SLAC-PUB-7415
February 1997

ON THE DISTINCTION BETWEEN CLASSICAL AND QUANTUM PROBABILITIES * †

H. Pierre Noyes
Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309

Abstract

We provide an alternative to Etter's link theory using our standard bit-string notation and relate his connection between classical and quantum probabilities to S-matrix quantum scattering theory.

Invited paper presented at the 13^{th} annual meeting
of the WESTERN CHAPTER of the
ALTERNATIVE NATURAL PHILOSOPHY ASSOCIATION
Cordura Hall, Stanford University, Stanford, California
February 15-17, 1997

---

*Work supported by Department of Energy contract DE–AC03–76SF00515.

†A slightly revised and extended version of the text which appeared in *Proc. ANPA West* **13**.

1 INTRODUCTION

In a sequence of recent papers, Tom Etter [6, 7, 8, 9, 10] has articulated the connection between classical and quantum probabilities from a point of view that I will call *Etter's link theory*, or *link theory* for short. The technical development is rigorous, but grounded in basic theories that are unfamiliar to most physicists, particularly those actively *using* quantum mechanics in their daily research. In the paper that we will from now on call *Systems*[10] he employs the standard terminology of classical probability theory. In *Merology*[8] he uses the science of parts and wholes. In *Meta-Law*[9] he uses both classical set theory from the foundations of mathematics literature and the data-base table language of contemporary computer science. The same theory is being discussed in all cases, but this is not obvious to the practicing physicist who is unfamiliar with these different scientific or technical languages. In this paper we examine the simplest case of "linking" when only two states are involved using the language of quantum scattering theory, but do not succeed in making a rigorous mapping onto Etter's theory. That problem will be re-examined in a subsequent paper [25, 27].

It turns out that the easiest language in which to attempt our task is *also* unfamiliar to most physicists, although it's starting point can be thought of as the orthonormal and complete basis for a finite Hilbert space, often used in quantum mechanics, defined by

$$< i|j >= \delta_{ij}; \quad \Sigma_{i=1}^{W}|i >< i| = \mathbf{1} \tag{1}$$

The conventional way in which quantum scattering theory is modeled is to use these orthonormal and complete basis "vectors" as basis vectors in a *linear algebra* in which $X|a > +Y|b >$ is a *ray* in Hilbert space for any complex, real values for X, Y. But this immediately frustrates any *simple* attempt to base quantum discreteness directly on finite measurement accuracy. We have discussed elsewhere[20] why we believe that the finite and discrete measurements on which *experimental* physics always rests is a better starting point for modeling the quantum discontinuity than the continuum mathematics of classical physics. Our theory[26] could be characterized as a *neo-operationalist* approach harking back to P.W.Bridgman[4, 5]. Our current program, which grew out of earlier work on the *combinatorial hierarchy* [2, 3], is based on the standard computer operation of XOR between ordered strings of 0's and 1's. Although this program has, in our view, had considerable success in providing approximate quantitative calculations of the fundamental masses, coupling constants and cosmological constants of particle physics and physical cosmology[21, 22, 23, 24], this unfamiliar approach has stood in the way of acceptance by most of the physics community.

We believe that Etter's link theory may help build the needed bridge between *bit-string physics* and conventional relativistic quantum mechanics. But in order to make our case, we have found it useful to start with the bit-strings and then relate them to Hilbert space rather than the other way around. Our next section will lay out the formalism we need. Although we will not have time to discuss why in this paper, this new work has allowed us to identify where the presentation of *bit-string geometry* at ANPA WEST 11 turned awry [19]. We then apply this formalism to describe the part of Etter's more general link theory we believe relevant to S-matrix scattering theory and exhibit the two-body one-channel scattering problem as an example. Our final section makes a cursory connection between the formalism and its physical interpretation.

2 BIT-STRING COORDINATES AND COMPONENTS

2.1 Bit-Strings

Given positive definite integers m, W and their product mW, we define a *bit-string* $\mathbf{a}(a; mW)$ with *length* mW and *Hamming measure* or *norm* a (defined below, cf. Eq.2) by its mW ordered elements $a_w \in 0, 1$. Here $0, 1$ are also *integers*, and we require that $m \geq 1$ and $w \in 1, 2, ...mW \geq 2$. This allows us to define the *complement* to $\mathbf{a}$, which is $\mathbf{a}$ with 0's and 1's interchanged and which we call $\bar{\mathbf{a}}(mW - a; mW)$, by its elements $\bar{a}_w = (1 - a_w)^2$ and compute the Hamming measure of any string or its complement by

$$a = \Sigma_{w=1}^{mW} a_w; \quad \bar{a} = \Sigma_{w=1}^{mW}(1 - a_w)^2 = mW - 2a + a = mW - a \tag{2}$$

where we have used the fact that $a_w^2 = a_w$. Clearly the norm of any bit-string so defined is a positive integer. We can also define the *null*, or *colorless*, string $\mathbf{0}(0; mW)$ by $0_w = 0$ and the *anti-null*, or *white*, string $\mathbf{I}(mW; mW)$ by $I_w = 1$.

Since we wish to relate this model to quantum mechanics, we use the strings to represent "vectors" in a finite Hilbert space with mW dimensions by defining *bra*'s ($< \,|$) and *ket*'s ($| \,>$) in the Dirac notation with inner and outer products

$$< \mathbf{a}|\mathbf{b} >= \Sigma_{w=1}^{mW} a_w b_w; \quad (|\mathbf{a} >< \mathbf{b}|)_{ww'} = a_w b_{w'} \tag{3}$$

Since we will eventually wish to distinguish "vectors" of integer norm in Hilbert space from "vectors" of integer magnitude in Euclidean and Minkowski space, we will call the entities used to define bras and kets *b-vectors*, and will try to avoid the ambiguous term "vector" altogether. Note that in this notation the Hamming measure is given by $< \mathbf{a}|\mathbf{a} >= a$. If $< \mathbf{a}|\mathbf{b} >= 0$ we say that $\mathbf{a}$ and $\mathbf{b}$ are *orthogonal*. Note also that whatever the values of a, m, W the vectors $\bar{\mathbf{a}}$ and $\mathbf{a}$ are always orthogonal because $< \mathbf{a}|\bar{\mathbf{a}} >= 0$. We also need the diagonal matrix $\mathbf{1}$ with matrix elements $1_{ww'} = \delta_{ww'}$. Here $\delta_{ww'}$ is the Kroeneker delta which is unity when the two indices are the same and otherwise zero.

It is well to realize that if all we know about a string is its Hamming measure a then there are $\frac{(mW)!}{a!(mW-a)!}$ distinct strings with the the same Hamming measure. If we pick any one of them and have some way to distinguish each of the a ordered positions at which $a_w = 1$, then there are still $a!$ similar strings for which $< \mathbf{a}|\mathbf{a} >= a$. For future reference we note that the von Neumann density matrix is proportional to $|\mathbf{a} >< \mathbf{b}|$, and is what he and Etter [9, 10] call a *pure state*.

Two bit-strings $\mathbf{a}, \mathbf{b}$ combine by "$\oplus$", called *discrimination*, to give a third $\mathbf{h_{ab}} = \mathbf{a} \oplus \mathbf{b}$ which is defined by its elements as follows:

$$(\mathbf{h_{ab}})_w = (\mathbf{a} \oplus \mathbf{b})_w = (a_w - b_w)^2 \tag{4}$$

Here this somewhat unconventional way of writing XOR works because $a_w, b_w \in 0, 1$ and we have specified "0" and "1" to be *integers*, so that $a_w^2 = a_w, b_w^2 = b_w$. The *basic bit-string theorem* which relates discrimination to the Dirac inner product for bit-string rays follows immediately:

$$|\mathbf{a} \oplus \mathbf{b}| + 2 < \mathbf{a}|\mathbf{b} >= a + b \tag{5}$$

where we have introduced yet another notation for the Hamming measure, namely $|\mathbf{a}| \equiv < \mathbf{a}|\mathbf{a} > \equiv a$. Note that

$$|\mathbf{a} \oplus \mathbf{b}| = a + b \Leftrightarrow < \mathbf{a}|\mathbf{b} >= 0 \tag{6}$$

Thanks to our definitions, we have that

$$\begin{aligned} \mathbf{0} \oplus \mathbf{0} &= \mathbf{0} \\ \mathbf{0} \oplus \mathbf{I} &= \mathbf{I} \\ \mathbf{I} \oplus \mathbf{0} &= \mathbf{I} \\ \mathbf{I} \oplus \mathbf{I} &= \mathbf{0} \end{aligned} \tag{7}$$

$$\tag{8}$$

demonstrating that "$\oplus$" is isomorphic with XOR, symmetric difference, addition modulo 2, ... Further, given any non-null string **a** which is *not* the anti-null string we have that

$$\begin{aligned} \bar{\mathbf{a}} &= \mathbf{a} \oplus \mathbf{I} \\ \mathbf{a} &= \mathbf{I} \oplus \bar{\mathbf{a}} \\ \mathbf{I} &= \mathbf{a} \oplus \bar{\mathbf{a}} \\ \mathbf{0} &= \mathbf{a} \oplus \bar{\mathbf{a}} \oplus \mathbf{I} = \bar{\mathbf{I}} \end{aligned} \tag{9}$$

Thus given any two of the three strings **a**, $\bar{\mathbf{a}}$, **I** discrimination *creates* the third. However, using *only* discrimination, the system closes forming what John Amson calls a *discrimination system*[1, 2], a concept we will articulate further below.

Define bit-string *concatenation* $\mathbf{a}\|\mathbf{b}$ by

$$\begin{aligned} (\mathbf{a}\|\mathbf{b})_k &\equiv a_k,\ k \in 1, 2, ..., m_a W_a \\ &\equiv b_i,\ i \in 1, 2, ..., m_b W_b, k = m_a W_a + i \end{aligned} \tag{10}$$

Where the lengths of the two strings are $m_a W_a$ and $m_b W_b$ respectively. We emphasize that this operation is obviously *non-commutative.* We can relate concatenation to the ordinary addition of integers by the obvious result

$$|\mathbf{a}||\mathbf{b}||\mathbf{c}||...| = a + b + c + ... \tag{11}$$

It is also obvious that concatenation is equivalent to taking the tensor product of states in the Dirac notation

$$|(\mathbf{a}||\mathbf{b}||\mathbf{c}||...) >= |\mathbf{a} > \bigotimes |\mathbf{b} > \bigotimes |\mathbf{c} > ... \tag{12}$$

2.2 Orthogonal Coordinate Basis

In order to construct *b-vectors* whose Hamming measures are positive integers in an mW-dimensional Hilbert space, we first define an orthogonal set of *basis strings* $\mathbf{w}_i(W; mW)$, $i \in 1, 2, ..., m$ of Hamming measure W and length mW as follows:

$$\mathbf{w}_i(W; mW) = \mathbf{0}([i-1]W)||\mathbf{I}(W)||\mathbf{0}([m-i]W) \tag{13}$$

Then, relative to this basis, we can define the *bit-string coordinates* a_i of any string $\mathbf{a}(W; mW)$ in the space spanned by this basis by

$$a_i \equiv < \mathbf{a}|\mathbf{w}_i > \tag{14}$$

Note that $0 \le a_i \le W$ and that for $m \ge 2$, we can, *independent* of the values of W, m, define an alternative representation of these coordinates $(a_{ij}^-(a_i, a_j), a_{ij}^+(a_i, a_j))$ by

$$\begin{aligned} a_{ij}^- = (a_i - a_j) &= < \mathbf{a}|\mathbf{w}_i > - < \mathbf{a}|\mathbf{w}_j >= \frac{1}{2}[|\mathbf{a} \oplus \mathbf{w}_j| - |\mathbf{a} \oplus \mathbf{w}_i|] \\ a_{ij}^+ = a_i + a_j &= < \mathbf{a}|\mathbf{w}_i \oplus \mathbf{w}_j > \end{aligned} \tag{15}$$

We can also define the *bit-string components* $\mathbf{a}_i(a_i; W)$, which in the full Hilbert space of mW dimensions are to be interpreted by the construction

$$\mathbf{a}_i(a_i; mW) = \mathbf{0}([i-1]W)||\mathbf{a}_i(a_i; W)||\mathbf{0}([m-i]W) \tag{16}$$

Then in the full space of mW dimensions, we have that

$$|\mathbf{a}_i(a_i; mW) \oplus \mathbf{a}_j(a_j; mW)| = a_i + a_j = a_{ij}^+; \quad < \mathbf{a}_i(a_i; mW)|\mathbf{a}_j(a_j; mW) >= 0 \tag{17}$$

But the components $\mathbf{a}_i(a_i; W)$ in the contracted space of only W dimensions can still have meaning when they combine by discrimination. In the absence of further information, all we know is that

$$|\mathbf{a}_i(a_i; W) \oplus \mathbf{a}_j(a_j; W)| \in |a_{ij}^-|, |a_{ij}^-| + 2, ..., a_{ij}^+ - 2, a_{ij}^+ \tag{18}$$

Note that we must use $|a_{ij}^-|$ in specifying the lower end of the range because a_{ij}^- is a signed integer which can be negative, whereas a_{ij}^+ is always a positive integer. Our next step is to supply a context in which this contracted space becomes meaningful.

2.3 Discriminate Closure and the Combinatorial Explosion

We say that n bit-strings $\mathbf{a}_i(a_i; W)$ are *discriminately independent* if and only if for all $i, j, k, .. \in 1, 2, 3, ...n$ we have that

$$\begin{aligned} \mathbf{a}_i \oplus \mathbf{a}_j &\neq \mathbf{0} \\ \mathbf{a}_i \oplus \mathbf{a}_j \oplus \mathbf{a}_k &\neq \mathbf{0} \\ &\cdot \\ &\cdot \\ &\cdot \\ \mathbf{a}_1 \oplus \mathbf{a}_2 \oplus ... \oplus \mathbf{a}_n &\neq \mathbf{0} \end{aligned} \tag{19}$$

It follows immediately that, given n discriminately independent bit-strings, one can form from them $2^n - 1$ non-null strings, because this is the number of ways one can choose $1, 2, ..., n$ distinct things taking them $1, 2, ..., n$ at a time. Further, once one has constructed all of these strings, the discrimination between any two of them will *necessarily* produce another one of them. Thus they form a set which *closes* under discrimination. This property of *discriminate closure* for any finite set of bit-strings was discovered by John Amson[1, 2]. If none of the strings $\mathbf{a}_i$ are the null string, we can always construct from any discrimination system a related one which closes with $2n + 1$ strings simply by including $\mathbf{I}$ in the set and requiring that

$$\mathbf{a}_1 \oplus \mathbf{a}_2 \oplus ... \oplus \mathbf{a}_n = \mathbf{I}(W) \tag{20}$$

An immediate consequence of this observation is that of all the $2^W W!$ possible bit-strings of length W where all the positions of the symbols in the string are assumed known, only W of them can be discriminately independent. In particular the orthonormal and complete basis often used in quantum mechanics, defined by

$$< i|j >= \delta_{ij}; \quad \Sigma_{i=1}^{W} |i >< i| = \mathbf{1} \tag{21}$$

also specifies W discriminately independent bit-strings. To see this, we define $(\mathbf{i}(1;W))_w = \delta_{iw}$, which has the requisite orthonormality property, and see that at the same time the property of discriminate independence is satisfied by these W strings.

As already noted, one conventional way in which Hilbert space is used to model quantum mechanics is to use these orthonormal and complete basis "vectors" as basis vectors in a *linear algebra* in which $X|a> +Y|b>$ is a *ray* in Hilbert space for any complex, real values for X, Y. We have chosen instead to introduce discrimination and discriminate closure into our Hilbert space by introducing the operator "$\oplus$". Thanks to the basic bit-string theorem given above (Eqn.7), we see that, if we confine ourselves to b-vectors whose norms are integers and to bras and kets whose inner products are always integers, any abstract relationship which can be written in the Dirac notation can be mapped onto an expression which only requires discrimination to compute all the relevant b-vectors. We claim [22, 23] that bit-strings of length $W = 256$ then give us a system of sufficient richness and numerical flexibility to describe relativistic quantum mechanics without going beyond integer arithmetic, yet capable of modeling both the standard model of quarks and leptons and big-bang cosmology to currently available empirical accuracy. The remainder of this paper will be devoted to demonstrating how some of the first steps in this program can be articulated.

3 BIT-STRING LINK THEORY

3.1 Discrimination as the Basic Interaction

Rather than employ the general link theory[8, 9, 10] we develop what we believe suffices for our discrete physics program[26] in the firm belief that our version can be embedded in Etter's broader context[27]. The reason we are so confident this can be done is that the S-matrix theory operationalist point of view due to Heisenberg and later workers clearly satisfies the requirements of link theory by cleanly separating the classical observables in terms of which the "in" and "out" states are expressed from the unobservable pieces of the remaining quantum mechanical formalism. For methodological reasons such as *Occam's Razor* these pieces of mathematical formalism should be kept as parsimonious as possible. This radical Copenhagen approach, which leaves us measuring only the velocities, momenta and energies of particles which can be treated using classical physics, is quite consistent with actual practice in high energy accelerator laboratories. Further, much of the work showing that bit-string models are particularly convenient for describing the limitations of measurement accuracy for these particulate observables at all currently accessible energies including *ultra-relativistic* ones has already been done [17, 12, 13].

With this understood we can assume from the start that the observables we need for the input and output states can be specified simply as bit-strings whose Hamming measures are known to the nearest integer, and model scattering by the transition from the state represented by bit-string $\mathbf{a}$ to the state represented by bit-string $\mathbf{b}$. Etter[10] links these two states, considered as columns in a table, by keeping only the rows where the entries agree, numbering $< a|b >$. If this is a link in a Markov chain, when the link is cut this leaves one end with all the information and the other only having white noise[7]. This illustrates the asymmetry between past and future of statistically causal theories and allows a standard "case count" probability calculation. In contrast, if this is a link in a quantum chain governed by the unitary transition operator the pure quantum state is completely symmetric between past and future. The cutting of this link leaves information on both sides of the cut, which still has to be discussed. To make a long story short, if an uncut system is linked back on itself, this part of the quantum world is completely cut off from the classical world. If a branch is provided leading to the classical world and information is removed, this information leaves the quantum world and participates in the irreversibility of the classical world by joining the fixed past. In Etter's treatment the probability calculation can then involve "negative case counts" or negative amplitudes, but only the positive quantity $< a|b >^2$ appears in the laboratory. Thus Born's "amplitude squared" rule is recovered. Barring new phenomena which can be tamed for scientific study, the laboratory world remains statistically causal.

Our approach to the problem is different, partly for historical reasons. We believe that *for physics* it will prove to be isomorphic to Etter's link theory, but this work is yet to be done. In the first published paper on the combinatorial hierarchy in which I participated[2], I made heuristic use of the idea that the discrimination operation $\mathbf{a} \oplus \mathbf{b}$ connecting two states to form a third is a vertex in a Feynman diagram. After 18 years, I believe that this idea can now be given the precision requisite for believable calculations. The basic idea is simply that we should form the link between **a** and **b** by counting the cases where the 1's disagree rather than where they agree, that is by using $|\mathbf{a} \oplus \mathbf{b}|$ rather than the $< \mathbf{a}|\mathbf{b} >$ which is the basic ingredient in Etter's link. Clearly one should be able to map one theory onto the other *in this context* because by the basic bit-string theorem (Eqn. 7) $|\mathbf{a} \oplus \mathbf{b}| = a + b - 2 < \mathbf{a}|\mathbf{b} >$.

Our choice of taking the disagreement — i.e. $|\mathbf{a} \oplus \mathbf{b}|$ — rather than the agreement — i.e. $< \mathbf{a}|\mathbf{b} >$ — between strings as the fundamental positive scalar measure for interaction can be made plausible by noting that when the discriminant of two string vanishes they are *identical* and the whole idea of interaction looses meaning. Consequently our proposal banishes from the outset the "self-energy diagrams" which produce one class of infinities in relativistic quantum field theory. It also means that when a vertex opens it cannot close on itself; one of the two entities must interact with a third before any process can take place. This is one way of understanding how Faddeev[11] succeeded in banishing the infinities from the quantum mechanical three body problem which had made such serious problems for Weinberg[29, 30, 31].

3.2 Single Channel, Two Body Scattering

Once we have taken the step of representing interaction by "$\oplus$" and we consider a system with only two strings, the total number of cases to include is the number in which they overlap ($< \mathbf{a}|\mathbf{b} >$) plus the number in which they do not overlap ($|\mathbf{a} \oplus \mathbf{b}|$). The fundamental interpretive postulate of what might be called *bit-string scattering* *theory* is then that the probability of two strings interacting, called p_{ab}, is given by

$$p_{ab} = \frac{|\mathbf{a} \oplus \mathbf{b}|}{< \mathbf{a}|\mathbf{b} > + |\mathbf{a} \oplus \mathbf{b}|} = \frac{|\mathbf{a} \oplus \mathbf{b}|}{N_{ab}} = \frac{a + b - 2 < \mathbf{a}|\mathbf{b} >}{a + b - < \mathbf{a}|\mathbf{b} >} \tag{22}$$

Then $1 - p_{ab}$ is the probability of the pair *not* interacting with each other; the residual $a + b - N_{ab}$ slots not participating in the specific scattering process are still available for interaction with other strings if they occur in the problem. This will, hopefully, become clearer when we work out the three-body problems already implied by our approach[28].

When $\mathbf{a}$ is orthogonal to $\mathbf{b}$ in the Dirac sense we see that it is certain that an interaction will occur. Eventually we will see that this corresponds to a situation in which *all* the possibilities we are considering are available. In conventional continuum quantum mechanics there would be an infinite number of such possibilities; they are the "vacuum fluctuations" allowed by the uncertainty principle, or second order perturbation theory. Our theory has the tremendous advantage here that we need only consider some finite number of explicitly known states, namely those generated by discriminate closure from the discriminately independent strings in terms of which the initial statement of the problem is posed.

At this point it becomes convenient to parameterize our problem *in the two-string context* by defining what will become the quantum mechanical "phase shift" δ_{ab} through its tangent.

$$\begin{aligned} tan\ \delta_{ab} &= \frac{|\mathbf{a} \oplus \mathbf{b}|}{< \mathbf{a}|\mathbf{b} >} = \frac{p_{ab}}{1 - p_{ab}} \\ p_{ab} &= \frac{tan\ \delta_{ab}}{1 +\ tan \delta_{ab}} \end{aligned} \tag{23}$$

which vanishes when the interaction vanishes and approaches infinity when the probability of scattering approaches unity. If p_{ab} is actually equal to unity, the two strings keep on scattering until some interaction not so far included in the problem intervenes. This we call a *two body bound state.* We emphasize that up to this point we have considered only classical probabilities despite the implied quantum mechanical interpretation of our language.

As already noted, the classical situation differs from the quantum situation in that, once the transition from the in to the out state has actually occured, the classical world becomes determinate and joins the fixed past. However, as also noted, the quantum mechanical situation allows both transitions to keep on occurring so long as probability is conserved. This property of *unitarity* is insured in conventional quantum mechanics by relating the transitions to a unitary operator called the *S-matrix* which is invertible, i.e. $\mathbf{S}\mathbf{S}^{-1} = \mathbf{1} = \mathbf{S}^{-1}\mathbf{S}$, and define the inverse by $\mathbf{S}^{\dagger} = \mathbf{S}^{-1}$. Here "$\dagger$" stands for complex conjugate transposed. ‡ In the conventional elementary treatment of the two body, one channel problem, taking $S = 1 + it$ with t a complex scalar, the unitarity condition becomes $i(t - t^*) = -|t|^2$. This condition is automatically satisfied by taking $t = 2ie^{i\delta_{ab}} sin\ \delta_{ab}$, corresponding to $S = e^{2i\delta_{ab}}$. Thus our parameterization tells us immediately how to go from the real, classical parameter t_{ab} to a unitary transition operator. We have already noted that $\frac{tan\ \delta}{1+tan\ \delta}$ is a classical probability. But elementary scattering theory tells us that (see below for more details) the scattering cross section is proportional to $sin^2\delta = \frac{tan^2\delta}{1+tan^2\delta}$. and hence to $|\mathbf{a} \oplus \mathbf{b}|^2$ rather than to $|\mathbf{a} \oplus \mathbf{b}|$. This makes sense from the Feynman diagram point of view because in the quantum case we must first form the intermediate state from the in state and then separate it into the appropriate two pieces of the out state, the probability of each process being proportional to finding $|\mathbf{a} \oplus \mathbf{b}|$ cases in the appropriate context.

3.3 The Relation between Classical and Quantum Probabilities

Returning to our bit-string model, having seen that $sin^2\delta$ is directly proportional to the experimental probability of scattering, we have reached our goal of relating *classical* to *quantum* probabilities using the same parameter. Calling the two p^C and p^Q respectively, we have that:

$$p^C_{ab} = \frac{\tan\ \delta_{ab}}{1 + tan\ \delta_{ab}}$$

‡Unfortunately, in his treatment Etter has used the symbol S for "state" and the symbol T for the (sometimes invertible) operator which relates two states S, S' by what he calls the "Schroedinger Equation" $S' = T^{-1}ST$. In order not to hopelessly confuse physicists I have chosen *not* to use his symbols and will use S and T for the "S-matrix" and "Transition-matrix" with the meanings explained in my text above.

$$p^Q_{ab} = \frac{\tan^2\delta_{ab}}{1+tan^2\delta_{ab}} = sin^2\delta_{ab} \qquad (24)$$

It was when Etter showed me this formula derived directly from link theory that I was sure that we were closing in on a bit-string quantum mechanics and decided to write this paper. On another occasion I will present the simple algebraic proof[22] of this formula starting from Etter's description of how to link two states in the classical and the quantum cases.

4 PHYSICAL INTERPRETATION

4.1 Breaking Scale invariance

In the abstract problem so far discussed, we can see from Eq.23 that the critical parameter $tan\ \delta_{ab}$ is a *ratio* of two integers, and hence remains the same when $a \rightarrow ma$, $b \rightarrow mb$ whatever the unit. However, if we have some way to set the scale parameter m, independent of our formalism, we can achieve an *absolute quantization* of probabilities. One such quantization unit has been known for a long time, namely the quantum of action $\hbar$, which is also the discrete unit for changes in angular momentum. A new proposal for the fundamental *physical* postulate lying at the foundations of *discrete physics* is that *mass is quantized*, the unit of mass Δm being in the same ratio to some physical mass m as that mass is to the Planck mass $M_{Planck} = [\frac{\hbar c}{G_N}]^{\frac{1}{2}}$, where G_N is Newton's gravitational constant. That is

$$\begin{aligned} M_P\Delta m &= m^2 \\ \Delta m &= m^2[\frac{G_N}{\hbar c}]^{\frac{1}{2}} \end{aligned} \qquad (25)$$

The testable hypothesis, which quantizes gravity, is then to identify the mass unit Δm with the finite (and indivisible) mass of the graviton.

4.2 The Handy-Dandy Formula

It is important to realize how easily this formalism can be related to scattering experiments. As any elementary treatment of quantum mechanical scattering shows that for structureless "point" particles there can be no angular momentum transfer. In the coordinate system where the two particles have equal and opposite momentum the scattering is isotropic, the total cross section is 4π times the differential cross section and the differential cross section $\sigma = (\frac{\lambda}{2\pi})^2 sin^2\delta_{ab}$. Here $\lambda = 2\pi\hbar/p$ is the relativistic deBroglie wavelength and p the momentum computed (in units where the velocity of light c=1) from the energy E and the mass m using $p^2 = E^2 - m^2$. We are starting to approach our ideal of relating our theoretical parameters directly to experimental numbers in simple, paradigmatic cases.

The next step is to define a parameter which represents the momentum of either of particles of mass m_a and m_b which, in the coordinate system in which they have equal and opposite momentum and energies e_a and e_b respectively, we call k_{ab}. In units such that $\hbar = 1 = c$ this parameter is given by

$$k_{ab}^2 = (e_a + e_b)^2 - (m_a + m_b)^2 \tag{26}$$

To introduce this dimensional unit into our formalism we define the dimensional scattering amplitude T_{ab} by

$$T_{ab} = \frac{e^{i\delta_{ab}} sin\ \delta_{ab}}{k_{ab}} = \frac{1}{k_{ab} ctn\ \delta_{ab} - ik_{ab}} \tag{27}$$

Then this will diverge when the relative momentum goes to zero unless we take $\delta_{ab} = k_{ab}L_{ab} + O(k^2)$. Here L_{ab} is called the scattering length and we see that the total cross section at low enough energy so that the energy dependence of the scattering amplitude can be neglected is given by

$$\sigma_{ab}^{tot}(k_{ab}^2 \to 0) = 4\pi[\frac{sin^2(k_a b L_{ab})}{k_{ab}^2} + O(k_{ab})]_{k_{ab}=0} = 4\pi L_{ab}^2 \tag{28}$$

That the low (kinetic) energy cross section approaches a constant value is one of the characteristic features of a quantum mechanical system. Our final step in relating our model to directly measurable experimental parameters is to assume that, in addition to scattering, the two particles form a *bound state* with mass $m_{ab} < m_a + m_b$. Then the four *empirical parameters* we need for our model are m_a, m_b, m_{ab}, L_{ab}.

If we only have the four parameters cited, we can use a minimal *zero range theory*[16] in which the scattering amplitude as a function of energy (cf. Eq 27) takes on the value of L_{ab} at elastic scattering threshold, $e_a + e_b = m_a + m_b$ and has a pole when the invariant energy $(e_a + e_b)^2 - k_{ab}^2$ takes on the value of m_{ab}^2, i.e. the invariant energy of the bound state at rest. Since the squared momentum k_{ab}^2 defined by Eq.26 is then negative and equal to $k_0^2(m_a, m_b, m_{ab}) = m_{ab}^2 - (m_a + m_b)^2$, this state cannot be reached starting from a physical scattering state in a process conserving both 3-momentum and energy without a third particle or quantum in the system (called the *spectator* in the Faddeev three-body theory) to carry off the appropriate momentum and the energy $|k_0|$. As we noted in our discussion of Eq.23, the residual bound state can be thought of as a system in which the probability of scattering is unity, $\delta_{ab} = \frac{\pi}{2}$ and the particles can never separate without the intervention of a third particle to supply the requisite energy and momentum.

As we discuss in more detail in our paper on zero range scattering theory[16], in this minimal model the inaccessability of the bound state from the physical scattering region is reflected by the fact that k_0^2 is negative, and corresponds to a "pole in the S-matrix" when that is taken to be an analytic function of k^2 continued to negative values. Following Weinberg[30] we note that the fact that the model requires exactly two particles to form the bound state then specifies the residue at the pole, fixing the relation between the scattering length L_{ab} and the binding energy $-k_0^2 = (m_a + m_b)^2 - m_{ab}^2$. Using the definition of "coupling constant" which comes from the S-matrix version of relativistic quantum mechanics, namely in our notation

$$\begin{aligned} T_{ab} &= \frac{g_{ab}^2 m_{ab}}{s_{ab}(k_{ab}^2) - s_0} - i\epsilon \\ s_{ab} &= k_{ab}^2 + (m_a + m_b)^2 \\ s_0 &= k_0^2 + (m_a + m_b)^2 = m_{ab}^2 \end{aligned} \tag{29}$$

Then we can relate the coupling constant, or "residue at the bound state pole" to the fourth experimental parameter by

$$T_{ab}(k_{ab}^2 = 0) = L_{ab} = \frac{g_{ab}^2 m_{ab}}{(m_a + m_b)^2 - m_{ab}^2} \tag{30}$$

This imposes a constraint on the experimental parameters which Weinberg uses, for example, to estimate how much of the bound state composed of a neutron and a proton (i.e. the deuteron, or nucleus of heavy hydrogen) is composite and how much is "elementary", i.e. arising from other constituents in the system not represented by the degrees of freedom included in this model for an interactin neutron and proton.

We have already noted this connection between masses and coupling constants in our treatment of the fine structure of the spectrum of the hydrogen atom[15] which is first derived there by combinatorial arguments and then related to S-matrix theory in a treatment that is essentially equivalent to that given here, so far as the underlying physics goes. This connection, which we sometimes call the *handy-dandy formula*, is

$$(g_{ab}^2 m_{ab})^2 = (m_a + m_b)^2 - m_{ab}^2 \tag{31}$$

The equivalent equation (to order $(e^2/\hbar c)^2$) was first written down by Bohr in his 1915 relativistic treatment of the hydrogen atom and forms the starting point for Sommerfeld's 1916 successful explanation of the fine structure of the spectrum of hydrogen. Sommerfeld's formula survived the creation of the "new quantum mechanics" and stood unchallenged until the measurement of the Lamb shift after World War II provided much of the impetus for the creation of renormalized quantum electrodynamics. Now that we are assured of its generality, the application of this formula to many problems in elementary particle physics will be vigorously pursued[28]..

References

[1] J.Amson, "A Discrimination System", 31 August 1965 (unpublished) and *International Journal of General Systems*, issue edited by K. Bowden on **General systems and the Emergence of Physical Structure from Information Theory**, (in press).

[2] T.Bastin, H.P.Noyes, J.Amson and C.W.Kilmister, "On the Physical Interpretation and Mathematical Structure of the Combinatorial Hierarchy", *Int'l J.Theor.Phys*, **18**, 445-488 (1979).

[3] T.Bastin and C.W.Kilmister, *Combinatorial Physics*, World Scientific, Singapore, 1995.

[4] P.W.Bridgman, *The Logic of Modern Physics*, MacMillan, New York, 1928.

[5] P.W.Bridgman, *The Nature of Physical Theory*, Princeton University Press, 1936.

[6] T.Etter, "Boolean Geometry", *ANPA West Journal* **6**, No. 1, 15-33 (1966).

[7] T.Etter, "How to Take Apart a Wire", *ANPA West Journal* **6**, No. 2, 16-28 (1966).

[8] T.Etter, "Quantum Mechanics as a Branch of Merology, *Proc. PhysComp96.*

[9] T. Etter, "Process, System, Causality and Quantum Mechanics", *International Journal of General Systems*, issue edited by K. Bowden on **General systems and the Emergence of Physical Structure from Information Theory**, (in press).

[10] T.Etter, "Quantum Mechanics as Meta-Law", submitted to *Physica D.*

[11] L.D.Faddeev, *Mathematical Aspects of the Three Body Problem in the Quantum Scattering Theory*, Daniel Davey and Co., Inc., New York, 1965.

[12] L.H.Kauffman and H.P.Noyes, "Discrete physics and the derivation of electromagnetism from the formalism of quantum mechanics", *Proc. Roy. Soc. Lond. A* **452**, 81-95 (1996).

[13] L.H.Kauffman and H.P.Noyes, "Discrete physics and the Dirac equation", *Physics Letters A*, **218**, 139-146 (1996).

[14] D.O.McGoveran and H.P.Noyes, "Foundation of a Discrete Physics", in *DISCRETE AND COMBINATORIAL PHYSICS; Proc. ANPA 9*, H.P.Noyes, ed., ANPA West, 112 Blackburn Ave., Menlo Park, CA 94025, 1988, pp 37-104, and SLAC-PUB-4526 (June, 1989).

[15] D.O.McGoveran and H.P.Noyes, *Physics Essays*, **4**, 115-120 (1991).

[16] H.P.Noyes, *Phys.Rev* **C 26**, 1858-1877 (1982).

[17] H.P.Noyes, "THE RQM TRIANGLE: A Paradigm for Relativistic Quantum Mechanics", in *Proc. ANPA WEST 8*, F.Young, ed., available from ANPA WEST, 112 Blackburn Ave, Menlo Park, CA 94025 and SLAC-PUB-95-5752 (Feb.1992).

[18] H.P.Noyes, "On the Lorentz Invariance of Bit-String Geometry", in *Proc. ANPA WEST 11*, F.Young, ed., available from ANPA WEST, 112 Blackburn Ave, Menlo Park, CA 94025 and SLAC-PUB-95-6760 (Sept. 1995),

[19] H.P.Noyes, "On the Lorentz Invariance of Bit-String Geometry", in *Proc. ANPA WEST 11*, F.Young, ed., available from ANPA WEST, 112 Blackburn Ave, Menlo Park, CA 94025 and SLAC-PUB-95-6760 (Sept. 1995),

[20] H.P.Noyes, "Operationalism Revisited", *Science Philosophy Interface*, **1**, 54-79 (1996).

[21] H.P.Noyes, "Are Partons Confined Tachyons?", in *Proc. ANPA WEST 12* and SLAC-PUB-7100, March, 1996.

[22] H.P.Noyes, "Some Thoughts on Discrete Physics and the Reconstruction of Quantum Mechanics", in *Logic, Mathematics and the Computer*, C.Gefwert, P.Orponen and J.Seppänen, eds., Finnish Society for Computer Science, Helsinki, 1996, pp 258-268.

[23] H.P.Noyes, "Some Remarks on Discrete Physics as an Ultimate Dynamical Theory", in *Philosophies: Proc. ANPA 17*, K.G.Bowden, ed.; published by ANPA, c/o C.W.Kilmister, Red Tiles Cottage, Barcombe, Lewes, BN8 5DH, United Kingdom, 1996, pp 26-37.

[24] H.P.Noyes, "Decoherence, Determinism and Chaos Revisited" in *Law and Prediction in the Light of Chaos Research*, P.Weingartner and G. Schurz, eds., Springer Lecture Notes in Physics, 1996, pp 152-188.

[25] H.P.Noyes, Lecture prepared for but not presented in Philosophy 242a, Stanford University, fall 1996.

[26] H.P.Noyes, *DISCRETE PHYSICS: A New Fundamental Theory*, J.C. van den Berg, ed. (in preparation).

[27] H.P.Noyes and T.Etter, *Bit-String Link Theory* (in preparation).

[28] H.P.Noyes and E.D.Jones, *Bit-String Scattering Theory* (in preparation).

[29] S.Weinberg, *Phys.Rev.*, **130**, 776 (1963).

[30] S.Weinberg, *Phys.Rev.*, **131**, 440 (1963).

[31] S.Weinberg, *Phys.Rev.*, **133**, B232 (1964).