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Rare Kaon Decays *

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Abstract

The phenomenology of rare kaon decays is reviewed, with emphasis on tests of standard model flavordynamics.

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1. INTRODUCTION

The physics of kaons has played a major role in the development of particle physics. The concept of strangeness, with its implications for the quark model, the discovery of P and CP violation and the anticipation of charm and the GIM mechanism have all emerged from the study of K mesons. Today, rare decays of kaons continue to be an active field of investigation. For several topics of current interest kaon physics holds the promise of providing important insights:

- Rare kaon decays probe the flavordynamics of the standard model (SM), i.e. the physics of quark masses and mixing. This part of the theory is related to electroweak symmetry breaking, the sector of the standard model that is least understood and contains most of the free model parameters.

- The sensitivity of rare kaon phenomena, like $K-\bar{K}$ mixing, to energies higher than the kaon mass scale proved to be a very useful source of information on the charm quark, even before it was discovered [1]. In quite the same spirit rare decays of K mesons allow to probe the physics of top quarks. Many of them are strongly sensitive to the top quark mass and can yield results on the CKM couplings V_{td} and V_{ts}, information hardly accessible directly in top quark decays.

- Among the most important current problems in high energy physics is the poorly understood topic of CP violation. All of the, very few, experimental facts known to date about this fundamental asymmetry derive from a handful of K_L decay modes ($K_L \to \pi\pi$, $\pi l\nu$, $\pi^+\pi^-\gamma$) and can so far be all accomodated by a single complex parameter ε_K. More detailed insight into CP violation will be possible, for instance, by the precision studies offered through measuring theoretically clean rare K decays, such as $K_L \to \pi^0\nu\bar{\nu}$.

- Beyond rare processes that are strongly suppressed in the standard model, one may search for decays that are entirely forbidden within this framework and therefore particularly clear signals of new physics. Promising examples are lepton-family-number violating modes like $K_L \to \mu e$. Experiments are planned to probe the corresponding branching ratios down to a level of $\sim 10^{-12}$. This accuracy may be translated into a sensitivity to scales of new physics of a few hundred TeV, although precise values are model dependent. Sensitivity to such high energy scales seems very difficult to attain by any other method.

- Some rare and radiative decay modes of K mesons (such as $K^+ \to \pi^+l^+l^-$ or $K_L \to \pi^0\gamma\gamma$) are dominated by long-distance hadronic physics and therefore less useful for the study of short-distance flavordynamics. Still these cases are of considerable interest to test the low energy structure of QCD as described within the framework of chiral perturbation theory.

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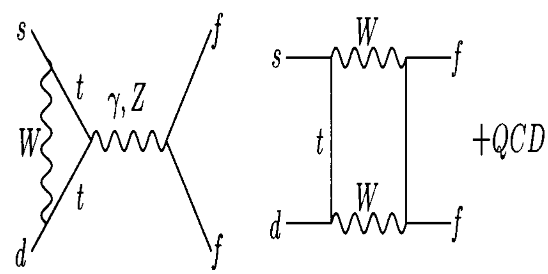

Figure 1. Typical diagrams inducing rare K decays in the standard model.

It is evident that the area of rare kaon decays covers a rather wide range of important topics. In the following talk we will concentrate on those processes that are sensitive to short-distance physics and probe the flavordynamics sector of the standard model.
After this introduction we briefly describe, in section 2, the necessary theoretical framework. The decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ form the subject of section 3. Sections 4, 5 and 6 deal with $K_L \to \pi^0 e^+e^-$, $K_L \to \mu^+\mu^-$ and $K^+ \to \pi^+\mu^+\mu^-$, respectively. A few further topics (chiral perturbation theory, additional possibilities to test CP violation, SM forbidden decays) are briefly addressed in section 7. We conclude with a summary in section 8.

2. THEORETICAL FRAMEWORK

Loop-induced flavor-changing neutral current (FCNC) processes, as they appear at higher order in the standard electroweak theory, can give rise to rare decays of K mesons (Fig. 1). The calculation of decay rates and branching fractions is based on the construction of low-energy effective Hamiltonians using operator product expansion (OPE) techniques. This method provides a systematic approximation to the full SM dynamics and is appropriate if the typical scale of external momenta is small compared to the mass scale of heavy virtual particles (M_W, m_t, m_c) in the loop. For K decays this condition is well satisfied. Schematically the effective Hamiltonians take the form

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM} C_i(M_W, \mu) \cdot Q_i \qquad (1)$$

where V_{CKM} denotes the appropriate CKM parameters, C_i the Wilson coefficients and Q_i local four-fermion operators. Generally the leading terms in the OPE, represented by four-fermion operators of lowest dimension (six), are sufficient for all practical purposes. For the example sketched in Fig. 1, the Q_i typically have the form $(\bar{s}d)_{V-A}(\bar{f}f)_{V-A}$. The analysis of higher-dimensional operators, corresponding to subleading contributions in the OPE, is discussed in [2]. The result of the formal OPE, eq. (1), has the intuitive interpretation of an effective, Fermi-type theory with the operators Q_i playing the role of interaction vertices and the coefficient functions C_i representing the corresponding coupling constants. Beyond being just a convenient approximation, the formalism based on (1) exhibits a very crucial conceptual feature: It provides a factorization of the full amplitude into a short-distance part, described by the Wilson coefficients C_i, and a long-distance contribution, contained in the hadronic matrix elements of local operators Q_i. The C_i comprise all the physics at short distances (above the factorization scale $\mu \approx 1GeV$), in particular the dependence on M_W, on heavy quark masses (m_t, m_c) and CKM couplings (V_{td}, V_{ts}). This part is calculable in perturbation theory, including the short-distance QCD effects, which may require renormalization group (RG) improvement if large logarithms (e.g. $\ln(M_W/m_c)$) are present. All the non-perturbative long-distance dynamics (below scale μ) is factored into the matrix elements of Q_i between the given external states and can be treated separately. In this manner the OPE helps to disentangle the complicated interplay of strong and weak interactions in FCNC decays.

Continuous progress in both theory and experiment has lead to an improved understanding of weak decays of hadrons. Simultaneously important SM parameters are becoming increasingly better under control. One of the more recent highlights is certainly the discovery of the top-quark, which has a crucial impact on the field of rare kaon processes. The top mass is already quite accurately determined. The pole mass directly measured in experiment reads $m_{t,pole} = (175\pm 6)GeV$ [3], which translates into a running $\overline{MS}$ mass $\bar{m}_t(m_t) \equiv m_t = (167\pm 6)GeV$. The latter definition is the one used in FCNC processes, where top appears as a virtual particle.

Further quantities important for analyzing rare K decays are the CKM matrix elements V_{cb} and $|V_{ub}/V_{cb}|$. From exclusive [4] and inclusive [5,6] $b \to c$ transitions one can extract $V_{cb} = 0.040\pm 0.003$. The situation with $|V_{ub}/V_{cb}|$ is more difficult and the uncertainty is still substantial, $|V_{ub}/V_{cb}| \simeq 0.08 \pm 0.02$. However the discovery of $B \to (\pi, \varrho) l\nu$ decays at CLEO [7] is encouraging and should eventually allow improvements on this issue.

During recent years progress has also been made in computing the effective Hamiltonians for weak decays. In leading order of RG improved perturbation theory the leading logarithmic QCD effects of the form $(\alpha_s \ln(M_W/\mu))^n$ are taken into account to all orders $n = 0, 1, 2, \ldots$ in evaluating the Wilson coefficients C_i. This resummation is necessary due to the presence of large logarithms $\ln(M_W/\mu)$, compensating the smallness of α_s, and these corrections are counted as terms of $\mathcal{O}(1)$. At the next-to-leading order (NLO) the corrections of relative order $\mathcal{O}(\alpha_s)$ $(\alpha_s(\alpha_s \ln(M_W/\mu))^n)$ are consistently included. These NLO calculations are by now available for essentially all important rare and CP violating processes.

Several quantitative as well as conceptual improvements are brought about by a full NLO analysis. First, going beyond the leading order result and including the relative $\mathcal{O}(\alpha_s)$ correction is necessary to assess the validity of the perturbative approach. Furthermore, without a NLO calculation a meaningful use of the scheme-specific QCD scale $\Lambda_{\overline{MS}}$ is not possible, since the distinction between various schemes shows up only at NLO. Likewise the ambiguity related to unphysical renormalization scale (μ) dependences, a theoretical error owing to the truncation of the perturbation series, can be reduced by including higher order corrections. In some cases the phenomenologically interesting top-quark mass dependence is altogether a NLO effect and thus requires the full NLO analysis to be included in an entirely satisfactory way. This is the case, for example, with $K_L \to \pi^0 e^+ e^-$. In other decays, like $K \to \pi\nu\bar{\nu}$, the m_t-dependence appears already at leading order. In this situation the NLO calculation is necessary for a meaningful distinction between various possible definitions of the top-quark mass, which differ at $\mathcal{O}(\alpha_s)$. As we have seen above, the difference between the running mass $m_t(m_t)$ and the pole mass $m_{t,pole}$, for instance, is about $8GeV$, a sizable amount that already exceeds the experimental error $\delta m_t \approx 6GeV$.

The subject of next-to-leading order QCD corrections to weak decays has been reviewed in detail in [8]. More information on this topic and references may be found in this article.

3. THE RARE DECAYS $K^+ \to \pi^+\nu\bar{\nu}$ AND $K_L \to \pi^0\nu\bar{\nu}$

The decays $K \to \pi\nu\bar{\nu}$ proceed through flavor changing neutral current effects. These arise in the standard model only at second (one-loop) order in the electroweak interaction (Z-penguin and W-box diagrams, Fig. 2) and are additionally GIM suppressed. The branching fractions are thus very small, at the level of 10^{-10}, which makes these modes rather challenging to detect. However, $K \to \pi\nu\bar{\nu}$ have long been known to be reliably calculable, in contrast to most other decay modes of interest. A measurement of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ will therefore be an extremely useful test of flavor physics. Over the recent years important refinements have been added to the theoretical treatment of $K \to \pi\nu\bar{\nu}$. These have helped to precisely quantify the meaning of the term 'clean' in this context and have reinforced the unique potential of these observables. Let us briefly summarize the main aspects of why $K \to \pi\nu\bar{\nu}$ is theoretically so favorable and what recent developments have contributed to emphasize this point.

- First, $K \to \pi\nu\bar{\nu}$ is a semileptonic decay. The relevant hadronic matrix elements

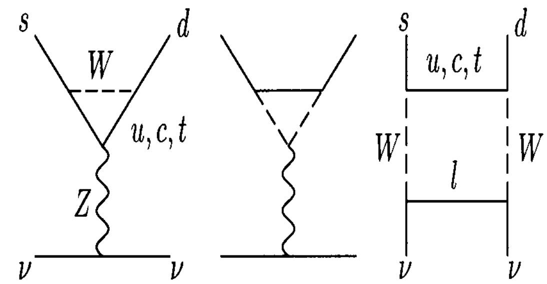

Figure 2. Leading order electroweak diagrams contributing to $K \rightarrow \pi\nu\bar{\nu}$ in the standard model.

$\langle\pi|(\bar{s}d)_{V-A}|K\rangle$ are just matrix elements of a current operator between hadronic states, which are already considerably simpler objects than the matrix elements of four-quark operators encountered in many other observables ($K - \bar{K}$ mixing, ε'/ε). Moreover, they are related to the matrix element

$$\langle\pi^0|(\bar{s}u)_{V-A}|K^+\rangle \qquad (2)$$

by isospin symmetry. The latter quantity can be extracted from the well measured leading semileptonic decay $K^+ \rightarrow \pi^0 l\nu$. Although isospin is a fairly good symmetry, it is still broken by the small difference between the masses of up- and down-quarks and by electromagnetism. These sources of isospin breaking manifest themselves in differences of the neutral versus charged kaon (and pion) masses (affecting phase space), corrections to the isospin limit in the formfactors and electromagnetic radiative effects. Marciano and Parsa [9] have analyzed these corrections and found an overall reduction in the branching ratio by 10% for $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and by 5.6% for $K_L \rightarrow \pi^0\nu\bar{\nu}$.

- Long-distance contributions are systematically suppressed as $\mathcal{O}(\Lambda^2_{QCD}/m_c^2)$ compared to the charm contribution (which is part of the short-distance amplitude). This feature is related to the hard ($\sim m_c^2$) GIM suppression pattern exhibited by the Z-penguin and W-box diagrams, and the absence of virtual photon amplitudes. Long-distance contributions have been examined quantitatively [10–14] and shown to be indeed negligible numerically (below $\approx 5\%$ of the charm amplitude).
- The preceeding discussion implies that $K \rightarrow \pi\nu\bar{\nu}$ are short-distance dominated (by top- and charm-loops in general). The relevant short-distance QCD effects can be treated in perturbation theory and have been calculated at next-to-leading order [15,16]. This allowed to substantially reduce (for K^+) or even practically eliminate (K_L) the leading order scale ambiguities, which are the dominant uncertainties in the leading order result.

In Table 1 we have summarized some of the main features of $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$.

The neutral mode proceeds through CP violation in the standard model. This is due to the definite CP properties of K^0, π^0 and the hadronic transition current $(\bar{s}d)_{V-A}$. The violation of CP symmetry in $K_L \rightarrow \pi^0\nu\bar{\nu}$ arises through interference between $K^0 - \bar{K}^0$ mixing and the decay amplitude. This mechanism is sometimes refered to as mixing-induced CP violation. By itself, it could a priori be attributed to a superweak interaction. However, any difference in the magnitude of this mixing-induced CP violation between two different K_L decay modes can not, and is therefore a signal of direct CP violation. Now, the effect of mixing-induced CP violation is already known for e.g. $K_L \rightarrow \pi^+\pi^-$. In this case it can be measured by $\eta_{+-} = A(K_L \rightarrow \pi^+\pi^-)/A(K_S \rightarrow \pi^+\pi^-) \approx \varepsilon_K$, which is of the order $\mathcal{O}(10^{-3})$. By contrast, $\eta_{\pi^0\nu\bar{\nu}} = A(K_L \rightarrow \pi^0\nu\bar{\nu})/A(K_S \rightarrow \pi^0\nu\bar{\nu})$ is of $\mathcal{O}(1)$ in the SM, far larger than η_{+-}. Thus the standard model decay $K_L \rightarrow \pi^0\nu\bar{\nu}$ is a signal of almost pure direct CP violation, revealing an effect that can not be attributed to CP violation in the $K - \bar{K}$ mass matrix alone (in which case

Table 1
Compilation of important properties and results for $K \to \pi\nu\bar{\nu}$

	$K^+ \to \pi^+\nu\bar{\nu}$	$K_L \to \pi^0\nu\bar{\nu}$
	CP conserving	CP violating
CKM	V_{td}	$\mathrm{Im}V_{ts}^* V_{td} \sim J_{CP} \sim \eta$
contributions	top and charm	only top
scale uncert. (BR)	$\pm 20\%$ (LO) $\to \pm 5\%$ (NLO)	$\pm 10\%$ (LO) $\to \pm 1\%$ (NLO)
BR (SM)	$(0.9 \pm 0.3) \cdot 10^{-10}$	$(2.8 \pm 1.7) \cdot 10^{-11}$
exp. limit	$< 2.4 \cdot 10^{-9}$ BNL 787 [17]	$< 5.8 \cdot 10^{-5}$ FNAL 799 [18]

$\eta_{+-} = \eta_{\pi^0\nu\bar{\nu}}$).
While already $K^+ \to \pi^+\nu\bar{\nu}$ can be reliably calculated, the situation is even better for $K_L \to \pi^0\nu\bar{\nu}$. Since only the imaginary part of the amplitude (in standard phase conventions) contributes, the charm sector, in $K^+ \to \pi^+\nu\bar{\nu}$ the dominant source of uncertainty, is completely negligible for $K_L \to \pi^0\nu\bar{\nu}$ (0.1% effect on the branching ratio). Long-distance contributions ($\lesssim 0.1\%$) and also the indirect CP violation effect ($\lesssim 1\%$) are likewise negligible. In summary, the total theoretical uncertainties, from perturbation theory in the top sector and in the isospin breaking corrections, are safely below $2-3\%$ for $B(K_L \to \pi^0\nu\bar{\nu})$. This makes this decay mode truly unique and very promising for phenomenological applications. (Note that the range given as the standard model prediction in Table 1 arises from our, at present, limited knowledge of standard model parameters (CKM), and not from intrinsic uncertainties in calculating $B(K_L \to \pi^0\nu\bar{\nu})$).

With a measurement of $B(K^+ \to \pi^+\nu\bar{\nu})$ and $B(K_L \to \pi^0\nu\bar{\nu})$ available, very interesting phenomenological studies could be performed. For instance, $B(K^+ \to \pi^+\nu\bar{\nu})$ and $B(K_L \to \pi^0\nu\bar{\nu})$ together determine the unitarity triangle (Wolfenstein parameters ϱ and η) completely (Fig. 3). The expected accuracy with $\pm 10\%$ branching ratio measurements is comparable to the one that can be achieved by CP violation studies at B factories before the LHC era [19]. The quantity $B(K_L \to \pi^0\nu\bar{\nu})$ by itself offers probably the best precision in determining $\mathrm{Im}V_{ts}^* V_{td}$ or, equivalently, the Jarlskog parameter

$$J_{CP} = \mathrm{Im}(V_{ts}^* V_{td} V_{us} V_{ud}^*) = \lambda \left(1 - \frac{\lambda^2}{2}\right) \mathrm{Im}\lambda_t \quad (3)$$

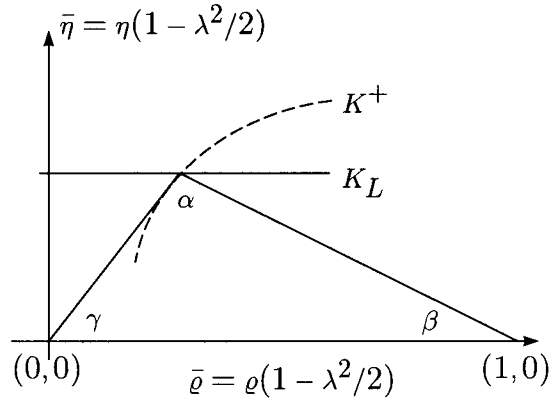

Figure 3. Unitarity triangle from $K \to \pi\nu\bar{\nu}$.

The prospects here are even better than for B physics at the LHC. As an example, let us assume the following results will be available from B physics experiments

$$\begin{aligned} \sin 2\alpha &= 0.40 \pm 0.04 \\ \sin 2\beta &= 0.70 \pm 0.02 \\ V_{cb} &= 0.040 \pm 0.002 \end{aligned} \quad (4)$$

The small errors quoted for $\sin 2\alpha$ and $\sin 2\beta$ from CP violation in B decays require precision measurements at the LHC. In the case of $\sin 2\alpha$ we have to assume in addition that the theoretical problem of 'penguin-contamination' can be resolved. These results would then imply $\mathrm{Im}\lambda_t = (1.37 \pm 0.14) \cdot 10^{-4}$. On the other hand, a $\pm 10\%$ measurement $B(K_L \to \pi^0\nu\bar{\nu}) = (3.0 \pm 0.3) \cdot 10^{-11}$ together with $m_t(m_t) = (170 \pm 3) GeV$ would give $\mathrm{Im}\lambda_t = (1.37 \pm 0.07) \cdot 10^{-4}$. If we are optimistic and take $B(K_L \to \pi^0\nu\bar{\nu}) = (3.0 \pm$

$0.15) \cdot 10^{-11}$, $m_t(m_t) = (170 \pm 1) GeV$, we get $\text{Im}\lambda_t = (1.37 \pm 0.04) \cdot 10^{-4}$, a truly remarkable accuracy. The prospects for precision tests of the standard model flavor sector will be correspondingly good.

The charged mode $K^+ \to \pi^+ \nu\bar{\nu}$ is being currently pursued by Brookhaven experiment E787. The latest published result [17] gives an upper limit which is about a factor 25 above the standard model range. Several improvements have been implemented since then and the SM sensitivity is expected to be reached in the near future [20]. For details see also [21]. Recently an experiment has been proposed to measure $K^+ \to \pi^+ \nu\bar{\nu}$ at the Fermilab Main Injector [22]. Concerning $K_L \to \pi^0 \nu\bar{\nu}$, a proposal exists at Brookhaven (BNL E926) to measure this decay at the AGS with a sensitivity of $\mathcal{O}(10^{-12})$ [20]. There are furthermore plans to pursue this mode with comparable sensitivity at Fermilab [23] and KEK [24,25]. It will be very exciting to follow the development and outcome of these ambitious projects.

4. $K_L \to \pi^0 e^+ e^-$

This decay mode has obvious similarities with $K_L \to \pi^0 \nu\bar{\nu}$ and the apparent experimental advantage of charged leptons, rather than neutrinos, in the final state. However there are a number of quite serious difficulties associated with this very fact. Unlike neutrinos the electron couples to photons. As a consequence the amplitude, which was essentially purely short-distance in $K_L \to \pi^0 \nu\bar{\nu}$, becomes sensitive to poorly calculable long-distance physics (photon penguin). Simultaneously the importance of indirect CP violation ($\sim \varepsilon$) is strongly enhanced and furthermore a long-distance dominated, CP conserving amplitude with two-photon intermediate state can contribute significantly. Treating $K_L \to \pi^0 e^+ e^-$ theoretically one is thus faced with the need to disentangle three different contributions of roughly the same order of magnitude.

- Direct CP violation: This part is short-distance in character, theoretically clean and has been analyzed at next-to-leading order in QCD [26]. Taken by itself this mechanism leads within the standard model to a $K_L \to \pi^0 e^+ e^-$ branching ratio of $(4.5 \pm 2.6) \cdot 10^{-12}$.
- Indirect CP violation: This amplitude is determined through $\sim \varepsilon \cdot A(K_S \to \pi^0 e^+ e^-)$. The K_S amplitude is dominated by long-distance physics and has been investigated in chiral perturbation theory [27–29]. Due to unknown counterterms that enter this analysis a reliable prediction is not possible at present. The situation would improve with a measurement of $B(K_S \to \pi^0 e^+ e^-)$, which could become possible at DAΦNE. Present day estimates for $B(K_L \to \pi^0 e^+ e^-)$ due to indirect CP violation alone give typically values of $(1-5) \cdot 10^{-12}$.
- The CP conserving two-photon contribution is again long-distance dominated. It has been analyzed by various authors [29–31]. The estimates are typically a few 10^{-12}. Improvements in this sector might be possible by further studying the related decay $K_L \to \pi^0 \gamma\gamma$ whose branching ratio has already been measured to be $(1.7 \pm 0.3) \cdot 10^{-6}$.

Originally it had been hoped for that the direct CP violating contribution is dominant. Unfortunately this could so far not be unambiguously established and requires further study.

Besides the theoretical problems, $K_L \to \pi^0 e^+ e^-$ is also very hard from an experimental point of view. The expected branching ratio is even smaller than for $K_L \to \pi^0 \nu\bar{\nu}$. Furthermore a serious irreducible physics background from the radiative mode $K_L \to e^+ e^- \gamma\gamma$ has been identified, which poses additional difficulties [32]. A background subtraction seems necessary, which is possible with enough events. Additional information could in principle also be gained by studying the electron energy asymmetry [29,31] or the time evolution [29,33,34].

5. $K_L \to \mu^+ \mu^-$

$K_L \to \mu^+ \mu^-$ receives a short-distance contribution from Z-penguin and W-box graphs similarly to $K \to \pi\nu\bar{\nu}$. This part of the amplitude

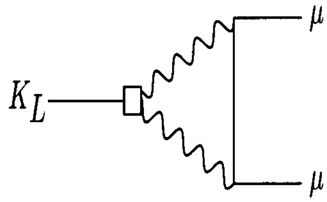

Figure 4. Long-distance contribution to $K_L \to \mu^+\mu^-$ from the two-photon intermediate state.

is sensitive to the Wolfenstein parameter ϱ. In addition $K_L \to \mu^+\mu^-$ proceeds through a long-distance contribution with two-photon intermediate state, which actually dominates the decay completely (Fig. 4). The long-distance amplitude consists of a dispersive (A_{dis}) and an absorbtive contribution (A_{abs}). The branching fraction can thus be written

$$B(K_L \to \mu^+\mu^-) = |A_{SD} + A_{dis}|^2 + |A_{abs}|^2 \quad (5)$$

Using $B(K_L \to \gamma\gamma)$ it is possible to extract $|A_{abs}|^2 = (6.8 \pm 0.3) \cdot 10^{-9}$ [32]. A_{dis} on the other hand can not be calculated accurately at present and the estimates are strongly model dependent [35–39]. This is rather unfortunate, in particular since $B(K_L \to \mu^+\mu^-)$, unlike most other rare decays, has already been measured, and this with very good precision

$$B(K_L \to \mu^+\mu^-) = \left\{ \begin{array}{ll} (6.9 \pm 0.4) \cdot 10^{-9} & \text{BNL 791 [40]} \\ (7.9 \pm 0.7) \cdot 10^{-9} & \text{KEK 137 [41]} \end{array} \right. \quad (6)$$

For comparison we note that $B(K_L \to \mu^+\mu^-)_{SD} = (1.3 \pm 0.6) \cdot 10^{-9}$ is the expected branching ratio in the standard model based on the short-distance contribution alone. Due to the fact that A_{dis} is largely unknown, $K_L \to \mu^+\mu^-$ is at present not a very useful constraint on CKM parameters. Some improvement of the situation might be expected from measuring the decay $K_L \to \mu^+\mu^-e^+e^-$, which could lead to a better understanding of the $K_L \to \gamma^*\gamma^*$ vertex. First results obtained at Fermilab (E799) give $B(K_L \to \mu^+\mu^-e^+e^-) = (2.9^{+6.7}_{-2.4}) \cdot 10^{-9}$.

6. $K^+ \to \pi^+\mu^+\mu^-$

The rare decay $K^+ \to \pi^+\mu^+\mu^-$ has recently been measured at Brookhaven (BNL 787) with a branching ratio [42]

$$B(K^+ \to \pi^+\mu^+\mu^-) = (5.0 \pm 0.4 \pm 0.6) \cdot 10^{-8} \quad (7)$$

This compares well with the estimate from chiral perturbation theory $B(K^+ \to \pi^+\mu^+\mu^-) = (6.2^{+0.8}_{-0.6}) \cdot 10^{-8}$ [43]. The branching ratio is completely determined by the long-distance contribution arising from the one-photon exchange amplitude. A short-distance amplitude from Z-penguin and W-box diagrams (similar to Fig. 1) also exists, but is smaller than the long-distance part by three orders of magnitude and does therefore not play any role in the total rate. However, while the muon pair couples via a vector current $(\bar{\mu}\mu)_V$ in the photon amplitude, the electroweak short-distance mechanism also contains an axial vector piece $(\bar{\mu}\mu)_A$. The interference term between these contributions is odd under parity and gives rise to a parity-violating longitudinal μ^+ polarization, which can be observed as an asymmetry $\Delta_{LR} = (\Gamma_R - \Gamma_L)/(\Gamma_R + \Gamma_L)$ [44–47]. $\Gamma_{R(L)}$ denotes the rate of producing a right- (left-) handed μ^+ in $K^+ \to \pi^+\mu^+\mu^-$ decay. The effect occurs for a μ^- instead of μ^+ as well, but the polarization measurement is much harder in this case.
Δ_{LR} is sensitive to the Wolfenstein parameter ϱ. It is a cleaner observable than $K_L \to \mu^+\mu^-$, although some contamination through long-distance contributions can not be excluded [45]. At any rate, Δ_{LR} will be an interesting observable to study if a sensitive polarization measurement becomes feasible. The standard model expectation is typically around $\Delta_{LR} \sim 0.5\%$.

7. FURTHER TOPICS

7.1. Tests of chiral perturbation theory

Chiral perturbation theory provides a systematic framework to treat the strong interactions of

kaons and pions at low energies. Long-distance dominated rare and radiative kaon decays, such as $K_S \to \gamma\gamma$, $K_L \to \pi^0\gamma\gamma$, $K^+ \to \pi^+\gamma\gamma$ or $K^+ \to \pi^+ l^+ l^-$, offer ample testing ground for this approach. These studies are interesting and important in their own right and may also be helpful for the extraction of short-distance effects. An example of the latter case is the muon polarization asymmetry Δ_{LR} in $K^+ \to \pi^+\mu^+\mu^-$ discussed in the previous section.
The application of chiral perturbation theory to rare K decays has been most recently reviewed in [43,48], where further references can be found. Other useful accounts of this subject, including experimental tests are given in [32,49].

7.2. CP Violation

In addition to the more commonly discussed observables of CP violation in K decays, like ε [50], ε'/ε [51] or $K_L \to \pi^0\nu\bar{\nu}$, other options for probing the CP asymmetry of nature in rare kaon processes have been proposed in the literature. For instance, any difference between the decay rates of $K^+ \to \pi^+\gamma\gamma$ and $K^- \to \pi^-\gamma\gamma$ would signal direct CP violation. The same applies to $K^+ \to \pi^+e^+e^-$ and $K^- \to \pi^-e^+e^-$. Although a theoretical treatment of these long-distance dominated modes is not easy, observation of a clear CP violating effect would certainly be very interesting.
Another example is the longitudinal μ^+ polarization in $K_L \to \mu^+\mu^-$, which violates CP. The standard model prediction is rather reliable in this case and one expects $(\Gamma_R - \Gamma_L)/(\Gamma_R + \Gamma_L) \approx 2 \cdot 10^{-3}$ [52].
More information on these topics may be found in the review articles [32,49,53].

7.3. SM forbidden decays

As mentioned in the introduction, lepton flavor violating decays of kaons can serve as sensitive, albeit indirect, probes of very high energy scales [32,49]. The current situation may be characterized by the following limits on branching ratios that have been established in experiments at Brookhaven and Fermilab [54]:

$$\begin{aligned} B(K_L \to \mu e) &< 3.3 \cdot 10^{-11} \quad \text{BNL 791} \\ B(K^+ \to \pi^+\mu^+e^-) &< 2.1 \cdot 10^{-10} \quad \text{BNL 777} \\ B(K_L \to \pi^0\mu e) &< 3.2 \cdot 10^{-9} \quad \text{FNAL 799} \end{aligned} \tag{8}$$

Forbidden in the standard model, those decays could be induced in extensions that violate lepton flavor. The above branching ratios would then behave typically as $BR \sim 1/M_X^4$, where M_X is the mass of an exotic heavy boson mediating the interaction at tree level. Improvements of the experiments mentioned before aim to reach a sensitivity in the branching ratios of $\sim 10^{-12}$. This corresponds to typically $M_X \sim 100 TeV$, which, although somewhat model dependent, is still quite an impressive figure. The window on such extremely short distances thus provided makes this class of decays certainly worth pursuing.

8. SUMMARY

The field of rare kaon decays offers a broad range of interesting topics, ranging from chiral perturbation theory, over standard model flavor dynamics to exotic phenomena, thereby covering scales from Λ_{QCD} to the weak scale (M_W, m_t) and beyond to maybe several hundred TeV. In the present talk we have focussed on the flavor physics of the standard model and those processes that can be used to test it. Several promising examples of short-distance sensitive decay modes exist, whose experimental study will provide important clues on flavordynamics. On the theoretical side, progress has been achieved over recent years in the calculation of effective Hamiltonians, which by now include the complete NLO QCD effects in essentially all cases of practical interest. The current status of four particularly important decay modes, $K_L \to \mu^+\mu^-$, $K_L \to \pi^0e^+e^-$, $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, is briefly summarized in Table 2. The SM predictions for the branching ratios are determined by the usual analysis that fits the CKM phase δ from the experimental value of the kaon CP violation parameter ε, and requires m_t, V_{cb}, $|V_{ub}/V_{cb}|$ and the kaon bag parameter B_K as main input. The SM numbers in Table 2 are from ref. [55] and use $\bar{m}_t(m_t) = (167 \pm 6)GeV$, $V_{cb} = 0.040 \pm 0.003$, $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ and (for the RG invari-

Table 2
Summary of present status of selected rare kaon decays.

	$K_L \to \mu^+\mu^-$	$K_L \to \pi^0 e^+ e^-$	$K^+ \to \pi^+\nu\bar{\nu}$	$K_L \to \pi^0\nu\bar{\nu}$
theoret. status	$--$	$+-$	$++$	$++$
BR (SM)	$(1.3 \pm 0.6) \cdot 10^{-9}$ (SD)	$(4.5 \pm 2.6) \cdot 10^{-12}$ (dir. CPV)	$(0.9 \pm 0.3) \cdot 10^{-10}$	$(2.8 \pm 1.7) \cdot 10^{-11}$
BR (exp)	$(7.2 \pm 0.5) \cdot 10^{-9}$	$< 4.3 \cdot 10^{-9}$	$< 2.4 \cdot 10^{-9}$	$< 5.8 \cdot 10^{-5}$

ant bag parameter) $B_K = 0.75 \pm 0.15$. For $K_L \to \mu^+\mu^-$ the branching ratio prediction refers only to the short-distance part, for $K_L \to \pi^0 e^+ e^-$ only to the contribution from direct CP violation. The decay $K_L \to \mu^+\mu^-$ is already measured quite accurately; unfortunately a quantitative use of this result for the determination of CKM parameters is strongly limited by large hadronic uncertainties. For $K_L \to \pi^0 e^+ e^-$ there are some theoretical and experimental difficulties, but improvements might be possible. The situation looks very bright for $K \to \pi\nu\bar{\nu}$. The charged mode is experimentally already 'around the corner' and its very clean status promises useful results on V_{td}. Finally the decay $K_L \to \pi^0\nu\bar{\nu}$ is a particular highlight in this field. It could serve e.g. as the ideal measure of the Jarlskog parameter J_{CP}. Measuring the branching ratio is a real experimental challenge, but definitely worth the effort.

It is to be expected that rare kaon decay phenomena will in the future continue to contribute substantially to our understanding of the fundamental interactions and it is quite conceivable that exciting surprizes await us along the way.

REFERENCES

1. M.K. Gaillard and B.W. Lee, Phys. Rev. **D10**, 897 (1974).
2. A. Pivovarov, these proceedings.
3. P. Tipton, talk presented at the International Conference on High Energy Physics, Warsaw (1996).
4. M. Neubert, Phys. Lett. **B338**, 84 (1994).
5. M. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. **D51**, 2217 (1995).
6. P. Ball, M. Beneke and V.M. Braun, Phys. Rev. **D52**, 3929 (1995).
7. J.P. Alexander *et al.* (CLEO), CLNS 96/1419, CLEO 96-9 (1996).
8. G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
9. W. Marciano and Z. Parsa, Phys. Rev. **D53**, R1 (1996).
10. D. Rein and L.M. Sehgal, Phys. Rev. **D39**, 3325 (1989).
11. J.S. Hagelin and L.S. Littenberg, Prog. Part. Nucl. Phys. **23**, 1 (1989).
12. M. Lu and M. Wise, Phys. Lett. **B324**, 461 (1994).
13. C.Q. Geng, I.J. Hsu and Y.C. Lin, Phys. Lett. **B355**, 569 (1995).
14. S. Fajfer, hep-ph/9602322.
15. G. Buchalla and A.J. Buras, Nucl. Phys. **B400**, 225 (1993).
16. G. Buchalla and A.J. Buras, Nucl. Phys. **B412**, 106 (1994).
17. S. Adler et al., Phys. Rev. Lett. **76**, 1421 (1996).
18. M. Weaver et al., Phys. Rev. Lett. **72**, 3758 (1994).
19. G. Buchalla and A.J. Buras, Phys. Rev. **D54**, 6782 (1996).
20. L. Littenberg and J. Sandweiss, eds., AGS-2000, Experiments for the 21st Century, BNL 52512
21. P. Meyers, these proceedings.
22. P. Cooper, M. Crisler, B. Tschirhart and J. Ritchie (CKM collaboration), EOI for measuring $B(K^+ \to \pi^+\nu\bar{\nu})$ at the Main Injector, Fermilab EOI 14, 1996
23. K. Arisaka et al., KAMI conceptual design report, FNAL, June 1991
24. T. Inagaki, T. Sato and T. Shinkawa, Experiment to search for the decay $K_L \to \pi^0\nu\bar{\nu}$ at KEK 12 GeV proton synchrotron, 30 Nov. 1991
25. N. Sasao, these proceedings.

26. A.J. Buras *et al.*, Nucl. Phys. **B423**, 349 (1994).
27. G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B303**, 665 (1988).
28. C. Bruno and J. Prades, Z. Phys. **C57**, 585 (1993).
29. J.F. Donoghue and F. Gabbiani, Phys. Rev. **D51**, 2187 (1995).
30. A.G. Cohen, G. Ecker and A. Pich, Phys. Lett. **B304**, 347 (1993).
31. P. Heiliger and L. Sehgal, Phys. Rev. **D47**, 4920 (1993).
32. L.S. Littenberg and G. Valencia, Ann. Rev. Nucl. Part. Sci. **43**, 729 (1993)
33. L.S. Littenberg, in *Proceedings of the Workshop on CP Violation at a Kaon Factory*, ed. J.N. Ng, TRIUMF, Vancouver, Canada (1989), p. 19.
34. G.O. Köhler and E.A. Paschos, Phys. Rev. **D52**, 175 (1995).
35. L. Bergström, E. Massó and P. Singer, Phys. Lett. **B249**, 141 (1990).
36. C.Q. Geng and J.N. Ng, Phys. Rev. **D41**, 2351 (1990).
37. G. Bélanger and C.Q. Geng, Phys. Rev. **D43**, 140 (1991).
38. P. Ko, Phys. Rev. **D45**, 174 (1992).
39. J.O. Eeg, K. Kumericki and I. Picek, hep-ph/9605337
40. A.P. Heinson *et al.*, Phys. Rev. **D51**, 985 (1995).
41. T. Akagi *et al.*, Phys. Rev. **D51**, 2061 (1995).
42. T. Shinkawa, *in* proceedings of the Workshop on K Physics, Orsay, France, 1996.
43. A. Pich, hep-ph/9610243
44. M. Savage and M. Wise, Phys. Lett. **B250**, 151 (1990).
45. M. Lu, M. Wise and M. Savage, Phys. Rev. **D46**, 5026 (1992).
46. G. Bélanger, C. Geng and P. Turcotte, Nucl. Phys. **B390**, 253 (1993).
47. G. Buchalla and A.J. Buras, Phys. Lett. **B336**, 263 (1994).
48. G. Ecker, hep-ph/9611346
49. J.L. Ritchie and S.G. Wojcicki, Rev. Mod. Phys. **65**, 1149 (1993).
50. J. Rosner, these proceedings.
51. M. Ciuchini, these proceedings.
52. G. Ecker and A. Pich, Nucl. Phys. **B366**, 189 (1991).
53. B. Winstein and L. Wolfenstein, Rev. Mod. Phys. **65**, 1113 (1993).
54. Particle Data Group, Phys. Rev. **D54**, 1 (1996).
55. A.J. Buras, M. Jamin and M.E. Lautenbacher, to appear.