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Pisin Chen

Stanford Linear Accelerator Center
Stanford, California 94309

Robert J. Noble

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

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Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

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CRYSTAL CHANNEL COLLIDER: ULTRA-HIGH ENERGY AND LUMINOSITY IN THE NEXT CENTURY

Pisin Chen
Stanford Linear Accelerator Center *
Stanford, California 94309 USA

Robert J. Noble
Fermi National Accelerator Laboratory †
Batavia, Illinois 60510 USA

ABSTRACT

We assume that, independent of any near-term discoveries, the continuing goal of experimental high-energy physics (HEP) will be to achieve ultra-high center-of-mass energies, possibly approaching the Planck scale (10^{28} eV), in the next century. To progress to these energies in such a brief span of time will require a radical change in accelerator and collider technology. High-gradient acceleration of charged particles along crystal channels and the possibility of colliding them in these same strong-focusing atomic channels have been separately investigated in earlier proposals. Here we expand further upon the concepts of emittance damping and plasma wave generation to explore a new paradigm for HEP machines early in the next century: the crystal channel collider. Energy and emittance limitations in natural crystal accelerators are determined. The technologies needed to begin experimental research on this accelerator concept are now emerging. The excitation of 1 to 100 GV/cm plasma waves in semiconductor and metal crystals by either the laser wakefield or side-injected laser techniques appears experimentally feasible with near-term lasers.

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INTRODUCTION

High-energy physics has progressed twelve orders of magnitude in energy during the last one-hundred years (1 eV to 10^{12} eV or 1 TeV). Modern high-energy colliders are both microscopes and time machines allowing us to probe fundamental physics at distances of 10^{-16} centimeters and hence understand the relevant phenomena 10^{-10} seconds after the Big Bang. It is thought by some that by advancing only one or two orders of magnitude higher in energy, experiments will place enough constraints on unified field theories to yield one consistent "Theory of Everything" including gravity. Machine builders instead assume that regardless of any intermediate discoveries, the continuing goal of experimental high-energy physics will be to achieve ultra-high center-of-mass energies, possibly approaching the Planck scale (10^{28} eV), in the next century. Electromagnetic acceleration is limited to about 10^{16} V/cm (the critical field for pair production), and a Planck-scale linear accelerator would then have a length of about one-tenth the Earth-Sun separation - not an inconceivable task for an advanced technological society. Still to reach these energies with their attendant high luminosity in such a brief span of time will require a radical change in accelerator and collider technology. In all likelihood more than one paradigm shift in accelerators will be needed. In this paper we investigate a concept that may enable us to reach energies of order 10^{18} eV early in the next century: the crystal channel collider.

For the next few decades, our sphere of technological influence will likely be limited to the near-Earth neighborhood. Accelerators with lengths of order 10^3 to 10^4 kilometers will probably become feasible. Very high gradients are the only avenue available then to attain truly "cosmic" energies in the immediate future. The energy density in an accelerator increases with the square of the acceleration gradient. To keep the total deposited energy manageable and maintain a small beam for high luminosity, an accelerator with small transverse dimensions is called for. The idea of using atomic structures to accelerate charged particles to high energy in a short distance was expounded early, notably by Hofstadter [1]. That paper remarkably contains the germ of several ideas which were independently discovered and developed by various workers over the following twenty years, including channeling to guide accelerated particles.

Ten years ago the present authors made a cursory study of a concept to accelerate positively-charged particles along crystal channels by the electron plasma waves in metals [2,3]. The maximum electric field of a plasma wave is of order $\sqrt{n_o}$ V/cm, where n_o is the electron number density in units of cm^{-3}. Acceleration gradients of 100 GV/cm or more were implied based on the electron densities in solids. The strong electrostatic focusing of the atomic channels combined with the high gradients were found to maintain low beam emittance in spite of multiple scattering on channel electrons. The technological demands to excite such large amplitude plasma waves with lasers or particle beams appeared daunting then, and crystal behavior at picosecond to femtosecond time scales and high power densities was uncertain at best. Some of the early estimates made for crystal survival were certainly too optimistic.

The development of ultra-short pulse-length lasers, nano-fabrication technology and a better experimental and theoretical understanding of high energy density effects in solids motivate us to return to the topic of a crystal channel accelerator. Recent work on radiative damping of channeled particle emittance has also opened up the possibility of achieving both high luminosity and high energy in a crystal collider [4,5]. An improved picture of a crystal accelerator emerges which allows us to further illucidate the advantages of crystals for acceleration and emittance control as well as point out the constraints imposed by the use of natural crystals as high-energy particle accelerators. Limits on high luminosity may ultimately be more difficult to overcome than achieving ultra-high energy.

CHANNELING ACCELERATION AND EMITTANCE DAMPING

The basic concept of crystal channel acceleration combines plasma wave acceleration [6] with the well known channeling phenomenon [7] to allow positively charged particles to be accelerated over long distances without colliding with nuclei in the accelerating medium. Positively charged particles are guided by the average electric fields produced by the atomic rows or planes in a crystal. The particles make a series of glancing collisions with many atoms and execute classical oscillatory motion along the interatomic channels. The condition for classical motion is that the transverse de Broglie wavelength $\hbar/p\psi$, where p is the total momentum and ψ is the channeling angle relative to an atomic row or plane, be much less than the typical atomic screening length (~ 0.1 Å) where single atomic collisions become important. In contrast negatively charged particles oscillate about the atomic nuclei and rapidly suffer large-angle Coulomb scattering [8].

Acceleration in the crystal is provided by an electron plasma oscillation [9] with phase velocity near the speed of light. The maximum electric field of a relativistic plasma wave is the wave-breaking field [10,11]

$$\mathcal{E}_{WB} = \sqrt{2(\gamma_p - 1)}\ \mathcal{E}_0\ , \tag{1}$$

where $\mathcal{E}_0 = m_e\omega_p c/e$, m_e is the electron rest mass, $\omega_p = (4\pi n_o e^2/m_e)^{1/2}$ is the electron plasma frequency, and γ_p is the usual relativistic factor for a wave with phase velocity v_p. In convenient units, $\mathcal{E}_0(V/cm) \simeq 0.96(n_o(cm^{-3}))^{1/2}$ and $\omega_p/2\pi(sec^{-1}) \simeq 9000(n_o(cm^{-3}))^{1/2}$. For phase velocities approaching the speed of light (needed for relativistic acceleration), extremely high electric fields could be achieved if the necessary power densities can be applied to the plasma in the correct geometry. Doped semiconductors typically have carrier densities of 10^{14} to 10^{18} cm^{-3} corresponding to $\mathcal{E}_0 = 10$ MV/cm to 1 GV/cm, the same as typical laboratory gas plasmas. Conduction electrons in metals have densities of 10^{22} to 10^{23} cm^{-3} while the total electron density of solids is of order 10^{24} cm^{-3} implying gradients of order 1 TV/cm.

A basic obstacle to accelerating particles over long distances in crystals is beam loss from dechanneling. The transverse momentum of channeled particles increases due to collisions with electrons in the interatomic channels. Dechanneling occurs when a particle's transverse kinetic energy $E\psi^2/2$, where E is the total particle energy, allows it to overcome the channel's potential energy barrier V_c ($\sim 10 - 1000\ ze$ volts for a particle of charge ze). At this point close encounters with atomic cores quickly scatter particles out of the channel. This defines the critical channeling angle $\psi_c = (2V_c/E)^{1/2}$. In many crystals the electron density n over most of the channel is roughly constant. From Poisson's equation the channel potential energy function in either plane is simply $V = K_c x^2/2$, where $K_c = 4\pi z e^2 n$ is the focusing strength. The channel half-width a corresponds to the point where $V = V_c = K_c a^2/2$.

The increase in the *rms* angular divergence per unit length (projected onto a plane) of a channeled particle due to electron multiple scattering can be written as

$$\frac{d\langle\psi^2\rangle_{ms}}{ds} = \frac{4\pi(ze^2)^2 n}{E^2}\ln(b_{max}/b_{min}) \equiv \frac{\psi_c^2}{2\ell_d}\,, \tag{2}$$

where n is the channel electron density (which is typically less than the average electron density n_o in the crystal), the impact parameters in the Coulomb logarithm are $b_{max} \simeq c/\omega_p$ and $b_{min} = \hbar/\gamma m_e c$, and γ is the relativistic factor for the channeled particle [12]. The characteristic dechanneling length is $\ell_d = \Lambda E/ze$, and the dechanneling constant $\Lambda = a^2/2e\ln(b_{max}/b_{min})$ is essentially only a function of the channel width. In natural crystals where $a \simeq 1$ to 3 Å, Λ is typically of order 1 to 10 μm/MV, consistent with experimental dechanneling lengths for MeV to 100 GeV particles [13]. Note that ℓ_d and Λ are independent of the electron density in the channel to first order.

In the harmonic potential approximation, each crystal channel acts like a smooth focusing accelerator with betatron focusing function (wavelength/2π of transverse oscillations)

$$\beta_F = (E/K_c)^{1/2} = a/\psi_c\,. \tag{3}$$

The normalized *rms* channel acceptance, $A_n \equiv \gamma a^2/2\beta_F = \gamma a\psi_c/2$, defines the available transverse phase space for a channeled particle. Multiple scattering in a transverse focusing system randomly excites betatron oscillations leading to growth in the normalized *rms* emittance $\varepsilon_n = \gamma\varepsilon = \gamma\sigma^2/\beta_F$, where ε is the geometric emittance, and σ^2 is the *rms* amplitude of the particle [14]. In this terminology, dechanneling occurs when the particle emittance exceeds the channel acceptance.

Particle dechanneling in a crystal accelerator is modified by several effects. Acceleration reduces multiple scattering with increasing energy as is evident from Eqn. (2). The presence of any transverse fields in addition to the natural channel forces will change the betatron focusing function and channel acceptance. For example a plasma wave of amplitude A and transverse size b has longitudinal and transverse fields near the central axis ($x \ll b$) $\mathcal{E}_z = A\cos\phi$ and $\mathcal{E}_x = -2A\sin\phi\, x/k_p b^2$, respectively, where k_p is the plasma wavenumber and ϕ is the particle phase with respect to the wave crest [15]. There is a phase region in which both acceleration and transverse focusing are possible. If this wave is centered on the crystal channel axis, then particles experience a total focusing strength K given by the sum of K_c and $K_p = 2zeA\sin\phi/k_p b^2$ of the plasma wave. Charged particles oscillating in a transverse focusing system radiate and make transitions to lower energy levels of the potential with an energy-independent decay constant $\Gamma_c = 2r_{cl}K/3mc$, where $r_{cl} = (ze)^2/mc^2$ is the classical particle radius [4,5]. The channeled particle is assumed to be in the so-called undulator radiation regime where $\gamma\psi \ll 1$ and dipole radiation dominates. These radiative transitions act to damp the particle's normalized emittance. Collisional energy loss to electrons in the channel can also damp emittance (ionization cooling) but with an energy-dependent decay parameter $E^{-1}(dE_{coll}/ds) = (E/m_ec^2)d\langle\psi^2\rangle_{ms}/ds$ for a relativistic particle.

Combining these effects, the evolution equation for the normalized emittance in a crystal channel accelerator is

$$\begin{aligned} \frac{d\varepsilon_n}{ds} &= -\frac{\Gamma_c}{c}(\varepsilon_n - \hbar/2mc) - \frac{1}{E}\frac{dE_{coll}}{ds}\varepsilon_n + \frac{\gamma\beta_F}{2}\frac{d\langle\psi^2\rangle_{ms}}{ds} \\ &= -\frac{\Gamma_c}{c}(\varepsilon_n - \hbar/2mc) - \frac{zeK_ca^2}{2m_ec^2\Lambda E}\varepsilon_n + \frac{zeK_ca^2}{4mc^2\Lambda(KE)^{1/2}}\,, \end{aligned} \tag{4}$$

where $E = E_i + zeGs$, E_i is the initial particle energy, and G is the (net) acceleration gradient which is assumed to be a constant. The term $\hbar/2mc$ is the minimum quantum emittance of a particle in the ground state of the transverse potential. In a natural crystal channel ($K = K_c$) with electron density $n = 10^{23}$ cm^{-3}, the focusing strength is $K_c \simeq 20$ eV/Å^2, and the radiative damping distance c/Γ_c is 15 cm for positrons, 6 km for muons and 500 km for protons.

Because of the different energy dependencies in the three terms of Eqn. (4), not all terms are equally important in an arbitrary energy regime. The effectiveness of ionization cooling clearly falls off rapidly with increasing energy. Ionization cooling dominates over radiation damping for energies

$E < 3m^2c^2K_ca^2/4zem_eK\Lambda$. For example, this corresponds to roughly 10 MeV positrons, 600 GeV muons and 50 TeV protons in a natural crystal when $K = K_c$. The solution to Eqn. (4) when radiation damping can be neglected is

$$\varepsilon_n = (\varepsilon_{ni} - \frac{(K_c/K)A_{ni}}{\Lambda G + K_ca^2/m_ec^2})(\gamma_i/\gamma)^{K_ca^2/2m_ec^2\Lambda G} + \frac{(K_c/K)A_n}{\Lambda G + K_ca^2/m_ec^2}, \quad (5)$$

where $A_n = \gamma(K/E)^{1/2}a^2/2$ is the normalized channel acceptance including all transverse focusing forces, and A_{ni} is the channel acceptance at energy E_i. In the special case of ionization cooling with no net acceleration, the first term in Eqn. (5) becomes a damped exponential, and the emittance approaches the equilibrium value $\varepsilon_n = (m_ec^2/Ka^2)A_n$. Only if $Ka^2 > m_ec^2 \simeq$ 511 keV is the particle's equilibrium emittance within the channel acceptance. This value far exceeds the channel potential energy barrier found in natural crystals and could only be obtained in an artificially wide channel (10 to 100 Å) or by added focusing such that $K \gg K_c$. In the opposite limit of high gradients, $G \gg K_ca^2/\Lambda m_ec^2 \sim 1$ MV/cm, ionization cooling has a negligible effect on the emittance evolution in the channel compared to acceleration, and the emittance becomes

$$\varepsilon_n = \varepsilon_{ni} + \frac{(K_c/K)A_n}{\Lambda G}(1 - (\gamma_i/\gamma)^{1/2}). \quad (6)$$

Accelerated particles remain indefinitely channeled provided $G \geq K_c/K\Lambda$. This corresponds to 1 to 10 GV/cm in natural crystal channels when $K = K_c$. Note that the equilibrium *rms* amplitude is $\sigma^2 = (K_c/K)a^2/2\Lambda G$.

Neglecting ionization cooling for very high-energy channeled particles, the solution to the differential equation (4) is

$$\begin{aligned}\varepsilon_n(s) &= \varepsilon_{ni}\exp(-\Gamma_c s/c) + (\hbar/2mc)(1 - \exp(-\Gamma_c s/c)) \\ &\quad + \exp(-\Gamma_c s/c)\int_0^s \exp(\Gamma_c s'/c)\frac{zeK_ca^2}{4mc^2\Lambda K^{1/2}}\frac{ds'}{(E_i + zeGs')^{1/2}}. \quad (7)\end{aligned}$$

The integral can be rewritten in terms of Dawson's integral [16]

$$D(\chi) \equiv \exp(-\chi^2)\int_0^\chi \exp(t^2)dt, \quad (8)$$

by the change of variable $\chi(s) = [(\Gamma_c/c)(s + E_i/zeG)]^{1/2}$. The solution becomes

$$\begin{aligned}\varepsilon_n(s) &= \varepsilon_{ni}\exp(-\Gamma_c s/c) + (\hbar/2mc)(1 - \exp(-\Gamma_c s/c)) \\ &\quad + \frac{zeK_ca^2}{2\Lambda mc^2}\sqrt{\frac{c}{zeGK\Gamma_c}}[D(\chi(s)) - \exp(-\Gamma_c s/c)D(\chi(0))]. \quad (9)\end{aligned}$$

For $\Gamma_c s/c \ll 1$, this solution reduces to Eqn. (6) as expected. The function $D(\chi)$ reaches a maximum value of approximately 0.541 at $\chi \simeq 0.924$, and asymptotically approaches $1/2\chi$ for $\chi > 1$. For distances $s \gg c/\Gamma_c$ and $E \gg E_i$, the normalized emittance can be written approximately as

$$\varepsilon_n(s) = \frac{\hbar}{2mc} + \frac{3mc^2(K_c/K)a^2}{8ze\Lambda(zeGKs)^{1/2}}\,, \tag{10}$$

which damps like $\gamma^{-1/2}$ provided the net gradient G can be maintained constant with the increasing radiative energy loss. Note that the *rms* amplitude σ^2 of the channeled particle damps like γ^{-1} in this regime. The presence of K_c in Eqn. (10) reflects the deleterious effect of electron multiple scattering, and prevents one from realizing the ideal quantum emittance in such a collective accelerator.

To obtain small emittances and high luminosity in a channeling accelerator then, it is advantageous to have a high acceleration gradient and strong transverse focusing such that $K \gg K_c$. In practice the available technology will limit the plasma wave amplitude G_0 that can be generated in a crystal channel accelerator. When the magnitude of the radiative energy loss rate $(dE/ds)_{rad} = -\Gamma_c\gamma^2 K\sigma^2$ becomes comparable to zeG_0, a limiting energy is reached. In the regime $\Gamma_c s/c < 1$ where Eqn. (6) is valid, the radiation rate is proportional to γ^2, and the limit is

$$E_{max} \simeq \sqrt{\frac{3\Lambda}{zea^2KK_c}}m^2c^4G_0\,. \tag{11}$$

The presence of K and K_c in Eqn. (11) reflects the competing effects of strong focusing and multiple scattering in the channel. This places a fundamental energy limit on natural crystal accelerators with $K = K_c$ because the electron density ($\sim K_c$) is fixed by the atomic structure. For example if $G_0 = 100$ GV/cm and $K = K_c = 20$ eV/Å^2, then the maximum energy is about 300 GeV for positrons, 10^4 TeV for muons and 10^6 TeV for protons. On the other hand if one can artificially arrange that $K_c < (4ze\Lambda/3a^2)K$, accelerated particles will enter the regime $s > c/\Gamma_c$ before the limit (11) is reached. Here the radiation rate is only proportional to γ, and the energy limit is $E_{max} \simeq 4m^2c^4\Lambda G_0/K_ca^2$. The channeled particle is assumed to be in the undulator radiation regime [4,5] which is true if $K_c < (4ze\Lambda/3a^2)K$. If this is not the case, the radiation rate is higher than in Eqn. (4), and particles will stop being accelerated earlier or be quickly damped back into the undulator regime.

EXCITATION OF PLASMA WAVES IN SOLIDS

Only for acceleration gradients $G \geq \Lambda^{-1} \simeq 1-10$ GV/cm will particles remain channeled over long distances in a natural crystal accelerator. Since Λ is proportional to a^2, it may be useful to consider artificially wide channels ($a > 3$ Å) to reduce the gradient demand, at least for early experiments where gradients may be limited. Still, a large amplitude plasma wave with a field of 100 GV/cm or more is ultimately desirable to shorten the accelerator and keep the emittance as low as possible.

Two regimes of the crystal accelerator can be distinguished based on whether the plasma wave amplitude or the fields used to excite the wave are greater than or less than $I/r_a \simeq 1-10$ V/Å, where I is the ionization energy of electrons in an atom of size r_a. For fields greater than this, the Coulomb potential of an atom is sufficiently deformed to induce significant tunneling ionization. For an oscillating electric field $\mathcal{E}$, electrons tunnel from atoms within a time $v_e/c\epsilon\omega$, where $v_e = (2I/m_e)^{1/2}$, $\epsilon = e\mathcal{E}/m_e\omega c$ is the normalized field strength, and ω is the frequency [17]. Typically v_e/c is of order the fine-structure constant $\alpha \simeq 1/137$, so for field strengths $\epsilon > 10^{-2}$, electrons escape the atom within an oscillation period. In this high field regime, the lattice ionizes, but does not yet dissociate, on this time scale. If an intense laser ($> 10^{14} - 10^{15}$ W/cm^2) is used to build up the plasma wave, the lattice will already be in this ionized state prior to plasma-wave formation.

For laser and plasma fields below 0.1 to 1 GV/cm, reusable crystal accelerators can probably be built which might survive multiple pulses, and many of our conclusions on crystal survival in References 2 and 3 probably still hold. Plasma wave decay is determined by interband transitions with a timescale of 10 to 100 $\omega_p{}^{-1}$ in this regime [18]. For fields above a GV/cm, only disposable accelerators, perhaps in the form of fibers or films, are possible. The lattice is highly ionized by the laser driver used to excite the plasma wave in a few optical periods, and the free electron density immediately increases to 10^{23} cm^{-3} or more for any solid. Plasma wave build-up and channeled particle acceleration must occur before the ionized lattice disrupts due to ion motion. The lattice dissociates by absorbing plasmon energy on a timescale determined by the inverse ion plasma frequency $\omega_{pi}^{-1} = (m_i/m_e)^{1/2}\omega_p^{-1} \sim 10^{-14}$ sec, where m_i is the ion rest mass. Within this time, the ions have not moved appreciably, and the lattice remains sufficiently regular to allow channeling.

The generation of large-amplitude plasma waves in a crystal requires an intense power source to supply the plasma-wave energy before the lattice dissociates. A gradient of 100 GV/cm corresponds to an energy density of 3×10^7 J/cm^3, and this must be created and used within ω_{pi}^{-1}. Because of the increased availability of high peak-power lasers, two excitation methods which may have promise, at least for initial experiments, are considered here: the side-injected laser [19] and laser wakefield [20] techniques.

In the side-injected laser method, a laser with frequency $\omega_o \simeq \omega_p$ impinges (perpendicular to the acceleration axis) on a plasma containing a spatially periodic density perturbation, either formed by an acoustic wave ($\omega_{ac} \ll \omega_p$) or a grating. The initial electron density follows this pattern. The period of the density perturbation is set at the desired plasma wavelength λ_p and defines a wavevector $\vec{k}_p$ in the plasma such that the phase velocity $v_{ph} = \omega_p/k_p \simeq c$. The laser is linearly polarized parallel to this wavevector and is of course near cutoff ($\vec{k}_o \simeq 0$) upon entry. The laser (ω_o, $\vec{k}_o \simeq 0$) and the density modulation ($\omega_{ac} \simeq 0$, $\vec{k}_p$) quasiresonantly excite forward and backward traveling plasma waves with $\omega = \omega_o \simeq \omega_p$ and $\vec{k} \simeq \pm\vec{k}_p$. For a crystal accelerator, the density modulation is probably easiest to form by epitaxially growing a superlattice (in the longitudinal direction) with period λ_p consisting of two alternating materials with different electron densities. The initial spatial electron density modulation $\delta n/n_o$ is then automatically formed with the desired periodicity.

For initial side-injected laser experiments in metal crystals, a low gradient of 1 GV/cm may be adequate to demonstrate some channeling acceleration. For example a 10 MeV positron would channel about 10 μm in a metal crystal gaining about 1 MeV in energy from the plasma wave. Assuming that little tunnel-ionization occurs, the free electron density is just the conduction value 10^{22} cm^{-3} typical of metals. The plasma and laser wavelengths are 0.3 μm (near UV). The plasma wave decays via interband transitions, and the plasma-wave amplitude ϵ_p saturates according to the usual damped oscillator expression $\epsilon_p = \epsilon_o(\delta n/n_o)\hbar\omega_p/2\Gamma_p$, where ϵ_o is the normalized laser strength, $\delta n/n_o$ is the initial electron density modulation, and Γ_p is the plasmon decay width. The gradient of 1 GV/cm corresponds to $\epsilon_p = 10^{-2}$. Taking $\delta n/n_o = 10^{-1}$ and $\Gamma_p \simeq \hbar\omega_p/20$, the required laser strength is $\epsilon_o = 10^{-2}$ giving an intensity of 10^{14} W/cm^2, consistent with minimal tunnel-ionization. In contrast, for later experiments with high gradients of 100 GV/cm, the crystal will become highly tunnel-ionized to a density of about 10^{23} cm^{-3}. Plasma wave saturation will probably be determined by relativistic frequency detuning according to $\epsilon_p \simeq (\epsilon_o\delta n/n_o)^{1/3}$ since there will be few interband transitions available for damping. For $\epsilon_p = 0.3$ (100 GV/cm if $n_o = 10^{23}$ cm^{-3}), the required laser strength is $\epsilon_o = 0.3$ corresponding to an intensity of 10^{18} W/cm^2 at a wavelength of 0.1 μm.

In the laser wakefield method, a series of short laser pulses with frequency $\omega_o \gg \omega_p$, each separated by about a plasma period, are directed onto a plasma collinear with the desired acceleration direction. The longitudinal ponderomotive force arising from each pulse's intensity gradient excites a plasma oscillation. The laser frequency must be much greater than the plasma frequency so that the laser group velocity is near the speed of light. This imprints the driven plasma wave with a phase velocity near c. Simulations are typically required to determine the optimal increase in pulse spacing as ω_p detunes relativistically as well as the change in pulse shape as the nonlinear plasma wave steepens. For estimating purposes we simply use the analytic square-pulse result for the plasma-wave amplitude arising from a series of N identical laser pulses of strength ϵ_o [21],

$$\epsilon_p = (1+\epsilon_o^2)^{N/2} - (1+\epsilon_o^2)^{-N/2} \simeq N\epsilon_o^2\,, \tag{12}$$

where the last equality applies when $\epsilon_o^2 \ll 1$. A semiconductor crystal with carrier density 10^{18} cm^{-3} may be suitable for use in an initial laser wakefield experiment to attain a gradient of 1 GV/cm. This corresponds to $\epsilon_p \simeq 1$ and $\lambda_p = 30$ μm at this density. We take the laser wavelength as 1 μm (near IR) and assume that the number of laser pulses (each about 50 femtosec long) is $N = 30$. The required laser amplitude is $\epsilon_o \simeq 0.17$ corresponding to an intensity of 3×10^{15} W/cm^2, so some tunnel-ionization of the crystal occurs.

THE CRYSTAL CHANNEL COLLIDER

Conceivably side-injected laser, laser wakefield or another driving mechanism (e.g. electron beam-plasma wakefield) could be used to excite plasma waves in a future crystal channel collider. For low gradients ($<$ 1 GV/cm) reusable accelerators probably would take the form of crystal slabs on some alignable substrate. For higher gradients replaceable films or fibers are more appropriate since these are expected to be vaporized on each pulse. Alignment is certainly problematic here, and awaits the invention of fast, repeatable atomic-scale positioning. This is needed to permit staging of crystal accelerator sections with atomic precision and maintain a straight accelerator. Dislocations, unintended crystal curvature, and misalignment between sections will likely be the practical limits to long crystal accelerators.

The emittance solutions above suggest that small beamlets can be maintained with a high acceleration gradient and strong transverse focusing in crystal channels. As noted in Ref. 5, the small beamlets can in principle be brought into collision with a high probability if the crystals of each collider arm can be aligned channel to channel. This improves the luminosity, but limitations are still reached because the bunch population cannot be made arbitrarily high, as is true in all accelerators with small transverse dimensions and short wavelengths. The crystal lattice disrupts after about 10^{-14} sec, or a hundred plasma oscillations, so the number of accelerated bunches in each channel is limited to $n_b \simeq 100$. The number of particles in each bunch is denoted by N. The bunches pass through all bunches of the oncoming train so the luminosity is proportional to $n_b^2 N^2$. Of course the accelerating crystal contains a huge number of parallel atomic channels, n_{ch}, each accelerating its own n_b bunches. The luminosity of this parallel array of accelerators is then

$$L = f_{rep} n_{ch} n_b^2 N^2 \gamma / 4\pi\beta^* \varepsilon_n. \tag{13}$$

Here f_{rep} is the repetition rate of the accelerator, and β^* is the channel beta function $(E/K)^{1/2}$ since no additional focusing at the crossing is assumed.

For the sake of discussion, let us assume a natural crystal with $K = K_c$, $a \simeq 1$ Å , and that the emittance is given by Eqn. (6) with an acceleration gradient $G = 10\Lambda^{-1} \simeq 100$ GV/cm. The number of accelerated particles in each plasma oscillation bucket is limited by beam loading [15] to a value $n_{ch}N \simeq n_{ch}A_{ch}G/8\pi e$, where $A_{ch} \simeq \pi$ Å^2 is the area of an atomic channel. This yields $N \simeq 10$, and the luminosity becomes $L(cm^{-2}sec^{-1}) \simeq 2 \times 10^{22} f_{rep}n_{ch}$. To use a proton collider for discovering new physics at a center-of-mass energy E_{cm} may require a luminosity $L(cm^{-2}sec^{-1}) \simeq 10^{29}(E_{cm}(TeV))^2$, although this may be an overestimate. This implies $f_{rep}n_{ch} \simeq 5 \times 10^{12}$ at 10^3 TeV and 5×10^{18} at 10^6 TeV. The average beam powers at these energies are 800 GW and 8×10^8 TW, respectively. These high powers result from the inherent disadvantage of having many parallel accelerators each with a small number of particles per bunch. The situation can be improved by having low electron density and/or strong focusing ($K \gg K_c$) in each channel so that particles would enter the radiation damping regime where σ^2 damps like γ^{-1}, thus increasing the luminosity. The method for doing this for each channel independently is unclear. Alternatively it may be simpler to add final focusing and combine the beamlets from the channels into a single high-density beam spot for collision, giving a large luminosity enhancement, as is done in conventional linear colliders.

CONCLUSION

Although further study is needed on this concept, the chief advantages of collective acceleration in crystal channels remain the avoidance of emittance growth due to multiple scattering on atomic nuclei and the potential for very high acceleration gradients. The crystal naturally provides a confined, uniform electron plasma for acceleration and a strong focusing system to maintain a small beam size and increase luminosity. In natural crystal accelerators, multiple scattering on channel electrons competes strongly with radiative emittance damping, and keeps the transverse particle amplitudes from being reduced to the quantum mechanical limit. The resulting radiative energy loss limits the maximum attainable energy which is then proportional to the acceleration gradient that can be generated. For a gradient of 100 GV/cm, proton energies of order 10^{18} eV are possible. Channels with low electron density and/or strong additional focusing are suggested to raise the energy limit. Some form of final focusing of the crystal beamlets will probably be required to increase the luminosity. Independent of the acceleration mechanism, the quest for higher luminosity may ultimately prove more difficult than that of reaching ultra-high energy in the next century.

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