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Parallel Computation of Transverse Wakes in Linear Colliders [1]

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Abstract

SLAC has proposed the detuned structure (DS) as one possible design to control the emittance growth of long bunch trains due to transverse wakefields in the Next Linear Collider (NLC). The DS consists of 206 cells with tapering from cell to cell of the order of few microns to provide Gaussian detuning of the dipole modes. The decoherence of these modes leads to two orders of magnitude reduction in wakefield experienced by the trailing bunch. To model such a large heterogeneous structure realistically is impractical with finite-difference codes using structured grids. We have calculated the wakefield in the DS on a parallel computer with a finite-element code using an unstructured grid. The parallel implementation issues are presented along with simulation results that include contributions from higher dipole bands and wall dissipation.

1. Introduction

In the Next Linear Collider (NLC) proposed by SLAC the detuned structure (DS) is being considered as a viable design to control the emittance growth of long bunch trains due to transverse wakefield effects in the linacs [1]. An off-axis beam transiting through the accelerator structure excites dipole modes which impart transverse kicks on subsequent bunches. This dipole wakefield can be greatly reduced if the modes are decohered by suitably tapering the cell dimensions to provide Gaussian detuning of their frequencies, while keeping the accelerating mode in synchronism with the beam.

At X Band, the NLC DS is a 1.8m long disk-loaded waveguide (DLWG) consisting of 206 cells each of which has the geometry shown in Fig. 1(a). The unit cell is formed between disks whose aperture a and thickness t vary along the structure according to the distributions shown in Fig. 1(b). The tapering profile for a leads to the Gaussian detuning of the 1st dipole band while that for t results in detuning of higher bands (3rd and 6th). The cell radius b allows the accelerating mode to be tuned to 11.424 GHz at 120 degree phase advance across the cell length L.

2. Transverse Wakefield Analysis

The wakefield at distances of the order of the bunch spacing is mainly determined by the resonant modes of the structure which are responsible for the long range effect. Hence,

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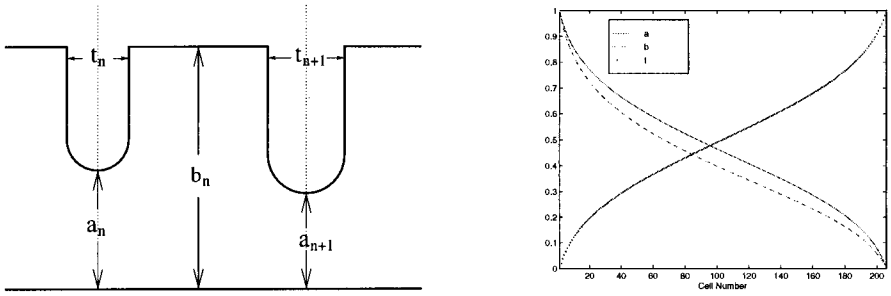


(a) (b)

Figure 1. (a) *The dimensions of a typical cell*; (b) *Variations of cell dimensions.*

the transverse wakefield at distance s behind the driving bunch is best calculated in the frequency domain using

$$W(s) = \frac{2}{206} \sum_{i=1}^{N} K_i \sin(\frac{\omega_i s}{c})$$

where ω_i, K_i are the frequency and the kick factor of mode i respectively. N is number of modes excited by the bunch and here N is large since there are 206 modes per dipole band. The kick factor is determined from the voltage

$$V_i = \int_{z_{min}}^{z_{max}} E_z(r_o, z)\ \exp\left(i\frac{\omega_i}{c}z\right)dz$$

and the stored energy

$$U_i = \frac{1}{2}\epsilon \int |E|^2\ dV + \frac{1}{2\mu}\int |B|^2\ dV$$

via the expression

$$K_i = \frac{|V_i|^2}{4(\omega_i/c)U_i\ r_o^2\ L}$$

To lowest order, the DS can be considered as a closed axisymmetric cavity if the end cell effects due to the power input/output coupler and the entry/exit beampipes are ignored. Although the problem is now reduced to 2D, the challenge remains as to how one can solve for a large number of modes in a long heterogeneous structure efficiently and accurately.

Many theoretical efforts have been devoted to the assessment of the effectiveness of the DS in suppressing wakefields. The calculations to date have all been carried out with approximate methods primarily because of the size of the problem. They include equivalent circuits [2], mode matching [3], and the open-mode approach [4]. Also, a simplified geometry is assumed by treating the ends of the disks as flat than round. Detailed accounts of these analysis can be found in the literature, and the results are generally found to be in qualitative agreement with experimental data.

3. Grid-based Simulation

It is of interest to find the wakefield from a direct simulation of the entire DS on a numerical grid. Such an approach till now has not been possible for the following reasons. Most standard field solvers, based on finite-difference (FD) schemes, use structured grids which are not efficient for modeling gradual tapers [5]. To fit the tapering in a and b alone would require two mesh lines per cell for a total of 512. Given the cell to cell variation in a and b is few microns while the cell radius is about 1 cm, the number of mesh points in the radial direction can be in the thousands. It suggests that if even a moderate resolution is adopted in the axial direction, to model the whole length of the structure would require prohibitively large number of mesh points (estimated to be tens of millions). This is not to mention that structured grids are not suited to model the rounded ends of the disks. One concludes from these considerations that grid-based simulations with FD codes are not feasible.

The finite-element (FE) method has been shown to be highly accurate and efficient in modeling accelerator structures with the use of unstructured grids [6]. The advantage of the unstructured grid is the ability of the mesh to fit small differences in geometry, and to refine locally around curvatures without significant increase in the total number of elements or degrees of freedom (DOF). The number of elements can be further reduced without loss in accuracy through the use of quadratic elements. Even with such improvements, however, the memory and CPU time required for a reasonable FE mesh is still far being practical to compute on any high-end workstation. It becomes apparent that this limitation can only be overcome by way of parallel processing.

4. Domain Decomposition and Parallel Implementation

The DS geometry is highly amenable to domain decomposition for parallel processing because the partitioning is straightforward. Each processing node is assigned a cell bounded by half-disks (subdomain) as shown previously in Fig. 1(a). A total of 207 nodes is used for the DS (domain) since the two ends are terminated in half-cells. In the single program multiple data (SPMD) model, global data such as meshes and matrices are distributed to the nodes where global operations can be processed locally. The communication between nodes is performed through message passing interface (MPI) which, in the present case, applies solely to the data along the radial boundaries that separate a particular cell from its neighbors.

The finite element formulation of the Maxwell's equation in 2D results in a generalized eigenvalue problem, $Kx = \lambda Mx$ where K is the stiffness matrix while M is the mass matrix. Both are large, sparse and symmetric. The eigenvalue problem can be solved using the Lanczos algorithm which has superior convergence properties, and is ideal for computing extremal eigenvalues of large problems [7]. In each Lanczos iteration, one solves the linear system $\tilde{K}x = b$ where the coefficient matrix $\tilde{K} = K - \sigma M$. σ is the shift introduced to accelerate convergence by better separating the eigenvalues.

To solve the linear system effectively on the parallel computer, one takes advantage of domain decomposition by separating the DOFs in the domain into "interior" DOFs that belong inside a subdomain, and those that reside on the boundaries between subdomains, the "interface" DOFs. Since there is no coupling between the "interior" DOFs from different subdomains, and the "interface" DOFs are far less in number, $\tilde{K}$ can be reordered in the form

$$\tilde{K} = \begin{pmatrix} A & C^T \\ C & F \end{pmatrix},$$

in which the "interface" DOFs are ordered after the "interior" ones. Now A is a block diagonal matrix with A_i being the submatrix from the ith subdomain, F contains only the "interface" DOFs, and C supplies the coupling between the two groups.

The next step is to apply the direct method and factorize $\tilde{K}$ using the Crout decomposition

$$\tilde{K} = \begin{pmatrix} L_A & 0 \\ CL_A^{-T}D_A^{-1} & L_G \end{pmatrix} \begin{pmatrix} D_A & 0 \\ 0 & D_G \end{pmatrix} \begin{pmatrix} L_A^T & D_A^{-1}L_A^{-1}C^T \\ 0 & L_G^T \end{pmatrix}$$

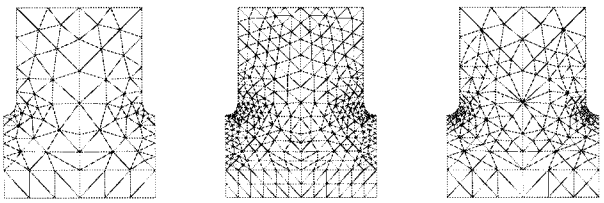
where $A = L_A D_A L_A^T, G = L_G D_G L_G^T$ are Crout decompositions of A, G respectively, and $G = F - CA^{-1}C^T$. Remembering A is block diagonal the decomposition of its submatrices can be done in parallel at each individual node. Because it constitutes the major block of $\tilde{K}$, this parallelization results in substantial gain in computation efficiency.

5. Adaptive Mesh Refinement

In finite element analysis, the accuracy of the solution improves as the mesh is refined. The straightforward refinement is to subdivide each triangular element over the whole mesh so the increase in DOFs scales with the number of subdivisions. It is computationally more cost effective to refine adaptively to keep the number of extraneous DOFs to a minimum. The criteria being used to determine if refinement is necessary is based on the local gradient in the stored energy. The refinement stops when a specified tolerance value is reached. This error indicator turns out to be the most effective in generating the optimal mesh for high accuracy in a given solution. Figs. 2 show an initial mesh, a uniformly refined mesh, and an adaptively refined mesh. The two refined meshes yield the same accuracy but the adaptively refined mesh has two times less the number of DOFs, thereby producing a significant savings in memory requirement and run time.

6. Simulation Results for NLC Detuned Structure

A new FE field solver using quadratic, mixed elements, and incorporating both parallelization and adaptive refinement features, has been running on the Intel's Paragon XP/S 150



(a) (b) (c)

Figure 2. (a) *Initial mesh*; (b) *Uniform refinement*; (c) *Adaptive refinement.*

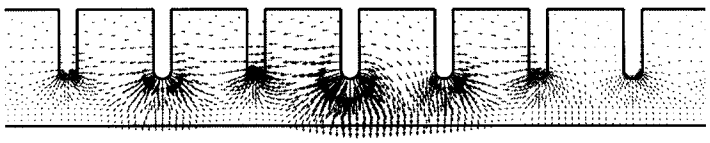


Figure 3. *Electric field pattern (from cell 163 to 170) of mode 166 .*

at Oak Ridge National Laboratory. This massively parallel computer has 1,024 node with three 75-MFLOPS i860 XP processors and 64 MBytes memory per node. The code is written in C++ and is compatible with any mesh generator. For the NLC DS simulation, the mesh data is generated with MODULEF. Fig. 3 shows the 166th mode in the first dipole band obtained from the simulation Fig. 4 compares the wakefield with the results from equivalent circuits and measurement including wall loss. The simulation data contains the contributions from 11 dipole bands for a total of 2266 modes versus 512 modes from the equivalent circuit analysis. The total CPU time consumed is about 15 hours which suggests that parallel computation can be practical for design and analysis.

7. Conclusion

A new 2D finite-element field solver is implemented on a parallel computer with adaptive mesh refinement. This enables the first-ever calculation of dipole modes in tapered accelerator structures using realistic geometries. The simulation results for the NLC DS include the contributions from higher bands and wall dissipation, and show good quantitative agreement with measured data for the wakefield. In terms of parallel computing efficiency the speedup with node increase is close to linear. The evaluation of the end-cell effects due to the input/output couplers and beampipes is in progress.

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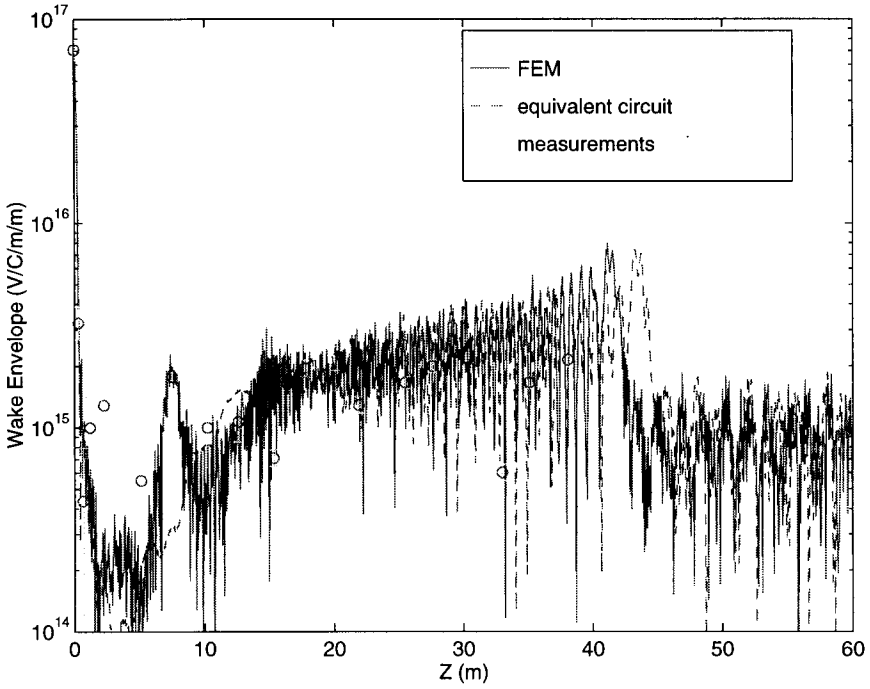

Figure 4. *Wake envelopes obtained from FEM and equivalent circuit calculations.*

valuable insights of the problem.

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