}}$ not commute with either of these matrices.

In the constrained MSSM, $A_{U_{LR}}$ is proportional to the unit matrix at the supersymmetry breaking scale, and is always Hermitian and commutes with $\lambda_U^\dagger \lambda_U$

even after including RGE running to a lower scale. The scenario of supersymmetric baryogenesis with diagonal squark masses [9] requires two small parameters beyond those of the constrained MSSM: a small supersymmetry breaking mass for \tilde{t}_R [$A_{U_{RR}}(3, 3)$], and a complex phase for the trilinear coupling A_t [$A_{U_{LR}}(3, 3)$].

Our approach is to notice that given the plausible assumption that at least part of the CP violation seen in $K - \bar{K}$ mixing is due to Standard Model box diagrams, $\lambda_U^\dagger \lambda_U$ in Eq. (8) already contains large phases and to try to use these for baryogenesis without introducing CP violation into the supersymmetry breaking sector. All we need are non-commuting $A_{U_{LR}}$ and $A_{U_{RR}}$. This is possible, for example, in models of horizontal symmetries due to the non-holomorphic terms discussed in [12, 13].

We now study a specific realization of the scenario discussed above, making a series of assumptions in order to gain predictive power and relate the CP violating phase responsible for baryogenesis to the one observed in low energy meson decays. We first assume that CP violation originates in the supersymmetry conserving part of the lagrangian, \mathcal{L}_{SUSY} . In the exact supersymmetric limit this corresponds to just one physical phase in the mixing matrix for 3 families of quarks. We will limit ourselves to this one phase, and make the following ansatz for the up and down type Yukawa coupling matrices:

$$\lambda_U = V^\dagger \hat{\lambda}_U V; \quad \lambda_D = \hat{\lambda}_D \quad (9)$$

where V is the CKM matrix, and $\hat{\lambda}_U$ and $\hat{\lambda}_D$ are diagonal matrices. Consistent with our assumption of no CP violation in the supersymmetry breaking sector, \mathcal{L}_{soft} , we set $W'(\Phi) = 0^1$, and insist that the parameters Z , Γ , and Λ in Eq. (3) are real. The relative phases between \mathcal{L}_{SUSY} and \mathcal{L}_{soft} will be responsible for baryogenesis. While for the most general case there are a large number of these relative phases that are physical, our assumption makes them all proportional to the one fundamental phase in the quark mixing matrix.

Notice, from Eqs. (5) that it is the $\Gamma_{U_{RR}}$ part of $M_{\tilde{u}_{RR}}^2 = m_0^2(Z_{U_{RR}} + \Gamma_{U_{RR}}^\dagger \Gamma_{U_{RR}})$ that is responsible for $A_{\tilde{u}_{LR}}$ not being proportional to λ_U . Thus, in order to reduce the number of arbitrary parameters we further set $Z_{U_{RR}} = 0$. Using the ansatz

$$\Gamma_{U_{RR}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & x \\ 0 & y & 1 \end{pmatrix} \quad (10)$$

¹In supergravity models with hidden sector supersymmetry breaking one has $W'(\Phi) = aW(\Phi)$. Thus a clear separation of the origin of CP violation demands $W'(\Phi) = 0$.

with $x, y \simeq 1$ in order to have large $\tilde{c}_R - \tilde{t}_R$ mixing ² leads by Eq. (5) to

$$M_{\tilde{u}_{RR}}^2 = m_0^2 A_{U_{RR}} = m_0^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1+x^2 & x+y \\ 0 & x+y & 1+y^2 \end{pmatrix} \quad (11)$$

and

$$A_{\tilde{u}_{LR}} = m_0 \lambda_U A_{U_{LR}} = m_0 \lambda_U \Gamma_{U_{RR}} \quad (12)$$

We let the rest of the scalar mass squared matrices be universal, and proportional to m_0^2 as in the constrained MSSM. For the rest of the parameters that define the softly broken MSSM we allow generic values

$$m_0 \simeq |\mu| \simeq M_3 \simeq M_2 \simeq 100 - 300 \text{ GeV}; \quad \tan \beta \simeq 1 \quad (13)$$

Eqs. (9 - 13) along with universality for the other scalar mass matrices define the model we will be studying.

We emphasize that we have made specific assumptions about soft supersymmetry breaking and CP violation that allows us to relate $A_{\tilde{u}_{LR}}$ to λ_U and $M_{\tilde{u}_{RR}}^2$ (W' , $Z_{U_{RR}} = 0$ and $\Gamma_{U_{RR}}$ is real). It would be interesting if such a model of supersymmetry breaking exists since, as we will show, it allows us to explain both the CP violation seen in K mesons and that required for baryogenesis in terms of one complex parameter in the MSSM super potential. In order to do so we introduced two additional parameters beyond those of the constrained MSSM: the squark mixing parameters x and y which could find their origin in models with horizontal symmetries. Even if nature were such that some of the assumptions about supersymmetry breaking and CP violation we have made did not hold, the rest of this paper serves as a useful study of the possibility of electroweak baryogenesis for the general case of non-universal and non-proportional supersymmetry breaking soft terms. In particular, all of our results would go through essentially unchanged if some of the parameters that we set to zero actually turn out to be non-vanishing but small.

3 Baryogenesis

First we will consider the strength of the EWPT and argue that a light up-type squark produced due to $\tilde{c}_R - \tilde{t}_R$ mixing serves to enhance the strength of the

²For the rest of the paper we will quote formulae relevant to the case of $\tilde{c}_R - \tilde{t}_R$ mixing. The physics for the case of $\tilde{u}_R - \tilde{t}_R$ mixing is similar, and can be trivially generalized by moving the parameters x and y from the (2,3) and (3,2) elements to the (1,3) and (3,1) elements of $\Gamma_{U_{RR}}$.

phase transition, possibly making it a first order one, and allowing any baryon asymmetry produced during the phase transition to persist to lower temperatures.

We then calculate the baryon asymmetry generated at the EWPT. We will not explicitly state the approximations made in this approach, and refer the reader to [9] for a discussion of the validity of the assumptions and approximations made. The procedure is quite simple: first we calculate a CP violating source term for axial baryon ($\tilde{c}_R + \tilde{t}_R$) number induced by the passage of the bubble wall through the plasma. Then we solve a set of coupled Boltzmann equations that include the effects of diffusion, rapid particle number changing interactions and the CP violating source. One finds then that there is an excess of $\tilde{c}_R + \tilde{t}_R$ number generated in the symmetric phase. This biases the electroweak sphalerons towards a production of net excess baryon number in the symmetric phase, which is then stored in the broken phase as the bubble sweeps through the plasma (the CP violating source term acts as the charge potential of earlier models [16], and the effects of diffusion are important in moving the $\tilde{c}_R + \tilde{t}_R$ asymmetry out into the symmetric phase where sphalerons efficiently convert this excess into a net excess baryon number [17]).

3.1 The Electroweak Phase Transition

The EWPT in the MSSM has been studied using two approaches: looking at the daisy-improved effective potential [18, 7] and by lattice simulation of a dimensionally reduced effective theory [8] with compatible results. The earlier studies [18] working with the assumption of universal scalar masses concluded that the situation in the MSSM was not much better than that in the Standard Model where a strong enough first order phase transition required an experimentally excluded Higgs boson mass.

Recently, however it has been shown [7, 8] that if $\tan \beta$ is small, and the right-handed stop, \tilde{t}_R is light due to a sufficiently small supersymmetry breaking mass, the order of the EWPT can be significantly enhanced over that in the Standard Model. The reason for this is quite simple. Scalars with vanishing or small Higgs independent mass contribute a cubic term to the scalar potential proportional to their Higgs dependent mass, resulting in a finite temperature Higgs potential of the form

$$V(h, T) = -\frac{M^2(T)}{2}h^2 - \delta T h^3 + \frac{\lambda(T)}{4}h^4 \quad (14)$$

with

$$\delta = \frac{1}{12\pi} \sum_i g_i^3 \quad (15)$$

where the sum is over all boson degrees of freedom with zero Higgs independent mass, and Higgs dependent mass defined by $m_i(h) = g_i h$. The critical temperature for the phase transition, T_0 is defined as the temperature where

$$\left. \frac{\partial^2 V}{\partial h^2} \right|_{h=0} = 0 \Rightarrow M^2(T) = 0. \quad (16)$$

This leads to a phase transition temperature $T_0 \sim 100$ GeV in both the Standard Model, as well as the MSSM. Minimizing the potential at the critical temperature,

$$\left. \frac{\partial V}{\partial h} \right|_{T=T_0} = 0 \Rightarrow -M^2(T_0) - 3\delta T_0 h + \lambda(T_0) h^2 = 0 \quad (17)$$

leads to the equation

$$\frac{h(T_0)}{T_0} = \frac{3\delta}{\lambda(T_0)} \quad (18)$$

for the field strength of the Higgs field h at the non-zero solution of Eq. (17). Thus we see that it is the presence of the cubic term δ which forces the existence of two degenerate minima at the critical temperature, and hence a first order phase transition.

Since the \tilde{t}_R couples to the Higgs boson with the large top quark Yukawa coupling, a light \tilde{t}_R can significantly enhance the strength of the electroweak phase transition. The quantity $h(T_0)/T_0$ is a measure of the strength of the phase transition. The requirement on the strength of the phase transition in order for the baryon asymmetry not to be erased by electroweak sphalerons after the phase transition is [19]

$$\frac{h(T_0)}{T_0} \geq 1 \quad (19)$$

To get a numerical estimate of the right-hand side of Eq. (18) one can use Eq. (15) for δ with a vanishing supersymmetry breaking mass for \tilde{t}_R , and approximate $\lambda(T_0)$ by $\lambda(0) = m_h^2/2v^2$ where m_h is the physical Higgs boson mass, and $v = 246$ GeV is the zero temperature vacuum expectation value (vev). This leads to the following estimate for the strength of the EWPT in the Standard Model and the MSSM for a Higgs boson mass of 75 GeV

$$\left(\frac{h(T_0)}{T_0} \right)_{SM} \simeq 0.5; \quad \left(\frac{h(T_0)}{T_0} \right)_{MSSM} \simeq 3. \quad (20)$$

Precisely this effect occurs if the lightest right-handed up-type squark is an admixture of \tilde{c}_R and \tilde{t}_R . The large mixing generates a light eigenstate with small Higgs independent mass without introducing any unnaturally small diagonal mass

parameters. In addition, this lightest eigenstate will still have couplings to the Higgs boson proportional to the top quark Yukawa (modified by a mixing angle of order 1), hence introducing a large cubic term in the Higgs potential, and ensuring a first order phase transition. We have confirmed these arguments for the model we propose and find a strongly first order phase transition over large regions of parameter space. For example using a Higgs mass of 75 GeV, $m_0 = 200$ GeV and $x = -1$, $y = 0$ in Eq. (6) leads to

$$\left(\frac{h(T_0)}{T_0}\right) \simeq 1.5 \quad (21)$$

using the approximations mentioned above and with appropriate modifications to Eqs. (14 - 18) in order to account for the non-zero supersymmetry breaking *stop* mass.

We should mention two effects we have neglected in the arguments of the previous two paragraphs that both tend to decrease the strength of the phase transition. The first is thermal masses for the squarks and gauge bosons which act like non-zero soft susy breaking masses for the squarks and tend to screen the cubic term. This effect has been included in both of [7]. The second is including $L - R$ mixing terms between the squarks which generally reduces the field dependent mass of the lightest eigenvalue, hence decreasing the magnitude of the cubic term. This effect was studied in the first of [7] where they find a large enough enhancement of the strength of the phase transition only if $|m_0 A_U + \mu/\tan\beta| \leq m_0$. Given this condition, both find a sufficiently first order phase transition for $\tan\beta \simeq 1$ and a *stop* mass ≤ 175 GeV. Although we have not studied the inclusion of these effects, we will adopt these results and insist that μ is negative since in our model both A_U and $\tan\beta \simeq 1$, and use as our criterion for a phase transition satisfying Eq. (19) the condition $m_{\tilde{u}_1} \leq 175$ GeV, where $m_{\tilde{u}_1}$ is the mass of the lightest up-type squark.

A final concern in models with small supersymmetry breaking squark masses is the possibility of a negative mass squared for one of the mass eigenstates resulting in a color and charge breaking vacuum. In order to avoid this possibility we insist that the mass squared for the lightest up-type squark be positive in the symmetric phase.

There has also been recent work suggesting that 2-loop QCD corrections associated with *stops* substantially enhances the strength of the phase transition [20]. Such an effect would imply that our constraints are conservative.

3.2 The CP Violating Source Term

In this sub-section we reproduce the salient points of the analysis in [9], which was limited to looking at the third generation of squarks only, generalized to account for $\tilde{c}_R - \tilde{t}_R$ mixing and non-proportional A terms.

We define the CP violating source term for axial squark number (actually quark + squark) as

$$\gamma_{\tilde{Q}} = \frac{v_w}{\Delta} [J_+(\vec{x}, t) + J_-(\vec{x}, t)]^0 \quad (22)$$

where J_{\pm}^{μ} are CP violating charge currents (in the plasma frame) generated in a thickness Δ that the wall sweeps through with velocity v_w . If we make an expansion for the currents in the wall velocity v_w , and if we approximate the phase space distribution for the squarks to be given by that in the symmetric phase, we get

$$\gamma_{\tilde{Q}}(z, t) = \gamma_w v_w N_c \frac{T^4}{4\pi^2} \times \text{ImTr}[\mathcal{J}_{\tilde{Q}_L} \mathcal{I}_L + \mathcal{J}_{\tilde{Q}_R} \mathcal{I}_R] \quad (23)$$

the quantities $\mathcal{J}_{\tilde{Q}_L} \mathcal{I}_L$ and $\mathcal{J}_{\tilde{Q}_R} \mathcal{I}_R$ can be related by a power series expansion to J_{CP} defined in Eq. (8) (with a trivial generalization to account for the left-handed squarks). The coordinate $z = 0$ defines the edge of the symmetric phase, and $z = w$ defines the edge of the broken phase, *i.e.* w is the thickness of the wall. $\mathcal{I}_{L,R}$ are 3×3 diagonal matrices with entries that can be fit by the formula

$$[\mathcal{I}_{L,R}]_{ii} = \frac{1}{50} \frac{T}{m_i} \frac{e^{m_i/T}}{(1 - e^{m_i/T})^2} \quad (24)$$

where m_i are the eigenvalues of $M_{\tilde{u}_{LL,RR}}^2$ in the symmetric phase. Further,

$$\mathcal{J}_{\tilde{Q}_L} = -\frac{1}{zT^5} U_L M_{\tilde{u}_{LR}}^2(z) M_{\tilde{u}_{LR}}^2(0)^\dagger U_L^\dagger; \quad \mathcal{J}_{\tilde{Q}_R} = \frac{1}{zT^5} U_R M_{\tilde{u}_{LR}}^2(z)^\dagger M_{\tilde{u}_{LR}}^2(0) U_R^\dagger \quad (25)$$

where

$$M_{\tilde{u}_{LR}}^2(z) = m_0 \lambda_U \Gamma_{U_{RR}} v_2(z) + \mu \lambda_U v_1(z) \quad (26)$$

and $U_{L,R}$ are the unitary matrices that diagonalize $M_{\tilde{u}_{LL,RR}}^2$. Ignoring the (LL) sector since it does not contribute to the CP violating source term in our model, we have

$$U_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \quad (27)$$

with \tilde{c} and s parametrizing the sine and cosine of the $\tilde{c}_R - \tilde{t}_R$ mixing angle. Making an expansion in derivatives of the mass matrix (the thick wall limit),

Eq. (26) for $M_{\tilde{u}_{LR}}^2$, gives us the following expression for the imaginary parts of the diagonal elements in $\mathcal{J}_{\tilde{Q}_R}$:

$$\begin{aligned} \text{Im}[\mathcal{J}_{\tilde{Q}_R}]_{ii} &= \frac{4m_0\mu M_W^2(z, T)\partial_z\beta}{g^2T^5} \text{Im}[U_R\Gamma_{U_{RR}}^\dagger \lambda_U^\dagger \lambda_U U_R^\dagger]_{ii} \\ &\simeq \frac{4m_0\mu M_W^2(z, T)\Delta\beta}{wg^2T^5} \text{Im}[U_R\Gamma_{U_{RR}}^\dagger V^\dagger \hat{\lambda}_U^2 V U_R^\dagger]_{ii} \end{aligned} \quad (28)$$

where we have explicitly removed all the dependence on the Higgs vevs to the prefactor, so $\hat{\lambda}_U$ is the diagonal matrix of up-type quark Yukawa couplings obtained from Eq. (9), and we have replaced $\partial_z\beta$ by $\Delta\beta/w$ where w is the wall thickness, and $\Delta\beta$ the variation of the Higgs vevs over the wall.

For ³

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3\sigma \\ -\lambda & (1 - \lambda^2/2) & A\lambda^2 e^{i\gamma} \\ A\lambda^3(1 - \sigma e^{-i\gamma}) & -A\lambda^2 & e^{i\gamma} \end{pmatrix} \quad (29)$$

We get

$$\text{Im}[\mathcal{J}_{\tilde{Q}}]_{ii} = \frac{4m_0\mu M_W^2(z, T)\Delta\beta}{wg^2T^5} \begin{pmatrix} 0 \\ A\lambda^2\lambda_t^2 \sin(\gamma)(xc^2 - ys^2) \\ -A\lambda^2\lambda_t^2 \sin(\gamma)(yc^2 - xs^2) \end{pmatrix} \quad (30)$$

where λ_t is the top quark Yukawa coupling, x and y are the off-diagonal entries in the matrix $\Gamma_{U_{RR}}$, and $\sin\gamma \simeq 1$ is the CP violating phase in the quark mass matrix (a similar result would hold in the case of $\tilde{u}_R - \tilde{t}_R$ mixing, with the CP violating source terms being proportional to $A\lambda^3$ instead of $A\lambda^2$). Eq. (30) is the reason for making the particular ansatz for supersymmetry breaking in Eq. (10) since it allows us to derive the baryon asymmetry from the same phase γ that is in the quark mass matrix, and that is (indirectly) measured in the Kaon decay experiments. Notice that in our ansatz the quantities x and y in $M_{\tilde{u}_{LR}}^2$ are related to c and s , the mixing angles in the Unitary matrix that diagonalizes $M_{\tilde{u}_{RR}}^2$ and for example, $x = y \Rightarrow c = s$ and the CP violating source term vanishes [this would correspond to $A_{U_{LR}}$ being proportional to $A_{U_{RR}}$ in Eq. (8)]. In a more general case, the parameters x and y present in $A_{U_{LR}}$ do not have to be related to c and s which come from diagonalizing $A_{U_{RR}}$. For example if the *stop* is light

³Notice that this differs from the Wolfenstein parametrization [21] by the trivial replacement $\sigma = \sqrt{\rho^2 + \eta^2}$ and by a rephasing of the third column. This then specifies the flavor basis in which CP violation is defined as originating in the supersymmetry conserving lagrangian. All low energy Standard Model calculations would be invariant under the change from this basis to any other, however in order for the full supersymmetric result to remain invariant, one would have to rephase also the squark mass matrices appropriately.

because of a small diagonal soft mass, and not due to mixing, we would have $c = 1$, $s = 0$.

3.3 The Rate Equations

Once again we follow the technology explicitly given in [9] adapting their results of no family mixing to allow for $\tilde{c}_R - \tilde{t}_R$ mixing.

The basic rate equation near equilibrium is given by

$$\dot{n}_i = -\Delta_i \frac{6\Gamma_{ij}}{T^3} \Delta_j \frac{n_j}{k_j} \quad (31)$$

where n_i is a particle density, Γ are the reactions that change n_i by Δ_i and n_j are the particle densities that participate in the reaction, changing an amount Δ_j in the process and k is a statistical factor. Assuming super equilibrium *i.e.* at least some of the gauginos and higgsinos are light enough to equilibriate quarks and squarks the particle densities of interest (those that participate in fast interactions due to the large top quark Yukawa coupling) are:

$$\begin{aligned} Q &= (t_L + b_L + c_L + s_L) + (\tilde{t}_L + \tilde{b}_L + \tilde{c}_L + \tilde{s}_L) \\ T &= (t_R + c_R) + (\tilde{t}_R + \tilde{c}_R) \\ H &= h + \tilde{h} \end{aligned} \quad (32)$$

It has been shown that strong sphaleron effects are important [22], so one has to consider the other quark species too. However, since strong sphalerons are the only interactions they feel, we can use the fact that strong sphalerons are rapid to constrain them.

$$(u_L + d_L) = -2u_R = -2d_R = -2s_R = -2b_R \equiv -2B = (Q + T) \quad (33)$$

in addition all the squarks of the above species are degenerate in our model, so we have

$$(k_{u_L} + k_{d_L}) = 2k_{u_R} = 2k_{d_R} = 2k_{s_R} = 2k_B \quad (34)$$

Assuming all the squarks are heavy except the one light $\tilde{c}_R - \tilde{t}_R$ admixture gives us the following statistical factors

$$k_Q = 12; \quad k_T = 12; \quad k_B = 3; \quad k_H = 12. \quad (35)$$

Now we set up the rate equations for the particle species Q , T , and H , including the effects of diffusion, the CP violating source term, and the “fast” interactions due to the top quark Yukawa, the top quark mass, the Higgs self interaction, and the strong sphalerons (Γ_m , Γ_y , Γ_h , and Γ_{ss} where we redefine $6\Gamma/T^3 \rightarrow \Gamma$ for convenience).

$$\begin{aligned}
\dot{Q} &= D_q \nabla^2 Q - \Gamma_y \left[\frac{Q}{k_Q} - \frac{H}{k_H} - \frac{T}{k_T} \right] - \Gamma_m \left[\frac{Q}{k_Q} - \frac{T}{k_T} \right] \\
&\quad - 6\Gamma_{ss} \left[4\frac{Q}{k_Q} - 2\frac{T}{k_T} + 3\frac{Q+T}{k_B} \right] + \gamma_{\bar{Q}} \\
\dot{T} &= D_q \nabla^2 T - \Gamma_y \left[-\frac{Q}{k_Q} + \frac{H}{k_H} + \frac{T}{k_T} \right] - \Gamma_m \left[-\frac{Q}{k_Q} + \frac{T}{k_T} \right] \\
&\quad + 3\Gamma_{ss} \left[4\frac{Q}{k_Q} - 2\frac{T}{k_T} + 3\frac{Q+T}{k_B} \right] - \gamma_{\bar{Q}} \\
\dot{H} &= D_h \nabla^2 H - \Gamma_y \left[-\frac{Q}{k_Q} + \frac{H}{k_H} + \frac{T}{k_T} \right] - \Gamma_H \frac{H}{k_H}
\end{aligned} \tag{36}$$

Further, since the rates Γ_y and Γ_{ss} are independent of the Higgs vev, these will always be fast, and we approximate the combination of particle species feeling these interactions to be given by their equilibrium values *i.e.*

$$\begin{aligned}
\frac{Q}{k_Q} - \frac{H}{k_H} - \frac{T}{k_T} &= 0 \\
4\frac{Q}{k_Q} - 2\frac{T}{k_T} + 3\frac{Q+T}{k_B} &= 0
\end{aligned} \tag{37}$$

which gives

$$\begin{aligned}
Q &= H \frac{k_Q}{k_H} \frac{3k_T - 2k_B}{2k_B + 3k_Q + 3k_T} \\
T &= -H \frac{k_T}{k_H} \frac{3k_Q + 4k_B}{2k_B + 3k_Q + 3k_T}
\end{aligned} \tag{38}$$

Plugging these equations into the rate equations, we can solve for the Higgs number H in the symmetric phase under the approximations detailed in [9] to get

$$H = A_H e^{v_w z / \bar{D}} \tag{39}$$

where v_w is the wall velocity and

$$A_H = \frac{\bar{\gamma} w}{\sqrt{\Gamma \bar{D}}} \tag{40}$$

and

$$\bar{D} = \frac{D_Q(3k_Q k_T + 2k_Q k_B + 8k_T k_B) + D_H k_H (2k_B + 3k_Q + 3k_T)}{3k_Q k_T + 2k_Q k_B + 8k_T k_B + k_H (2k_B + 3k_Q + 3k_T)} \tag{41}$$

$$\bar{\Gamma} = (\Gamma_m + \Gamma_h) \frac{3k_Q + 3k_T + 2k_B}{3k_Q k_T + 2k_Q k_B + 8k_T k_B + k_H(2k_B + 3k_Q + 3k_T)} \quad (42)$$

$$\bar{\gamma} = \gamma_{\bar{Q}} \frac{k_H(2k_B + 3k_Q + 3k_T)}{3k_Q k_T + 2k_Q k_B + 8k_T k_B + k_H(2k_B + 3k_Q + 3k_T)} \quad (43)$$

with [9]

$$\begin{aligned} D_Q &\simeq \frac{6}{T} & D_H &\simeq \frac{110}{T} \\ \Gamma_m &\simeq \frac{4M_W^2 \lambda_t^2 \sin^2 \beta}{21g^2 T} & \Gamma_h &\simeq \frac{M_W^2}{35g^2 T} \end{aligned} \quad (44)$$

Using Eqs. (33) and Eqs. (38) we can solve for the left-handed quark density

$$n_L = Q + (u_L + d_L) = H \frac{3k_Q k_T - 4k_Q k_B - 4k_T k_B}{k_H(2k_B + 3k_Q + 3k_T)} = \frac{2}{13} H \quad (45)$$

Finally we turn on the 'slow' weak sphaleron rate Γ_{ws} and use the well known fact that an excess of left-handed baryons acts as a chemical potential that biases the baryon number violating weak sphalerons to produce a net baryon asymmetry. If we assume that weak sphalerons are active in the symmetric phase, and shut off abruptly at the bubble wall, then the solution of the rate equation for baryon number gives

$$\rho_B = -\frac{3\Gamma_{ws}}{v_w} \int_{-\infty}^0 n_L(z) dz = -\frac{6}{13} \frac{A_H \bar{D}}{v_w^2} \Gamma_{ws} \quad (46)$$

where we have used Eq. (39) to get the final result. Finally, using $\lambda_t = 1$, $\Gamma_{ws} = \alpha_w^4 T$, $|V_{cb}| = 0.04$, $\tan \beta = 1$, and $s = 55T^3$ we get the numerical result

$$\frac{\rho_B}{s} = 4 \times 10^{-9} \frac{T}{m} \frac{e^{m/T}}{(1 - e^{m/T})^2} \frac{m_0 \mu \gamma_w}{T^2 v_w} \Delta\beta [\sin \gamma (xs^2 - yc^2)] \quad (47)$$

where m is the mass of the lightest right-handed up-type squark in the symmetric phase, and the rest of the parameters have been defined before. T , γ_w , v_w and $\Delta\beta$ are parameters relating to the EWPT and cannot be measured. The rest of the parameters in Eq. (47) will presumably be measured in terrestrial experiments. T is pretty robust around 100 GeV. Estimates for the wall velocity are $v_w \simeq 0.1 - 0.3$. $\Delta\beta$ is harder to estimate: in general we would expect some suppression depending on the ratio of physical Higgs boson masses and we naively estimate $\Delta\beta = (m_h/m_A)^2$, where m_A is the mass of the pseudoscalar Higgs boson.

3.4 Results

In Fig. 1 we plot the regions in the $x - y$ plane that generate sufficient baryon asymmetry for $\tilde{c}_R - \tilde{t}_R$ or $\tilde{u}_R - \tilde{t}_R$ mixing for two different values of m_0 . We use $T = 100$ GeV, $\Delta\beta = 0.25$, $v_w = 0.1$, and set the CP violating phase $\gamma = \pi/4$. For the allowed region we require $\rho_B/s > 1 \times 10^{-11}$ and $85 \text{ GeV} < m_{\tilde{u}_1} < 175$ GeV, where \tilde{u}_1 is the lightest up-type squark.

Notice that although the magnitude of the CP violating source term is suppressed by a factor of the Cabibbo angle for $\tilde{u}_R - \tilde{t}_R$ mixing, the allowed region is similar in size for the two cases. This tells us that the dominant constraint is coming from the requirement on the strength of the EWPT, and that the CP violation is sufficient in both cases. Another feature is the symmetry under $\tilde{c}_R \leftrightarrow \tilde{u}_R$, $(x, y) \leftrightarrow -(x, y)$. This is because in Eq. (47) the dependence on $\sin \gamma$ changes sign as one goes from $\tilde{c}_R - \tilde{t}_R$ mixing to $\tilde{u}_R - \tilde{t}_R$ mixing.

It is clear from Fig. 1 that raising m_0 significantly decreases the allowed region. This is due to a combination of raising the mass of the lightest squark above the range where the EWPT is strongly first order and the Boltzman suppression in Eq. (47). Along these lines, if the gauginos and higgsinos are much heavier than the phase transition temperature, $T_0 \sim 100$ GeV, we do not expect supersymmetry to play a significant role at the EWPT. This leads us to put an upper bound on m_0 and on the gaugino and higgsino masses of ~ 300 GeV, above which we do not expect any scenario of supersymmetric baryogenesis to be viable.

The Higgs bosons affect this calculation in two ways: through the limit placed on the lightest Higgs boson mass by the requirement of a first order phase transition, a Higgs boson mass larger than ~ 100 GeV would make a first order phase transition less plausible, and by the dependence of the baryon asymmetry on $\Delta\beta$, the variation of the Higgs vevs over the bubble wall. We have estimated $\Delta\beta \sim (m_h/m_A)^2$ and used $\Delta\beta = 0.25$ which results from $m_h = 75$ GeV and $m_A = 150$ GeV. If a more accurate calculation shows a stronger dependence of $\Delta\beta$ on the ratio of Higgs masses [23], the net result would be a reduction in the baryon asymmetry.

Another possible source of suppression is the weak sphaleron rate Γ_{ws} . We have used $\Gamma_{ws} = \alpha_w^4 T^4$ [24]. If, as suggested in some recent work [25], $\Gamma_{ws} \propto \alpha_w^5 T^4$, this would significantly reduce the possibility of electroweak baryogenesis in most models.

Suppressing the baryon production by a factor of 10 through any of the possible mechanisms discussed above results in a smaller but still substantial allowed region for the cases we have presented, except for the case of $\tilde{u}_R - \tilde{t}_R$ mixing with $m_0 = 250$ GeV, where no allowed region remains (this is due to the additional Cabibbo suppression in the CP violation).

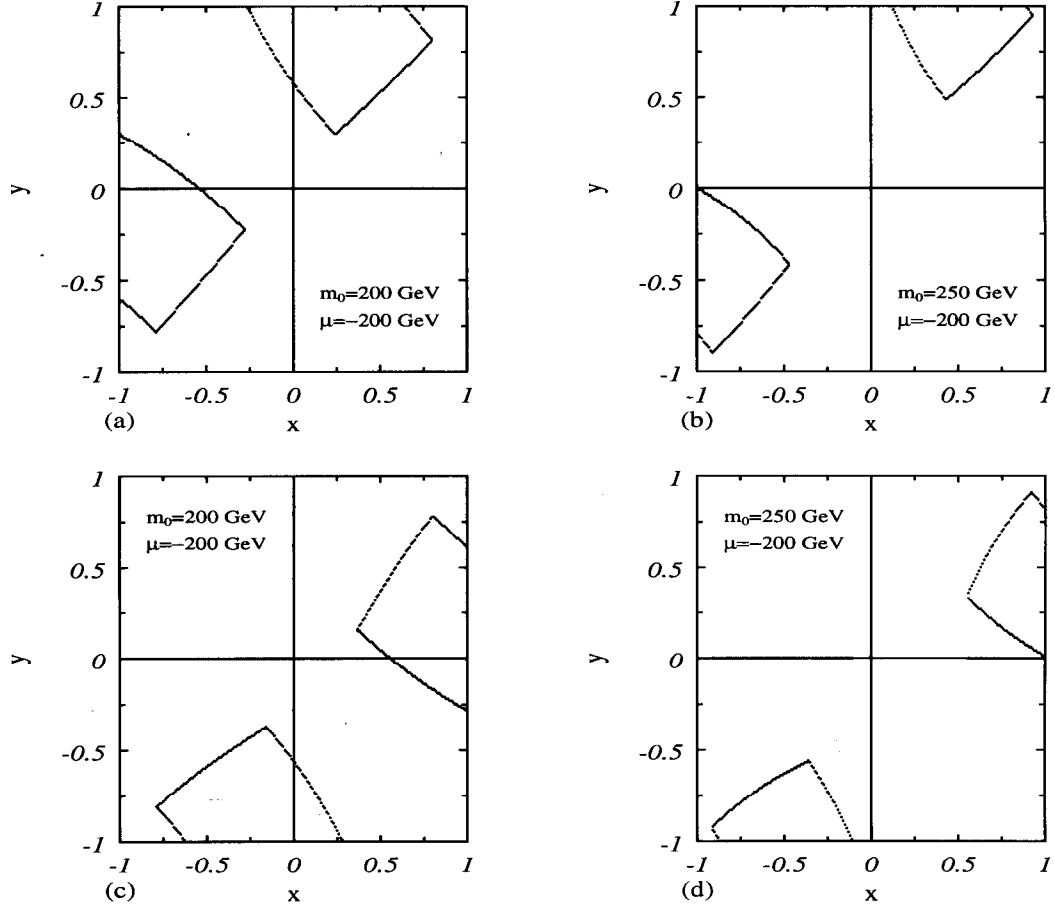


Figure 1: Allowed regions for the mixing parameters x and y that generate a sufficient baryon asymmetry. Figs. (a) and (b) are for $\tilde{c}_R - \tilde{t}_R$ mixing while Figs. (c) and (d) are for $\tilde{u}_R - \tilde{t}_R$ mixing. The values of m_0 and μ are given in each plot, with the rest of the MSSM parameters as in the text.

4 Flavor Physics

The largest effect in low energy flavor physics is a large contribution to $D - \bar{D}$ mixing from gluino box diagrams. This results in an enhancement of the mixing by many orders of magnitude over the Standard Model prediction. This is due to a combination of two factors. The first is the large supersymmetry breaking $\tilde{c}_R - \tilde{t}_R$ or $\tilde{u}_R - \tilde{t}_R$ mixing. The second is the fact that the Cabibbo angle is generated in the up-type quark mass matrix [Eq. (9)] coupled to the non-degeneracy of the squark masses [12]. It is large over most of the allowed regions in Fig. 1, and in fact the experimental upper bound excludes a significant amount of the parameter space. Using $\Delta(m_D) < 1.3 \times 10^{-13}$ GeV, $f_D B_D^2 = 160$ MeV, $m_D = 1.86$ GeV and a gluino mass $m_{\tilde{g}} = 150$ GeV, we plot in Fig. 3 the allowed regions from Fig. 1 but with the added constraint coming from $\Delta(m_D)$.

Notice that a significant allowed region only remains for one case: $\tilde{c}_R - \tilde{t}_R$ mixing and $m_0 = 200$ GeV. Once again, suppressing the baryon production by a factor of 10 reduces this region slightly, but still leaves a substantial allowed area. In the allowed regions, $\Delta(m_D)$ ranges from $1 \times 10^{-14} - 1 \times 10^{-13}$ GeV, thus it is always within one order of magnitude of the current upper bound, and consequently should either be observed or rule the model out soon.

Another robust prediction of this model is a large EDM for the up quark from one-loop gluino exchange diagrams and for the down quark from higgsino exchange. Approximating the neutron EDM by the larger of the of the up or down quark EDM's, we find that it ranges from 1×10^{-27} e-cm to 1×10^{-28} e-cm. once again always within a couple of orders of magnitude of the experimental upper bound $d_n < 1 \times 10^{-25}$ e-cm.

The rate for the decay $b \rightarrow s\gamma$ is usually enhanced over that in the Standard Model, and improvements in the experimental error, or Standard Model theoretical uncertainty would put significant constraints on the model.

The effects on neutral K and B meson mixing are less pronounced, with the dominant contribution coming from chargino boxes, with the result that one requires a smaller CP violating angle γ and $|V_{td}|$ to account for the observed CP violation in neutral K decays and the magnitude of $B_d - \bar{B}_d$ mixing respectively, changing the shape of the Unitarity triangle. A sub leading effect is that the B-physics experiments no longer precisely measure the angles of the Unitarity triangle. However, this effect is pronounced only in very specific regions of the parameter space, and even then hard to observe experimentally [26]. Moreover penguin effects that could help detect new physics beyond the Standard Model [27] are negligible in this model.

If we fix the rest of the CKM matrix parameters (which don't vary much anyway) to $\lambda = 0.22$, $A = 0.78$, $\sigma = 0.023$, and fit for γ using the inputs

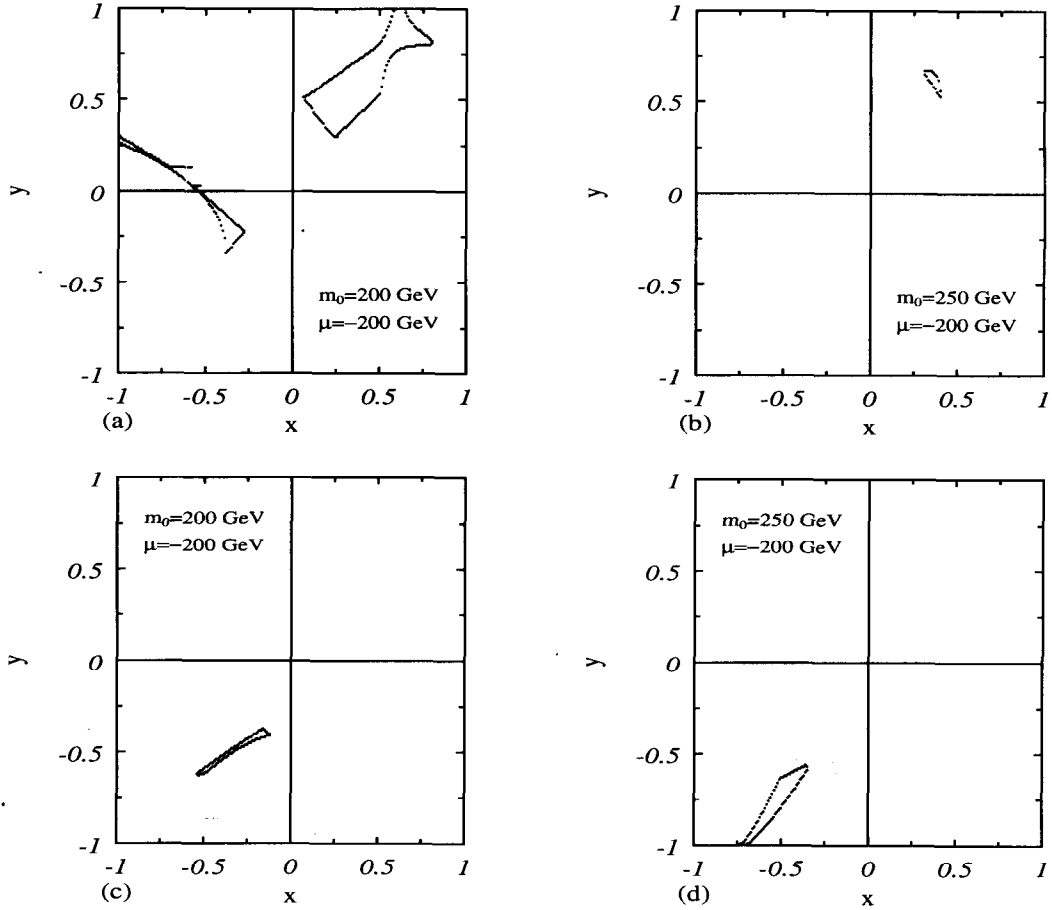


Figure 2: Regions for the mixing parameters x and y that generate a sufficient baryon asymmetry and are consistent with the experimental bound on $D - \bar{D}$ mixing. Figs. (a) and (b) are for $\tilde{c}_R - \tilde{t}_R$ mixing while Figs. (c) and (d) are for $\tilde{u}_R - \tilde{t}_R$ mixing. The values of m_0 and μ are given in each plot, with the rest of the MSSM parameters as in the text.

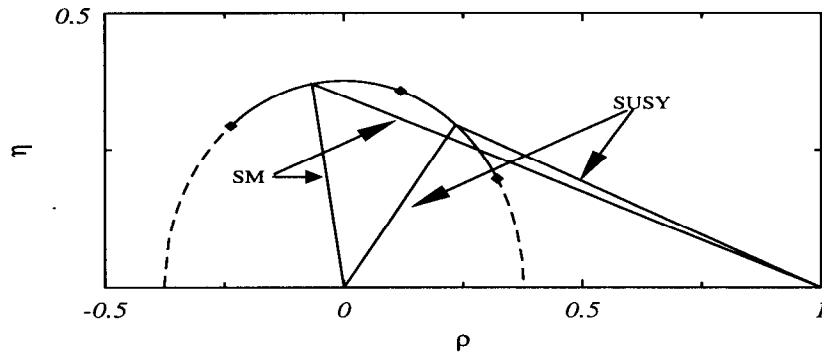
given in Table 1 of [26], we get $\gamma \simeq \pi/2$ for the Standard Model, and $\gamma \simeq \pi/4$ in our model over all of the allowed $x - y$ space. Unfortunately, given the current hadronic uncertainties, the errors on these values for γ are so large that a measurement of γ would not distinguish these two scenarios. In addition, because of a conspiracy between these preferred values for γ and $|V_{ub}|$, the benchmark CP asymmetry measurement $B_d \rightarrow \Psi K_S$ will not be able to distinguish these two scenarios either since the angle β turns out to be similar in both cases. We show this in Fig.3(a) where we plot the semi circle determined by $|V_{ub}|$ (on which the apex of the triangle is constrained to lie), and the Unitarity triangle corresponding to the central value for γ in the Standard Model, and for a typical value of γ in our model. The solid parts of the semi circle correspond to the 1σ allowed values for γ in both models. An improvement in calculation of hadronic matrix elements would render the situation a little less bleak, and we plot in Fig. 3(b) the same information as in Fig. 3(a) assuming the central values for experimental quantities don't shift, but that the hadronic uncertainties are reduced. This corresponds to Table 2 of [26]. In this case a direct measurement of γ , or alternatively $\alpha = \beta + \gamma$ could differentiate the models. $\sin 2(\beta + \gamma)$ measured in $B_d \rightarrow \pi\pi$ is clearly negative in the Standard Model and a positive value would indicate new physics as in our model.

5 Conclusions

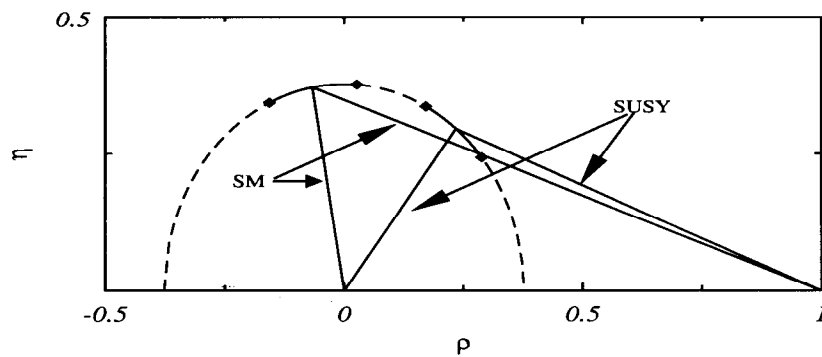
We have studied a particular ansatz for super symmetry breaking with off diagonal soft masses for the up-type squarks such as generically occur in models with Abelian flavor symmetries. The motivation for this is that it results in a first order EWPT, and makes the one physical phase allowed in the MSSM super potential also responsible for baryogenesis. In this scenario, the phase resides in the up-type quark mass matrix and has already been observed in Kaon decay experiments, and will be well measured at the asymmetric B factories. The model predicts $D - \bar{D}$ mixing and a neutron EDM close to the experimental upper bounds. The expectations for CP violating asymmetries in B decays are different in this model than in the Standard Model, and can help distinguish the two with an improvement in hadronic calculations.

Acknowledgements

The author wishes to acknowledge A. Grant's collaboration on parts of this work. In addition he would like to thank Y. Grossman, P. Huet, M. Shaposhnikov and S. Thomas for useful discussions.



(a)



(b)

Figure 3: The Unitarity triangle in the Standard Model and in the supersymmetric extension proposed here. The dashed semi circle is $|V_{ub}|$, and the apex of the triangle is constrained to lie on it. The solid parts of the semi circle delineated by diamonds at the ends display the 1σ allowed values for the angle γ in the Standard Model and the new model. Fig. (a) is with the current hadronic uncertainties. Fig. (b) assumes the theoretical improvements discussed in [28].

References

- [1] A. D. Sakharov, *JETP Lett.* **5**, 24 (1967).
- [2] V. Kuzmin, V. Rubakov and M. Shaposhnikov, *Phys. Lett.* **B155**, 36 (1985).
- [3] G. Farrar and M. Shaposhnikov, *Phys. Rev. Lett.* **70**, 2833 (1993);
Phys. Rev. **D50**, 774 (1994).
- [4] P. Huet and E. Sather, *Phys. Rev.* **D51**, 379 (1995).
- [5] For a review see A. Cohen, D. Kaplan and A. Nelson,
Ann. Rev. Nucl. Part. Sci. **43**, 27 (1993).
- [6] See for example G. Kane *et al.*, *Phys. Rev.* **D49**, 6173 (1994).
- [7] M. Carena, M. Quiros and C. Wagner, *Phys. Lett.* **B380**, 81 (1996); D.
Delepine *et al.*, *Phys. Lett.* **B386**, 183 (1996).
- [8] J. Cline and K. Kainulainen, hep-ph/9605235; M. Laine, *Nucl. Phys.* **B481**,
43 (1996).
- [9] P. Huet and A. Nelson, *Phys. Rev.* **D53**, 4578 (1996).
- [10] R. Garisto and J. Wells, SLAC-PUB7286, hep-ph/9609511.
- [11] F. Gabbiani *et al.*, *Nucl. Phys.* **B477**, 321 (1996).
- [12] M. Leurer, Y. Nir and N. Seiberg, *Nucl. Phys.* **B420**, 468 (1994).
- [13] Y. Nir and R. Rattazzi, *Phys. Lett.* **B382**, 363 (1996).
- [14] S. Dimopoulos and A. Pomerol, *Nucl. Phys.* **B453**, 83 (1995).
- [15] By Minimal Supersymmetric Standard Model we mean the supersymmetric
extension of the standard model with minimal particle content. The con-
strained MSSM includes the assumptions of coupling constant unification,
universal scalar and gaugino masses and universal trilinear terms at a high
“unification” scale.
- [16] A. Cohen, D. Kaplan and A. Nelson, *Phys. Lett.* **B263**, 86 (1991).
- [17] A. Cohen, D. Kaplan and A. Nelson, *Phys. Lett.* **B336**, 41 (1994).
- [18] S. Myint, *Phys. Lett.* **B287**, 325 (1992); J. Espinosa, M. Quiros and F.
Zwirner, *Phys. Lett.* **B307**, 106 (1993).

- [19] M. Shaposhnikov, *JETP Lett.* **44**, 364 (1986).
- [20] J. Espinosa, *Nucl. Phys.* **B475**, 273 (1996).
- [21] L. Wolfenstein, *Phys. Rev. Lett.* **51**, 1945 (1983).
- [22] G. Giudice and M. Shaposhnikov, *Phys. Lett.* **B326**, 118 (1994).
- [23] M. Carena *et al.*, CERN-TH/96-242; hep-ph/9702409.
- [24] J. Ambjorn and A. Krasnitz, *Phys. Lett.* **B362**, 97 (1995).
- [25] P. Huet and D. Son, UW-PT-96-20; hep-ph/9610259; P. Arnold, D. Son and L. Yaffe, UW-PT-96-19; hep-ph/9609481.
- [26] M. Worah, *Phys. Rev.* **D54**, 2198 (1996).
- [27] Y. Grossman and M. Worah, SLAC-PUB-7351; hep-ph/9612269, to be published in *Phys. Lett. B*.
- [28] G. Buchalla, A. Buras and M. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996).