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STUDY OF MICROWAVE INSTABILITY*

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Abstract

Longitudinal stability of a bunch is studied above the threshold of microwave instability using a moment expansion approach. We derive a system of nonlinear equations describing bunch dynamics and study it in computer simulations.

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1 Introduction

The microwave instability is usually described by linearizing Vlasov equation in the angle-action variables I, ϕ and assuming that the interaction of azimuthal harmonics $\rho_n(I)$ of the distribution function ρ is weak, where

$$\rho(I, \phi, t) = \sum_n \rho_n(I, t) e^{in\phi} \tag{1}$$

The argument implied here is that the Hamiltonian flow smears out particles over invariant tori characterized by the action variables, and the remaining azimuthal dependence of the distribution function is small. Indeed, such an approach successfully describes bunch spectrum and the threshold of the microwave instability. However, recently there have been interesting observations of bunch centroid and bunch shape oscillations above instability threshold at LEP [1] and the damping ring at SLAC [2]. There are also indications that the oscillations sometimes occur in localized region in the longitudinal coordinate instead of affecting the entire longitudinal distribution as one expects by an action-angle analysis.

In this paper we describe an alternative approach to the problem of bunch stability using decomposition of the Fokker-Plank equation in the system of nonlinear equations for the moments of the distribution function. In particular, this approach allows us to avoid the conventional action-angle decomposition. The physical quantities we are interested in, the moments, are expressed in the Cartesian $z - \delta$ phase space. To close the infinite hierarchy

of moments equations, we assume that higher order correlations are small. Although both the action-angle and the Cartesian languages must be equivalent before truncation, they may have different speed of convergence depending on the problem being studied. It is hoped that Cartesian expansion approach would converge faster for the cases corresponding to those observed recently above threshold. This approach, is well known in kinetic theory. It has been used recently by Wurtele et al. citeWurtele in their study of the threshold of the instability. The recent experimental observations made us interested in it again. This note is a progress report of our work.

2 Moments of the distribution function

Longitudinal motion of a particle is described in terms of the distance z of a particle from the position of bunch center in the RF bucket ($z > 0$ for a particle in the head of a bunch), and the deviation $\delta = p - p_0$ of the momentum from the momentum p_0 of an equilibrium particle at the zero current. We use the dimensionless coordinates $q = z/\sigma_0$, $p = -\delta/\delta_0$ and dimensionless time $s = \omega_s t$, where σ_0 is the rms bunch length, δ_0 is the rms energy spread, and $\omega_s/2\pi$ is synchrotron frequency at the zero current, $\omega_s\sigma_0 = \alpha\delta c_0$.

We start with the Fokker-Plank equation for the distribution function $\rho(q,p,s)$, (with $\int dqdp\rho(q,p,s) = 1$),

$$\frac{\partial\rho}{\partial s} + p\frac{\partial\rho}{\partial q} - \frac{\partial u(q)}{\partial q}\frac{\partial\rho}{\partial p} = \gamma\frac{\partial}{\partial p}[T_0\frac{\partial\rho}{\partial p} + p\rho] \tag{2}$$

where γ is radiative decrement, and the equilibrium temperature $T_0 = 1$ in the dimensionless variables we choose. The self-consistent potential $u(q)$ is

$$\frac{du(q)}{dq} = q - \lambda\int_q^\infty dq' f(q',s)W^\delta[(q'-q)\sigma_0]. \tag{3}$$

Here $f(q,s) = \int dp\rho(q,p,s)$, and $W^\delta(z)$ is the wake-field, $W^\delta(z) = 0$ at $z < 0$, describing the energy loss $\Delta pc_0 = -e^2W^\delta(z)$ of a trailing particle due to the field excited by a point-like leading particle with the separation $z > 0$ between particles. The coefficient λ is proportional to the number of particles per bunch, N_b:

$$\lambda = \frac{N_b r_e}{\gamma\omega_s\delta_0 T_{rev}}. \tag{4}$$

Let us define the average of an arbitrary function $\Phi(q,p,s)$ with the distribution function:

$$\overline{\Phi}(q,s)f(q,s) = \int dp\rho(q,p,s)\Phi(q,p,s) \tag{5}$$

In particular, for $\Phi = p^n$ we get for $n = 0,1,..,4$ correspondingly

$$\frac{\partial f}{\partial s} + \frac{\partial}{\partial q}(\bar{p}f) = 0, \tag{6}$$

$$\frac{\partial\bar{p}f}{\partial s} + \frac{\partial}{\partial q}(\overline{p^2}f) + \frac{\partial u}{\partial q}f = -\gamma\bar{p}f, \tag{7}$$

$$\frac{\partial \overline{p^2} f}{\partial s} + \frac{\partial}{\partial q}(\overline{p^3} f) + 2\bar{p}\frac{\partial u}{\partial q} f = -2\gamma(\overline{p^2} - 1) f, \tag{8}$$

$$\frac{\partial \overline{p^3} f}{\partial s} + \frac{\partial}{\partial q}(\overline{p^4} f) + 3\overline{p^2}\frac{\partial u}{\partial q} f = -3\gamma(\overline{p^3} - 2\bar{p}) f, \tag{9}$$

$$\frac{\partial \overline{p^4} f}{\partial s} + \frac{\partial}{\partial q}(\overline{p^5} f) + 4\overline{p^3}\frac{\partial u}{\partial q} f = -4\gamma(\overline{p^4} - 3\overline{p^2}) f \tag{10}$$

This derivation is quite analogous to the derivation of the equations of hydrodynamics from the kinetic equation. The first equation is the continuity equation assuring conservation of mass, the second equation corresponds to the conservation of momentum, etc.

The system of Eqs. (6)-(10) has a steady-state solution $f(q)$, $\partial f/\partial s = 0$,

$$\frac{\partial f(q)}{\partial q} = -\frac{du}{dq} f, \tag{11}$$

$$\bar{p} = 0, \quad \overline{p^2} = T_0 = 1, \quad \overline{p^3} = 0, \quad \overline{p^4} = 3, \tag{12}$$

This solution corresponds to the Haissinski solution [4]

$$\rho_H(q, p, s) = \rho_0 e^{-[\frac{p^2}{2} + u(q)]}, \qquad f_H(q) = f_0 e^{-u(q)}, \tag{13}$$

and results for $\overline{p^n}$ corresponds to the averaging $\overline{p^n} = \int dp p^n e^{-p^2/2} / \int dp e^{-p^2/2}$, i.e. $\overline{p^{2m}} = 2^m \Gamma(m + 1/2)/sqrt(\pi)$ for even $n = 2m$ and is equal to zero otherwise.

Define functions $v(q, s)$, $\tau(q, s)$, $\zeta(q, s)$, $\zeta_5(q, s)$:

$$v(q, s) = \bar{p} \tag{14}$$

$$T = 1 + \tau(q, s) = \overline{(p - \bar{p})^2} \tag{15}$$

$$\zeta(q, s) = \overline{(p - \bar{p})^3} \tag{16}$$

$$3 + r(q, s) = \overline{(p - \bar{p})^4} \tag{17}$$

$$\zeta_5 = \overline{(p - \bar{p})^5} \tag{18}$$

The average moments can be written now as

$$\bar{p} = v(q, s) \tag{19}$$

$$\overline{p^2} = 1 + \tau + v^2 \tag{20}$$

$$\overline{p^3} = \zeta + 3v + 3v\tau + v^3 \tag{21}$$

$$\overline{p^4} = 3 + r + 4v\zeta + 6v^2 + 6v^2\tau + v^4 \tag{22}$$

$$\overline{p^5} = \zeta_5 + 15v + 5vr + 10v^2\zeta + 10v^3 + 10v^3\tau + v^5. \tag{23}$$

Note that these moments are functions of the Cartesian longitudinal position q and time s.

3 Basic equations

Define $\psi(q,s)$ as a perturbation of the distribution function around the Haissinski solution Eq. (13) according to

$$f(q,s) = f_H[1+\psi(q,s)]. \tag{24}$$

Note that conservation of number of particles gives

$$\int dq f_H(q)\psi(q,s) = 0 \tag{25}$$

for all s.

The potential Eq. (3) has two parts $u = u_0 + u_1$ where u_0 is the Haissinski self-consistent potential

$$\frac{du_0(q)}{dq} = q - \lambda \int_q^\infty dq' f_H(q') W^\delta[(q-q')\sigma_0] \tag{26}$$

and

$$\frac{du_1(q)}{dq} = -\lambda \int_q^\infty dq' \psi(q',s) f_H(q') W^\delta[(q-q')\sigma_0]. \tag{27}$$

The system of Eqs. (6-10) can be continued indefinitely. To close the system, we need some way to define the higher moments $\tau, \zeta, r, \zeta_5, ..$ in terms of the lower moments. The obvious criterion is the requirement that the truncated system of equations has to have correct spectrum of coherent bunch oscillations at least in the case of the zero beam current.

To close the system of two equations with variables ψ, v it is sufficient to put $\tau = 0$. Similarly, taking $\zeta = 0$ we get the system for three variables ψ, v, and τ. To get a closed system of four equations, we can define $r(q,s)$ by the condition

$$3 + r = \overline{(p-\bar{p})^4} = 3[\overline{(p-\bar{p})^2}]^2 = 3(1+\tau)^2. \tag{28}$$

The coefficient here is defined by the number of pairs $(p-\bar{p})^2$ which can be chosen out of four multipliers $(p-\bar{p})^4$.

Similarly, to get a system of five equations, we can define ζ_5 by the condition

$$\overline{(p-\bar{p})^5} = 10\overline{(p-\bar{p})^2}\ \overline{(p-\bar{p})^3} = 10(1+\tau)\zeta. \tag{29}$$

where the coefficient is chosen in the same way.

After some algebra, for example, the system of four Eqs. (6)-(10), with the definitions of ψ and r given by Eqs. (20)-(24), takes the form

$$\frac{\partial\psi}{\partial s} + v' - u_0' v = -(\psi v)' + u_0' v \psi, \tag{30}$$

$$\frac{\partial v}{\partial s} + \psi' + \tau' - u_0'\tau + u_1' + \gamma v = -v v' + \frac{\psi - \tau}{1+\psi}\psi', \tag{31}$$

$$\frac{\partial \tau}{\partial s} + 2v' + \zeta' - u_0'\zeta + 2\gamma\tau = -v\tau' - 2\tau v' - \frac{\zeta}{1+\psi}\psi', \tag{32}$$

$$\frac{\partial \zeta}{\partial s} + 3\tau' + 3\gamma\zeta = -3\tau\tau' - 3\zeta v' - v\zeta', \tag{33}$$

where prime means taking derivative with respect to q.

Similarly, to derive the system of five equations, we use Eq. (29). One find that Eqs. (31-33) remain the same, while Eq. (34) should be replaced by

$$\frac{\partial \zeta}{\partial s} - 3\tau' + 6u_0'\tau + r' - u_0'r + 3\gamma\zeta = 3\tau\tau' - v\zeta' - 3v'\zeta - 3\tau^2 u_0' + \frac{\psi'}{1+\psi}[6\tau + 3\tau^2 - r], \tag{34}$$

$$\frac{\partial r}{\partial s} + 12v' - 6u_0'\zeta + 10\zeta' + 4\gamma(r - 3\tau) = -vr' - 4rv' - 10\tau\zeta' - 6\zeta\tau' + 6\zeta\tau u_0' - \frac{6\psi'\zeta}{1+\psi}(1+\tau). \tag{35}$$

To understand how many equations are needed to describe the instability, let us consider the system in the simplest case with $\tau = 0$ neglecting nonlinear terms in the RHS of equations. This gives the system of two equations

$$\frac{\partial \psi}{\partial s} + \frac{\partial v}{\partial q} - u_0' v = 0, \tag{36}$$

$$\frac{\partial v}{\partial s} + \frac{\partial \psi}{\partial q} + u_1' = -\gamma v, \tag{37}$$

which in turn can be reduced to one equation for $V(q)$, $v(q,s) = V(q)\exp[q^2/4 - i\Omega s]$. At zero current and neglecting damping, we get the Schrodinger-type equation

$$V'' + [\Omega^2 + \frac{1}{2} - \frac{q^2}{4}]V = 0 \tag{38}$$

For an arbitrary Ω, one of two solutions grows as $V \propto e^{q^2/4}$ at large q, giving $vf_H \to$ const, which is unphysical. To avoid such a solution, Ω^2 has to be quantized, giving the spectrum of bunch oscillations at zero current: $\Omega_n^2 = n$, $n = 0, 1, ...$ Compared with what is expected under the condition, namely $\Omega_n = n$, one notes that only the lowest frequency ($n = 0$ and 1) give the correct result. This is the result of taking into account only two equations for ψ and v. To get correct frequencies of multipole oscillations, it is necessary to take into account more variables.

Keeping three equations in the system of Eqs.(6-10) we were able to get correct spectrum of dipole and quadrupole oscillations, four equations would give correct spectrum including sextupole oscillations, etc. To minimally describe the experimental observations, one must include at least three or four variables.

For the sake of simplicity, let us take an impedance as a sum of pure resistive and inductive terms (delaying the subtle question of divergence of the impedance at $\omega \to \infty$). In CGS units,

$$Z(\omega) = R - i\frac{L\omega}{c_0^2}. \tag{39}$$

Here c_0 is velocity of light. The wake for this model is

$$\lambda W^\delta(q\sigma_0) = R_e\delta(q) + L_e\delta'(q), \tag{40}$$

where the effective R_e and L_e are

$$R_e = 4\pi \frac{\lambda}{\sigma_0} \frac{R}{Z_0}, \quad L_e = \frac{\lambda L}{\sigma_0^2}. \tag{41}$$

The model has been studied before in terms of the interaction of the azimuthal modes of the distribution function and results are available [5]. In this model,

$$u_0' = q - R_e f_H(q) + L_e f_H'(q); \quad u_1' = -R_e f_H \psi + L_e (f_H \psi)'. \tag{42}$$

The Haissinski solution for the model is defined by

$$f_H' = -u_0' f_H = \frac{R_e f_H - q}{1 + L_e f_H}, \tag{43}$$

and, normally, has only one maximum $f_H' = 0$. Correspondingly, $u_0(q)$ has only one minimum and does not satisfy the assumption of the Baartman-Dyachkov model [6] about a potential having two minima.

For such a model, the linearized system of three variables can be reduced to a second-order equation. Without radiation damping and for the time dependence $e^{-i\Omega s}$ it takes the form

$$v'' - a(q)v' + b(q)v = 0, \tag{44}$$

where

$$a(q) = u_0' + \frac{R_e f_H + u_0' L_e f_H}{3 + L_e f_H} \tag{45}$$

$$b(q) = \frac{1}{3 + L_e f_H}[\Omega^2 - u_0''(1 + L_e f_H) + R_e f_H u_0' + L_e f_H (u_0')^2]. \tag{46}$$

The new function $V(q)$,

$$v(q) = V(q) e^{\frac{1}{2}\int_0^q dq' a(q')} \tag{47}$$

satisfies the Schrodinger-like equation:

$$V'' + \frac{1}{3 + L_e f_H}[\Omega^2 - u_{eff}]V = 0, \tag{48}$$

with the effective potential

$$u_{eff} = u_0''(1 + L_e f_H) - (R_e + L_e u_0') u_0' f_H + \frac{3 + L_e f_H}{4} a^2(q) - \frac{3 + L_e f_H}{2} a'(q). \tag{49}$$

At large distances, $|q| \to \infty$, Eq. (49) is similar to Eq. (38)

$$V'' + [\frac{\Omega^2 + 1/2}{3} - \frac{q^2}{4}]V = 0, \tag{50}$$

giving spectrum $\Omega^2 = 3n + 1$, $n = 0, 1, \ldots$ The eigen functions define the asymptotic behavior of $v(q)$ in terms of the Hermitian polynomials $\hat{H}_n(x)$

$$v(q) \propto \hat{H}_n(\frac{q}{\sqrt{2}}). \tag{51}$$

where

$$G^{\Lambda}_{\Lambda',\Lambda''} = \sum_{l,m,k} Y^{\Lambda}_{l} a^{l}_{m,k} X^{\Lambda'}_{m} X^{\Lambda''}_{k}. \tag{59}$$

In the linear approximation, the RHS should be dropped. The eigen values Λ define the mode frequencies $\omega_n = \mathrm{Im}\Lambda_n$ and the decrements $\gamma_n = \mathrm{Re}\Lambda_n$ of the n-th mode. The number of modes is equal to $n_v \times h$. Fig. 1 shows the mode frequencies for different h and n_v in the case when the wake is turned down, $R_e = L_e = 0$. For the fixed $n_v = 3$, the number of modes increases with h, but all new modes either increase degeneracy of the existing modes or add new spurious modes with higher frequencies, see Fig 1(a). For this reason, we choose in the following $h = n_v$. Fig. 1 (b) shows modes for $n_v = 3, 4, 5$ variables, and $h = n_v$. The number of correct modes increases with n_v as it was mentioned above with one exception of $f = 1.77$ at $n_v = 5$.

Variation of the frequencies with the wake-field parameter λ is shown in Fig. 2 for $n_v = h = 4$ and the radiation damping $\gamma = 0.01$. Variation of the damping γ_n with λ is shown in Fig. 3.

The set of Eqs. (59-60) can be split in two subsets, one for oscillating modes with $\mathrm{Im}\Lambda \neq 0$ and another one for a quasi-static modes with $\mathrm{Im}\Lambda = 0$. The oscillating modes come in pairs with the same $\mathrm{Re}\Lambda$. We indicate pairs of oscillating modes with $v, \bar{v}$, $v, \bar{v} = 1, .., n_v$: $\Lambda_v = \Lambda^*_{\bar{v}}$, $X^v = (X^{\bar{v}})^*$. The quasi-static modes are indicated with the index c, $c = 1, .., n_c$.

To simplify solution of the system of equations, we can average out fast oscillating terms. Although the resonance interaction between modes is possible at certain parameters L_e, R_e and may describe interesting physics of mode splitting and of mode recombination (see Fig. 2), we are not interested in such special cases because the saw-tooth instability does not have resonance character taking place at some range of currents rather than at one certain current.

Averaging out oscillating terms, we get

$$\frac{dg_c}{ds} + \Lambda_c g_c = -\sum_{c_1,c_2}^{n_c} G^{c_1}_{c_1,c_2} g_{c_1} g_{c_2} - \sum_{v=1}^{n_v} [G^{c}_{v,\bar{v}} + G^{c}_{\bar{v},v}] g_v g_{\bar{v}} \quad c = 1, .., n_c, \tag{60}$$

$$\frac{dg_v}{ds} + \Lambda_v g_v = -\sum_{c=1}^{n_c} [G^{v}_{v,c} + G^{v}_{c,v}] g_v g_c, \quad v = 1, .., n_v \tag{61}$$

$$\frac{dg_{\bar{v}}}{ds} + \Lambda_{\bar{v}} g_{\bar{v}} = -\sum_{c=1}^{n_c} [G^{\bar{v}}_{\bar{v},c} + G^{\bar{v}}_{c,\bar{v}}] g_{\bar{v}} g_c, \quad \bar{v} = 1, .., nv. \tag{62}$$

The coefficients $G^{v}_{v,c} + G^{v}_{c,v} = [G^{\bar{v}}_{\bar{v},c} + G^{\bar{v}}_{c,\bar{v}}]^*$. Therefore, $g_{\bar{v}} = g^*_v$, and Eqs. (60)-(62) can be written for the variables g_c, $|g_v|^2$.

$$\frac{dg_c}{ds} + \Lambda_c g_c = -\sum_{c_1,c_2}^{n_c} G^{c_1}_{c_1,c_2} g_{c_1} g_{c_2} - \sum_{v=1}^{n_v} [G^{c}_{v,\bar{v}} + G^{c}_{\bar{v},v}] |g_v|^2 \quad c = 1, .., n_c, \tag{63}$$

$$\frac{d|g_v|^2}{ds} + (\Lambda_v + \Lambda^*_v)|g_v|^2 = -\sum_{c=1}^{n_c} [G^{v}_{v,c} + G^{v}_{c,v} + c.c] g_c |g_v|^2, \quad \bar{v} = 1, .., n_v. \tag{64}$$

There is always a trivial solution $g_c = |g_v|^2 = 0$ corresponding to the Haissinski solution. Small fluctuations around such a solution may lead to instability only if $\mathrm{Re}\Lambda_v < 0$. In this case, g_v grows with time

$$|g_v|^2 = g_v(0)^2 \exp\{-\int ds[2\mathrm{Re}\Lambda_v + \kappa_c g_c]\}, \quad \kappa_c \equiv [G^v_{v,c} + G^v_{c,v} + c.c] \tag{65}$$

provided $g_v(0) \neq 0$. That drives growth of g_c and the growing g_c may stop growth of g_v.

Numerical analysis shows, at least for the impedance model Eq. (39), that, for $R_e > 0$, and $n_v = h = 4$ there is only one pair of unstable modes with $\mathrm{Re}\Lambda_v < 0$ and two c-type modes. Retaining only these modes, we get the system of three equations for $X = g_{c1}$, $Y = g_{c2}$, and $Z = |g_v|^2$

$$\frac{dX}{ds} + \Lambda_1 X = -\alpha_{11}X^2 - \alpha_{12}XY - \beta_1 Z, \tag{66}$$

$$\frac{dY}{ds} + \Lambda_2 Y = -\alpha_{22}Y^2 - \alpha_{21}XY - \beta_2 Z, \tag{67}$$

$$\frac{dZ}{ds} + \Lambda_3 Z = -\kappa_1 XZ - \kappa_2 YZ, \tag{68}$$

where $\Lambda_{1,2} = \Lambda_{c1,c2}$, $\Lambda_3 = \Lambda_v + \mathrm{c.c.}$, $\alpha_{11} = G^{c1}_{c1,c1}$, $\alpha_{22} = G^{c2}_{c2,c2}$, $\alpha_{12} = G^{c1}_{c1,c2}$, $\alpha_{21} = G^{c2}_{c2,c1}$, $\beta_{1,2} = G^{c1,2}_{v,\bar{v}} + = G^{c1,2}_{\bar{v},v}$, $\kappa_{1,2} = [G^v_{v,c_{1,2}} + G^v_{c_{1,2},v} + \mathrm{c.c}]$. The system reminds the system of equations for the Lorenz attractor [7] and can describe quite different motion depending on parameters.

The trivial solution $X = Y = Z = 0$ corresponds to the Haissinski solution. Generally, there are other fixed points $\dot{X} = \dot{Y} = \dot{Z} = 0$ in the 3-D phase space X, Y, Z. In the vicinity of a fixed points, stability depends on the eigen values of the system linearized around the fixed point. Variation of the 3-D volume around a fixed point with time depends on the trace of the Jacobian $J_{\alpha\beta}(X, s) = DX_\alpha(s)/DX_\beta(0)$. For the system Eq. (67-69),

$$TrJ(s, X) = \Lambda_1 + \Lambda_2 + \Lambda_3 + (2\alpha_{11} + \alpha_{21} + \kappa_1)X + (\alpha_{12} + 2\alpha_{22} + \kappa_2)Y. \tag{69}$$

For the impedance model Eq. (40), for large R_e, Λ_1 and Λ_2 are positive while Λ_3 is negative. In this case, the fixed point at the origin is instable and the instability starts with exponentially growing Z. The growing Z drives X and Y and their growth can stop the instability when $|\kappa_1 X + \kappa_2 Y| > |\Lambda_3|$. After that, Z exponentially decay while X and Y can stay about constant at their maximum X_{max}, Y_{max} until Z becomes so small that the driving terms βZ in the equations for X and Y can be dropped. After that X and Y decrease and the system can go back to the origin. Unfortunately, at least for the case $n_v = h = 4$ we were unable to find the proper parameters. The dynamics we found is shown in Fig. 4. The time dependence of Z and X corresponds to our expectations. However, the system does not come back to the initial conditions we used, $X(0) = Y(0) = 0$, $Z(0) = 10^{-6}$. It is not clear at the present time whether this is related to the way we truncate the system or something else.

References

[1] D. Brandt et al., "Measurements of Single Bunch Stability in LEP", LEP Performance Note 35, September 1990.

[2] P. Krejcik et al., Proc. IEEE Part. Accel. Conf., Washington D.C., 1993, p. 3240.

[3] X. T. Yu and J.S. Wurtele, Calculation of the Bunch Lengthening Threshold, PAC, V. 5, p.3327, (1993)

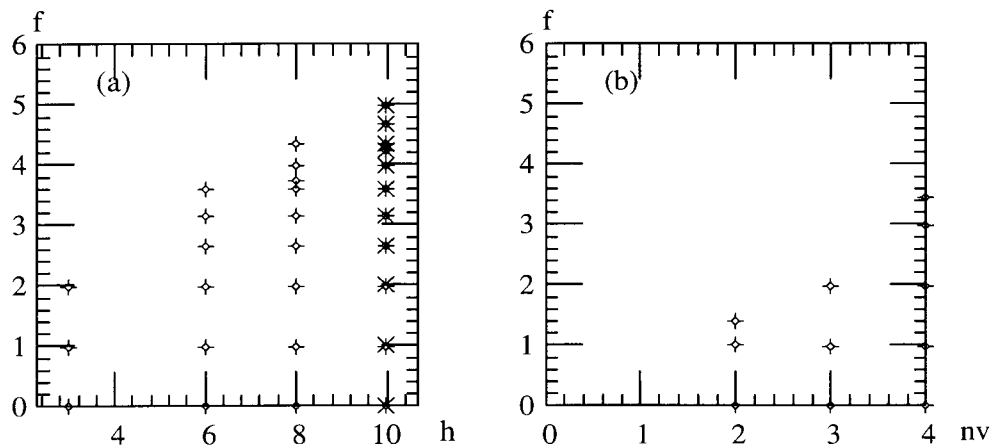
[4] J. Haissinski, Nuovo Cimento 18B, 72, (1973)

[5] K. Oide, A New Mechanism of Longitudinal Single-Bunch Instability in Storage Rings, KEK, 1995 (unpublished).

[6] R. Baartman and M. Dyachkov, Proc. IEEE Part. Accel. Conf., Dallas, (1995)

[7] E.N. Lorenz, J. Atmosph. Sci., V. 20, p.130, (1963)

Figure 1:



a)Frequencies vs rank of the polynomial. nv=3, γ=0.01

Also shown γ=0, h=10 (crosses).

b)Frequencies vs number of variables nv, h=nv, γ=0.01

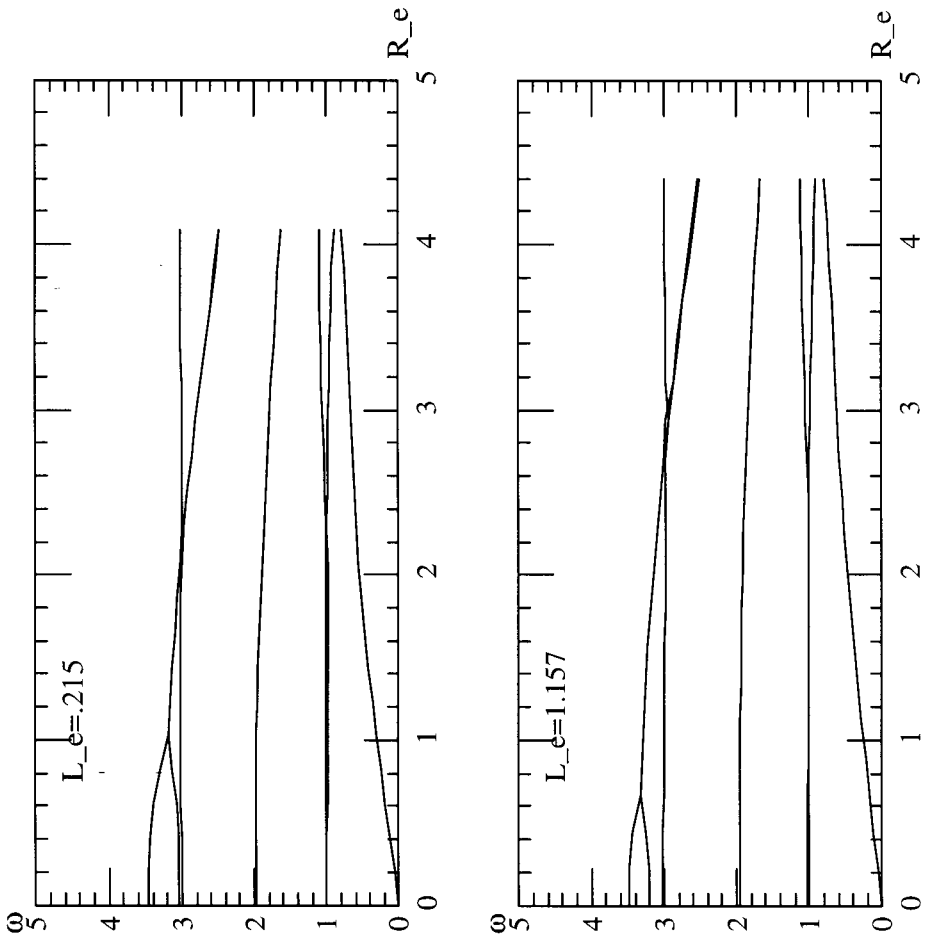


Figure 2: Frequencies of the modes vs R_e for $L_e = 0.215$ and $L_e = 1.157$. $n_v = 4$, $h = 4$.

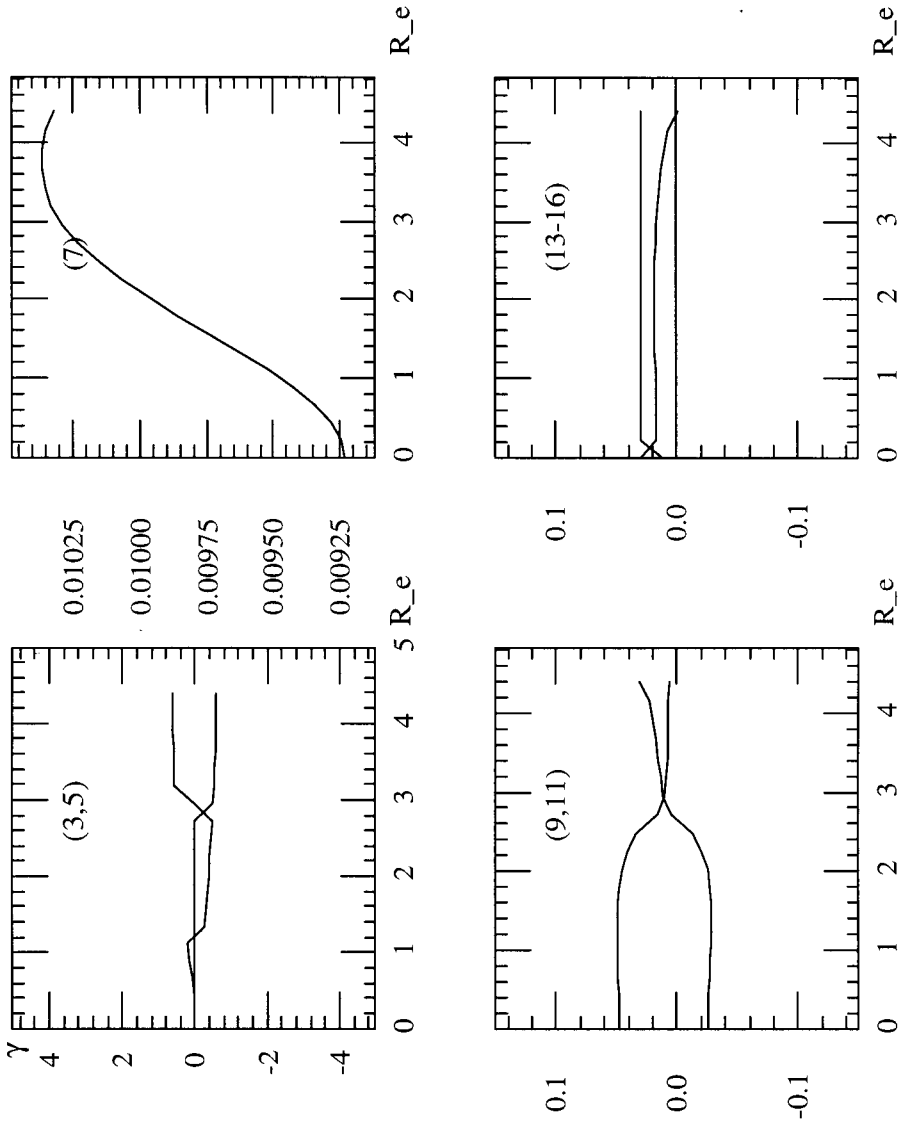


Figure 3: ReΛ vs R_e. Damping corresponds to Re$\Lambda > 0$. $n_v = 4$, $h = 4$, $L_e = 1.157$.

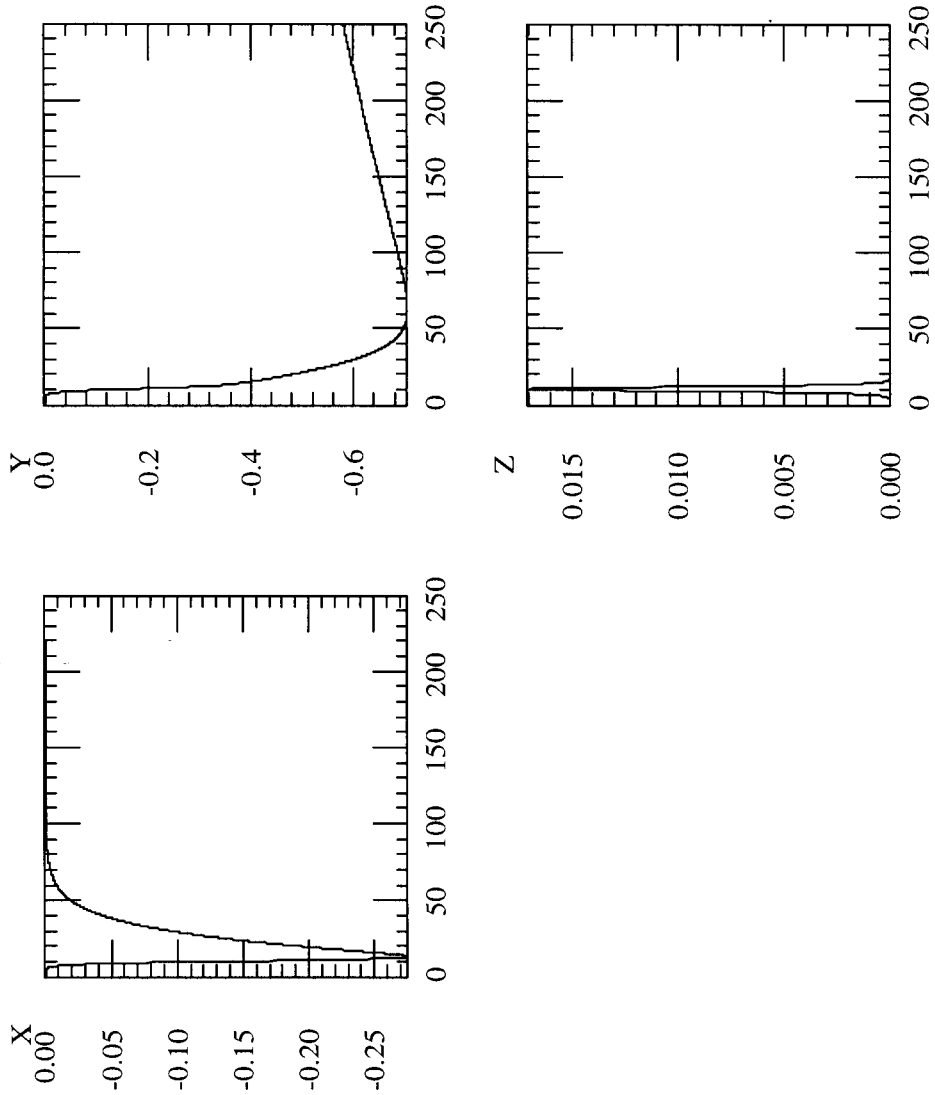


Figure 4: Variation of the parameters X, Y, Z in time, see text.