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RELOCATION

- A Concept for Dealing with Measurement Object Expansion or Contraction -[1]

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The Measurement Object Expansion and Contraction Problem

Over the last decade, industrial metrology has experienced a tremendous improvement in measurement sensors. First, optical tooling was made obsolete by theodolite systems, which in turn are now being displaced by laser trackers. Furthermore, there are already first indications that laser trackers, as far as static measurements are concerned, will have to compete very hard with the next generation of motorized total stations. These technological advances have made it possible to measure even large objects with high resolution. Now, as it turns out, the factor limiting the achievable accuracy is not equipment related but is imposed by atmospheric effects. These effects are refraction, scintillation, and temperature related object expansion and contraction. Due to temperature changes, large objects, like an airplane's fuselage or wing, will significantly expand or contract during the course of a laser tracker or theodolite system measurement. Typical data analysis software does not model these changes and hence, will absorb them in the least squares process. This leads to inaccuracies in the individual target point coordinates and their corresponding standard deviations, as well as to a warping of the coordinate system. This paper will propose a method for dealing with measurement object expansion and contraction effects.

Traditional Mitigation Approach

Traditionally, when it is not possible to avoid the expansion/contraction problem by keeping the ambient temperature stable, the mitigation approach calls for reducing the model size and limiting the measurement time per station. This scheme subdivides the total measurement project into smaller slices, with only acceptable temperature excursions within each slice. These slices need to overlap in order to generate identical points for the subsequent coordinate transformation. While this approach reduces the temperature related noise in each slice, it creates a significant burden for the data analysis. First, each slice has to be analyzed individually, only then can all the slices be transformed with a conformal projection into a common coordinate system. Not only is this approach very time consuming, it more importantly also weakens the global geometry and results in the loss of a rigorous error propagation.[2]

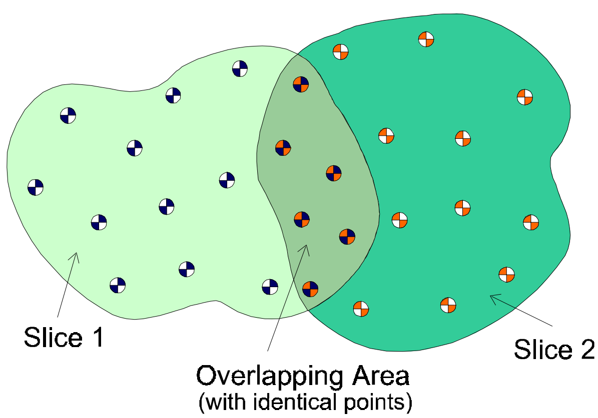

Fig. 1 Breaking measurement object into slices

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[2] None of the commercially available software packages provides the tool (S-Transformation) to also transform the standard deviations of estimated parameters from each slice into the global frame.

The Relocation Approach

With *relocation* we introduce an extension to the mathematical model of the geodetic/bundle adjustment to filter temperature related effects. The filter information is derived from the grouping of points into *lumps* and observations into *epochs*. *Lumps* are clouds of points on the measurement object or in the space surrounding the object which are expected to react to temperature changes in a similar manner. The first lump or *world* space combines all points which are not part of the object and serve as tie points or network reference stations. Epochs are defined as periods of time during which the measurement conditions have not changed beyond a set threshold. The relocation process will first extract all points related to the world space and then sort the remaining points into lump groups. These groups are subsequently further divided by epochs into lump-epoch subgroups. The individual behavior of the lump-epoch observation groups with respect to the world space is expressed as an extension to the traditional mathematical model of the geodetic/bundle adjustment. As a result we obtain coordinates from which the bias caused by object expansion or contraction is, to a large extent, removed.

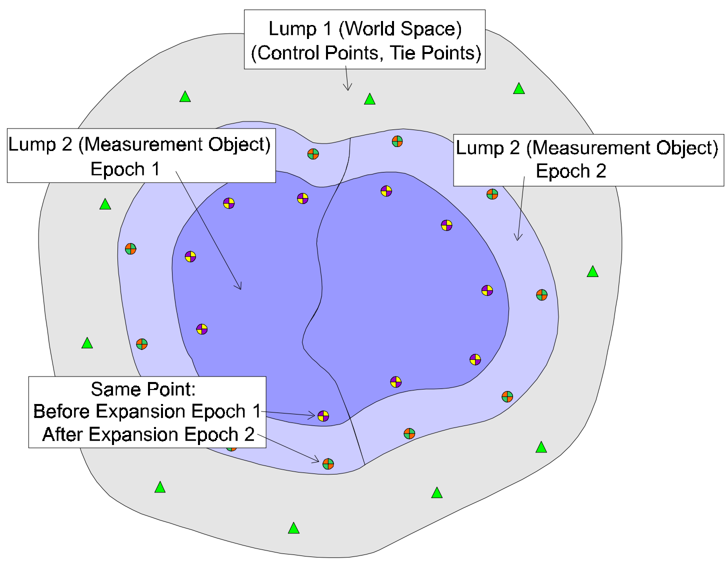

Fig. 2 Relocation Approach, Lumps, and Epochs: An object (lump 2) sits on a stable floor (lump 1). The object has expanded over the course of the measurements. Therefore, the points are grouped into two epochs. The symbols and mark the same point. At the beginning of the measurements (epoch 1), the points were located here , then due to expansion they moved to there . During epoch 1 the points on the left of the epoch demarcation line were measured and subsequently during epoch 2 the points on the right side of the line were measured.

Since all observations are fitted in one block, one obtains a totally integrated least squares estimation of all observations from all stations. This also ensures a rigorous stochastic treatment of uncertainties and errors, allowing a meaningful chi-square test to validate the goodness of the least squares fit. And most importantly, allows the computation of realistic standard deviations for all point coordinates as well as for all observations. Since all the data is treated in one block, this approach also significantly reduces the time required for the data analysis.

The Mathematical Formulation

The expansion/contraction[3] of an object is completely described by the displacement of each point in the object. The study of the displacement of a point may be conducted in any reference system. Given one reference system called the global system, let **X** and **X'** be respectively the position vector of the point before and after expansion. Then the displacement vector δ of the point during expansion is simply:

$$\boldsymbol{\delta} = \mathbf{X}' - \mathbf{X}$$

This relationship δ is a function of the initial position **X** and the time dependent motion. Because we are primarily interested in eliminating thermal effects during a measuring session and not in modeling them, we may simply adopt an instantaneous geometric seven parameter transformation to express the change between **X** and **X'**:

$$\mathbf{X}' = k\ \mathbf{M}\,\mathbf{X} + \mathbf{t}$$

where

k is the instantaneous scale factor ,
$\mathbf{t} = (t_1, t_2, t_3)^t$ is the instantaneous translation vector ,
$\mathbf{M} = (m_{i\,j})$ is the instantaneous rotation matrix corresponding to 3 successive elementary rotations r_3 , r_2 and r_1 :

$$m_{11} = \cos r_2 \cos r_3$$
$$m_{12} = \cos r_2 \sin r_3$$
$$m_{13} = -\sin r_2$$
$$m_{21} = \sin r_1 \sin r_2 \cos r_3 - \cos r_1 \sin r_3$$
$$m_{22} = \sin r_1 \sin r_2 \sin r_3 + \cos r_1 \cos r_3$$
$$m_{23} = \sin r_1 \cos r_2$$
$$m_{31} = \cos r_1 \sin r_2 \cos r_3 + \sin r_1 \sin r_3$$
$$m_{32} = \cos r_1 \sin r_2 \sin r_3 - \sin r_1 \cos r_3$$
$$m_{33} = \cos r_1 \cos r_2$$

All the points in the object (or lump with our notations) have the same set of parameters for a given expansion. These seven parameters may be grouped into a vector **c** representative of our model of expansion:

$$\mathbf{c} = (k, t_1, t_2, t_3, r_1, r_2, r_3)^t$$

In general, the mathematical model for a geodetic observation l between an instrument I and a point P is given by a function $\boldsymbol{f}$. The parameters of this function can be grouped into three categories: the coordinates of the target P ($\mathbf{X}_P$), the coordinates of the instrument I ($\mathbf{X}_I$) and some instrument parameters, specific of the type of observation ($\mathbf{p}_I$):

$$l = \boldsymbol{f}\,(\mathbf{X}_P, \mathbf{X}_I, \mathbf{p}_I)$$

If we neglect expansion effects, all these parameters are time independent. When expansion is considered, the same formalism stands true but everything has to be referred to the actual time of the observation. Since it is fair to assume that the expansion motion encountered in metrology is slow and regular, the observation time span can be divided into periods. referred to as epochs. Although, there may be expansion within the time frame of an epoch, it is assumed to be small enough to be neglected.

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[3] Subsequently in this paper, the motion caused by thermal expansion or contraction is only referred to as expansion. From a mathematical modeling point of view there is no difference between expansion and contraction.

At a given epoch, a general observation has two end points belonging to possibly two different lumps. To model this observation, we must first analyze whether or not each of the two lumps is active, i.e. experiences expansion. An active lump is, by our definition, a lump surveyed during at least two different epochs. The first epoch is defined as the reference epoch of a lump. A series of seven parameter transformations is used to express the change between subsequent epochs and the reference epoch. Observing a lump at different epochs does not necessarily imply that each point on the lump has been observed at each individual epoch. However, because of analytical constraints, each epoch must include at least three points. The described formalism effectively projects points in subsequent epochs back into the reference frame of the first epoch.

Naming A and B the lumps of point P and of instrument I, respectively, an observation l at epoch e may be represented by:

$$l = \boldsymbol{f}\,(\mathbf{X}_P^{\,refA}, \mathbf{X}_I^{\,refB}, \mathbf{p}_I^{\,e}, \mathbf{c}_A^{\,e}, \mathbf{c}_B^{\,e})$$

where

refA is the reference epoch of lump A,

$\mathbf{X}_P^{\,refA}$ is the position vector of point P at epoch refA,

$\mathbf{c}_A^{\,e}$ is the vector of the expansion parameters for lump A, corresponding of the change between epochs e and refA, and allowing the computation of $\mathbf{X}_P^{\,e}$ (position vector of the point P at the time of the observation). If the epochs are identical, then the expansion parameter vector is the null vector and $\mathbf{X}_P^{\,e} = \mathbf{X}_P^{\,refA}$, otherwise

$\mathbf{c}_A^{\,e} = (k_A^{\,e}, t_{A\,1}^{\,e}, t_{A\,2}^{\,e}, t_{A\,3}^{\,e}, r_{A\,1}^{\,e}, r_{A\,2}^{\,e}, r_{A\,3}^{\,e})^t$ and

$\mathbf{X}_P^{\,e} = k_A^{\,e}\, \mathbf{M}_A^{\,e}\, \mathbf{X}_P^{\,refA} + \mathbf{t}_A^{\,e}$,

$\mathbf{p}_I^{\,e}$ is the vector of the instrument parameters at epoch e,

refB, $\mathbf{X}_I^{\,refB}$ and $\mathbf{c}_B^{\,e}$ are the equivalent of refA, $\mathbf{X}_P^{\,refA}$ and $\mathbf{c}_A^{\,e}$ for instrument I on the lump B.

The mathematical formulation is simplified when a survey project lay-out permits set-up of the instrument on lump1 (the by definition stable world space) versus on an active lump.

Field Test

Measurement data was used to test the relocation approach and to validate the algorithms. Since no large measurement object was available which would have produced significant thermal expansion, a test set-up was conceived. The set-up consisted of two aluminum plates, 4' and 2' square, respectively. Both plates were fitted with fiducial points on three rings at 45º radial spacing (see fig. 3). Tooling ball bushings were used to establish the fiducial points. The three points on each radius simulate expansion; the "original" point on the inner position "moves" because of expansion to the more outwards positions. In the context of the above explained formalism, the three points an a radius simulate three epochs. Because of the overall dimensions and the minimum spacing possible between points, the test represented a 1000ºC temperature change for aluminum. While this temperature differential is certainly extreme and exaggerates realistic measurement conditions 20 fold, it will also exaggerate any problem that the relocation algorithm has to deal with. It is therefore believed, that, if the procedure and algorithm can deal with these extreme temperature changes, they will be able to handle every days measurement conditions.

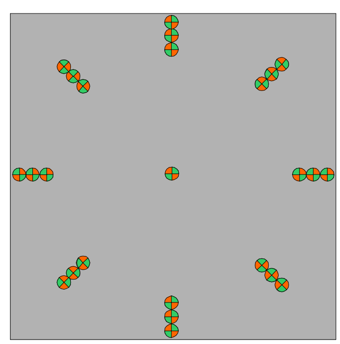

Fig. 3 Point lay-out on plate

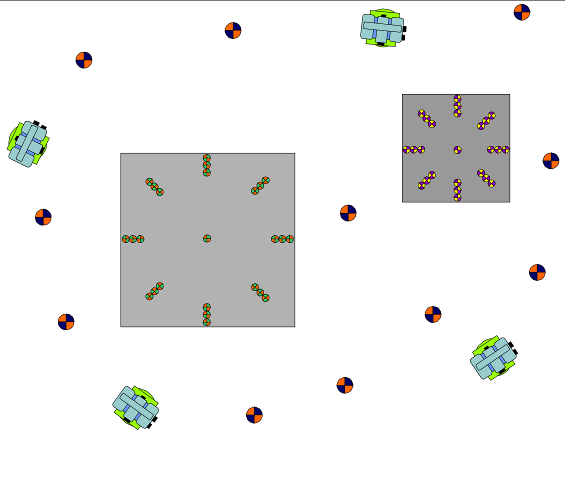

Fig. 4 Measurement Lay-out
Lump 1 World Space
Lump 2 Large Plate
Lump 3 Small Plate

Fig. 5 Large test plate with tracker

The plates were measured with a CMS3000 laser tracker from four stations. To avoid any possible residual systematic effects, the tracker was calibrated before and after the measurements, and the observations were taken in the front and reverse face positions. The distance and angle data from the four stations were combined and analyzed in one relocation enabled Bundle adjustment. The sequence of observations is documented in the following table (see Table 1).

Lump 2 (large plate)		Lump 3 (small plate)	
Station	**Epoch**	**Station**	**Epoch**
1	1	2	1
2	2	3	2
3	3	4	3

Table 1 Measurement sequence

Data Analysis

The measurement data was adjusted in two ways. First, the baseline run did not consider possible motion, i.e. the three points on one radius were given the same point number. Since these points have a significant physical spacing, considerable residuals and standard deviations were expected. Then secondly, the same data set was fitted with a relocation enabled adjustment. The data interpretation followed the traditional sequence. A first adjustment was performed, and the observation residuals were analyzed for possible outliers. After necessary corrections, the adjustment was re-run, and a chi-square test was calculated to check the adjustment's integrity.

The validity of the relocation approach was judged by examining the standard deviations of the coordinates and the residuals of the observations. As pointed out earlier, the statistics were calculated in a rigorous way directly from the inverted normal matrix. Tables 2, 4 and 6 show the standard deviations of the X, Y, and Z coordinates from the first traditional run, and Tables 3, 5, and 7 show the same coordinates from the relocation enabled adjustment. A similar comparison of the residuals of the horizontal angles, vertical angles and distances, respectively, is presented in Tables 8 through 13.

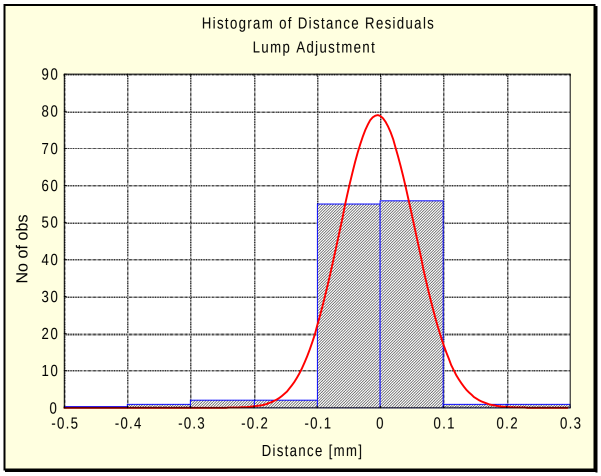

Table 2 Distance residuals from relocation adjustment

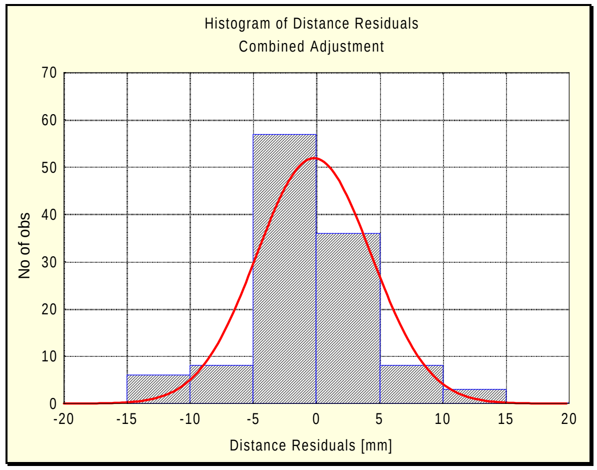

Table 3 Distance residual from traditional adjustment

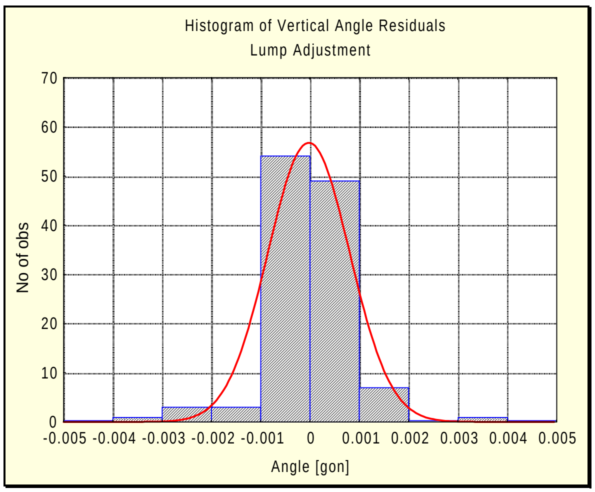

Table 4 Vertical Angle residuals from relocation adjustment

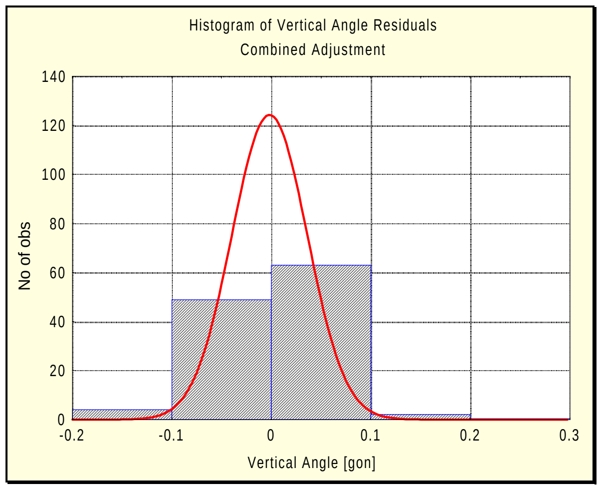

Table 5 Vertical angle residual from traditional adjustment

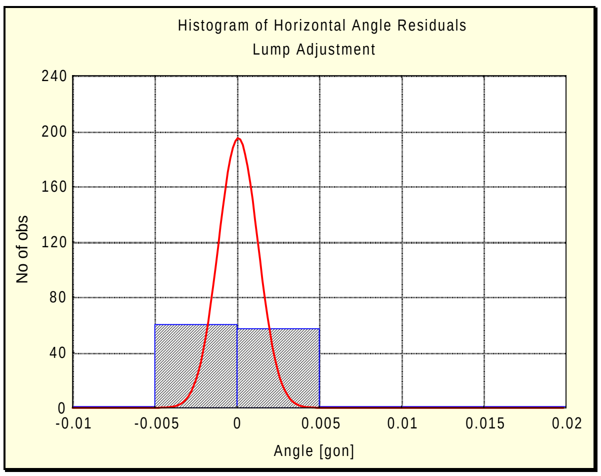

Table 6 Horizontal Angle residuals from relocation adjustment

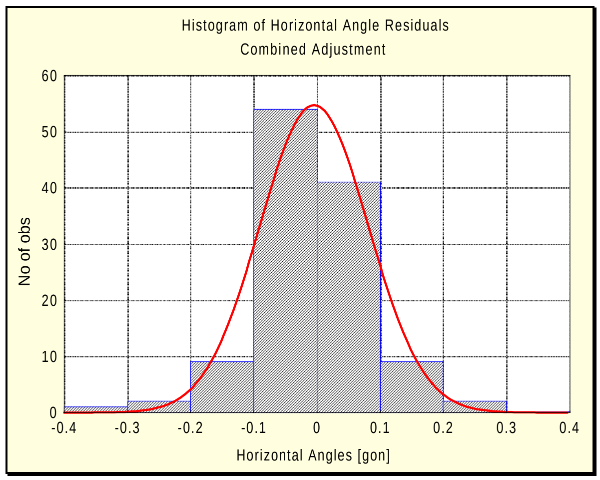

Table 7 Horizontal angle residual from traditional adjustment

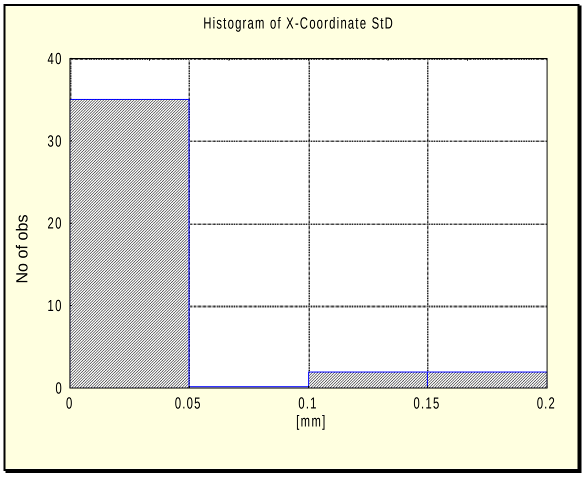

Table 8 X-Coord. StD. From relocation adjustment

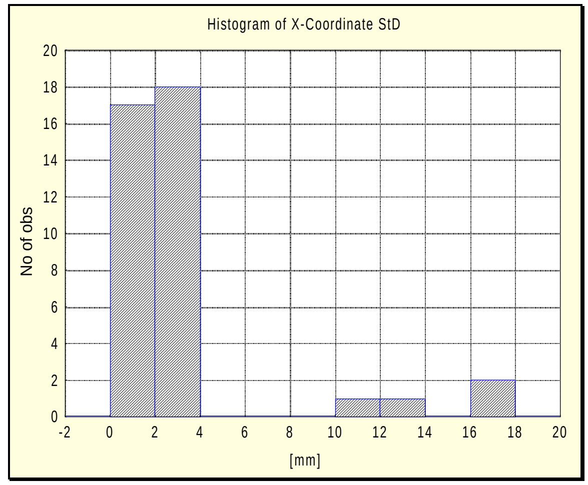

Table 9 X-Coord. StD. From traditional adjustment

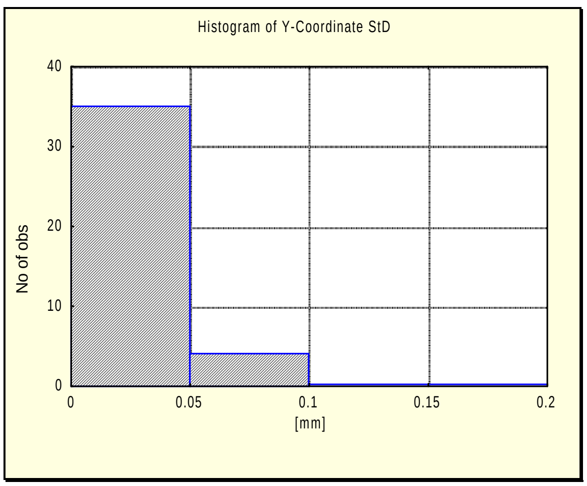

Table 10 Y-Coord. StD. From relocation adjustment

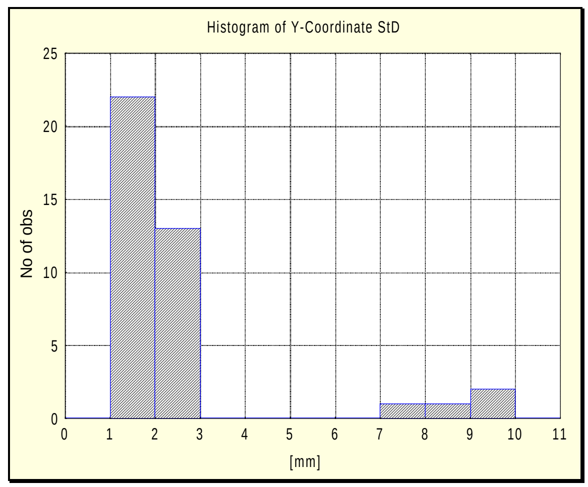

Table 11 Y-Coord. StD. From traditional adjustment

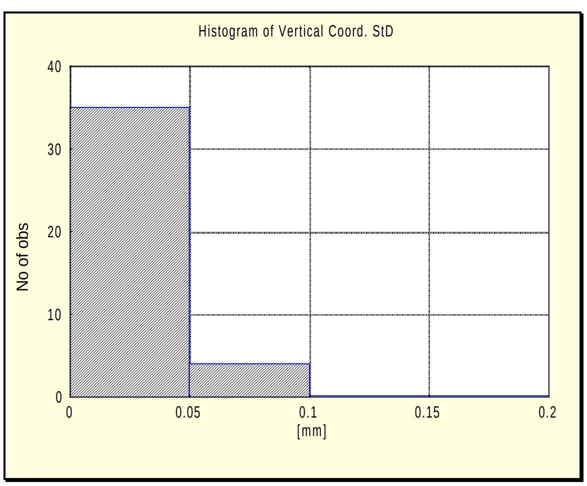

Table 12 Z-Coord. StD. From relocation adjustment

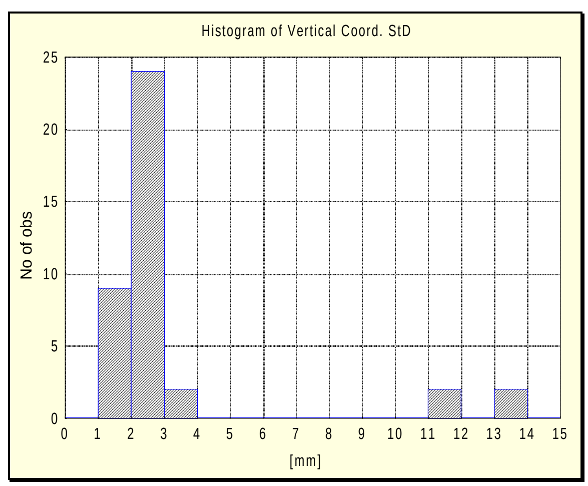

Table 13 Z-Coord. StD. From traditional adjustment

The adjustment of all points without the relocation algorithm shows large residuals and standard deviations. This was to be expected, since the three epoch instances of each point have a clear physical separation and therefore distinct, different coordinates. If these points now are addressed with the same point number, a classical adjustment can only try to minimize the discrepancies. However, this effect mimics only in an exaggerated way what happens when a measurement object expands and the resulting point motion is not considered in the measurement procedure and modeled in the adjustment algorithm.

On the other hand, as the small residuals and excellent standard deviations from the relocation enabled adjustment prove, the relocation approach is obviously able to deal with the exact same data set properly. Although the three epoch instances of each point have still the same point number, the relocation algorithm permits the least squares process to filter out the point motion caused by the expansion. Consequently, the least squares minimization does not become biased with what would otherwise be interpreted point motion, and retains its ability to estimate accurate parameters despite the presence of expansion.

Conclusion and Outlook

From the above comparisons it can be clearly seen that the relocation approach effectively filters thermal expansion or contraction motion from measurement data. This avoids the otherwise inevitable warping of the coordinate system, and as a result, one can obtain more accurate and reliable coordinates. In addition, the relocation process also allows the rigorous calculation of statistical information.

It should also be pointed out, that the same algorithm can be used to successfully measure objects which are not in a fixed position. The object could either translate or rotate slowly and regularly, or move in jumps between epochs.

More work is necessary to streamline the relocation formalism and to automate the rejection of statistical insignificant epoch groupings.

The described algorithm will be implemented in Version 2.0 of the SMX Insight data analysis package.

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