CP Violation in K Decays
and Rare Decays *

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Abstract

The present status of CP violation in decays of neutral kaons is reviewed. In addition selected rare decays of both K and B mesons are discussed. The emphasis is in particular on observables that can be reliably calculated and thus offer the possibility of clean tests of standard model flavor physics.

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1 Introduction

The violation of CP symmetry is one of the most important issues in contemporary particle physics. First, it is a topic of fundamental interest in itself. Together with C violation, CP violation provides one with an absolute definition of matter versus antimatter. It is also one of the three necessary conditions for the generation of a baryon asymmetry in the universe. Studies of this phenomenon allow one furthermore to probe standard model flavordynamics, which is the part of this theory that is least understood and contains most of the free model parameters, including the single CP violating CKM phase δ. Therefore CP violation is also closely linked to the open question of electroweak- and flavor symmetry breaking.

The experimental information on CP violation, on the other hand, is still very limited. Thirty-two years after the discovery in $K_L \to \pi^+\pi^-$ decays, its observation has so far been restricted exclusively to decays of neutral kaons, where it could be identified in just a handful of modes ($K_L \to \pi^+\pi^-$, $K_L \to \pi^0\pi^0$, $K_L \to \pi\mu\nu$, $K_L \to \pi e\nu$ and $K_L \to \pi^+\pi^-\gamma$). All of these effects are described by a single parameter ε. The question of direct CP violation in $K \to \pi\pi$, measured by the parameter ε'/ε is not yet resolved conclusively.

It is clear from these remarks that further detailed investigations of CP violation in kaon decays are highly desirable. This includes ε, ε'/ε, but also further possibilities in rare decays such as $K_L \to \pi^0 e^+e^-$ or $K_L \to \pi^0\nu\bar{\nu}$. Since CP violation and flavordynamics are intimately related, important additional and complementary information on this topic will come from studying rare decays in general, which may or may not be CP violating. Interesting opportunities are given by $K^+ \to \pi^+\nu\bar{\nu}$ and rare B decays such as $B \to X_s\gamma$, $B \to X_s\nu\bar{\nu}$, $B \to X_s l^+l^-$, $B \to l^+l^-$, and it is natural to include them in a discussion of CP violation in K decays.

Crucial tests of CP violation will also be conducted by studying CP asymmetries in decays of B mesons. This very important and interesting field is covered by the contribution of M. Gronau (these proceedings) and will therefore not be discussed in this talk.

The outline is as follows. After these introductory remarks we briefly summarize the theoretical framework that is employed to describe CP violating and rare decay processes. In section 3 we review the theoretical status of CP violation in $K \to \pi\pi$ decays, described by the parameters ε and ε'/ε. The rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, the latter of which probes CP violation, are discussed in section 4. Section 5 briefly summarizes the status of $K_L \to \pi^0 e^+e^-$ and $K_L \to \mu^+\mu^-$. Section 6 addresses the radiative decay $B \to X_s\gamma$ and the rare decay modes $B \to \mu^+\mu^-$ and $B \to X_s\nu\bar{\nu}$ are described in section 7. Our emphasis in discussing rare decays is on short-distance dominated and theoretically clean processes, which offer excellent prospects for future precision tests of SM flavor physics. A selection of further interesting modes is briefly mentioned in section 8. We conclude with a summary in section 9.

2 Theoretical Framework

In the standard model rare and CP violating decays are related to loop-induced flavor changing neutral current (FCNC) processes. This is illustrated in Figure 1 which shows the underlying electroweak transitions at the quark level. However, quarks come only in hadronic boundstates. The treatment of FCNC processes is thus in general a very complex theoretical
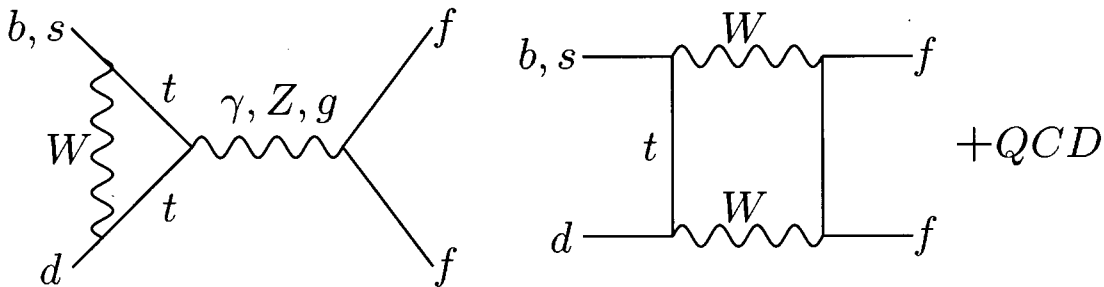
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Figure 1: *Typical diagrams for FCNC processes in the standard model.*

problem: It involves electroweak loop transitions at high (M_W, m_t) and intermediate (m_c) energies in conjunction with QCD radiative effects at short and long distances, including non-perturbative strong interaction boundstate dynamics. To make this problem tractable a systematic approximation scheme is necessary that allows one to disentangle the interplay of strong and weak interactions. Such a tool is provided by the operator product expansion (OPE). It can be used to write the quark level transition in the full theory, illustrated in Fig. 1, in the following form

$$\sum_i \frac{G_F}{\sqrt{2}} V_{CKM} \; C_i(M_W, \mu) \cdot Q_i \equiv \mathcal{H}_{eff} \tag{1}$$

where the Q_i are a set of local four fermion operators (usually of dimension six), C_i are the associated Wilson coefficient functions and V_{CKM} denotes schematically the relevant CKM parameters. The detailed form and number of the relevant operators depends on the process under consideration. Operators of dimension higher than six are suppressed by inverse powers of the heavy mass scale (e.g. M_W, m_t) and can usually be neglected for low energy B and K meson decays.

Using a somewhat less formal language, operators and Wilson coefficients are in essence nothing more than effective interaction vertices and effective couplings, respectively. The expression on the lhs of (1) can be viewed as a (low energy) effective Hamiltonian, approximating FCNC interactions among quarks and leptons at energies far below the M_W scale. The crucial feature of the OPE approach is that it provides a factorization of short distance and long distance contributions. The short distance physics from scales $\mathcal{O}(M_W)$ down to $\mu \gtrsim 1 \, GeV$ is factorized into the Wilson coefficients, which can be calculated in perturbation theory, including QCD effects. The contribution from long distance scales below μ on the other hand, is isolated into the matrix elements of the operators Q_i between physical hadron states. These have to be treated non-perturbatively, for instance in lattice QCD (see S. Güsken, these proceedings). The scale μ that separates the short distance and long distance regime is arbitrary in principle. It has to cancel between the Wilson coefficients and the matrix elements. For practical purposes, however, one would like to choose μ rather low in order to include as much of the physics as possible into the calculable coefficient function. On the other hand it is essential for the present approach that QCD be still perturbative at scale μ, otherwise the calculation of C_i would break down. Therefore μ must not be too low. Preferably it should also be close to the relevant scale in the hadronic matrix elements, without of course violating the requirement of perturbativity. Valid choices are $\mu = \mathcal{O}(m_b)$ for B decays and $\mu \gtrsim 1 \, GeV$

Table 1: *Important input parameters.*

V_{cb}	$|V_{ub}/V_{cb}|$	$\bar{m}_t(m_t)$	B_K
0.040 ± 0.003	0.08 ± 0.02	$167 \pm 6\,GeV$	0.75 ± 0.15

for K decays.

There has been continuing progress during recent years in our understanding of FCNC processes within the standard model. First of all, relevant input parameters have become better known due to ongoing progress in both theory and experiment. The most important quantities that enter in constraining the CKM phase δ from the measured value of ε (kaon CP violation) are the CKM angles V_{cb}, $|V_{ub}/V_{cb}|$, the top quark mass m_t, and the hadronic bag parameter B_K.

V_{cb} is already quite well known from exclusive and inclusive semileptonic B decay, based on heavy quark effective theory (HQET) [1] and heavy quark expansion techniques [2, 3], respectively (see also T. Mannel, these proceedings). For $|V_{ub}|$ the situation is less favorable, but the recent observation of $B \to (\pi, \varrho)l\nu$ at CLEO [4] is promising for future improvements on this topic. The rather precise determination of m_t by CDF and D0 [5] is a remarkable achievement, in particular for the field of rare decays as it fixes one of the most important input parameters. Note that the pole mass value $m_t^{pole} = 175 \pm 6\,GeV$ measured in experiment corresponds to a running $(\overline{MS})$ mass of $\bar{m}_t(m_t) = 167 \pm 6\,GeV$. The latter mass definition is more suitable for FCNC processes where top appears only as a virtual particle. The value of B_K from lattice calculations is still not very precise at present, but systematic uncertainties are becoming increasingly better under control [6, 7]. In Table 1 we summarize the values of the input parameters that were used for most of the results to be presented below. The standard model predictions quoted in sections 4, 5 and 7 are based on [8] as updated in [9].

Further progress has been achieved over the past several years through the calculation of next-to-leading order (NLO) QCD corrections in renormalization group (RG) improved perturbation theory to the Wilson coefficients for most of the rare and CP violating FCNC processes. At leading order, leading logarithmic corrections of the form $(\alpha_s \ln(M_W/\mu))^n$, which are contributions of $\mathcal{O}(1)$ due to the large logarithm multiplying α_s, are resummed to all orders, $n = 0, 1, \ldots$. At NLO relative $\mathcal{O}(\alpha_s)$ corrections of the form $\alpha_s(\alpha_s \ln(M_W/\mu))^n$ can be systematically included. This topic is reviewed in [8], where more details and references can be found. Here we would just like to summarize the main points that motivate going beyond the leading logarithmic approximation in weak decay hamiltonians.

- First of all, the inclusion of NLO corrections is necessary to test the validity of perturbation theory.

- Without NLO QCD effects a meaningful use of the scheme-specific QCD scale parameter $\Lambda_{\overline{MS}}$ is not possible.

- Unphysical scale dependences can be reduced by going beyond LO.

- The Wilson coefficients by themselves are unphysical quantities and in general scheme dependent. This scheme dependence is an $\mathcal{O}(\alpha_s)$ (NLO) effect, that is important for a proper matching to lattice matrix elements.

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- In some cases the phenomenologically interesting m_t-dependence is, strictly speaking, a NLO effect (e.g. for ε'/ε, $K_L \to \pi^0 e^+ e^-$, $B \to X_s e^+ e^-$).

- If the m_t-dependence enters already at leading order (as is the case e.g. for $K \to \pi\nu\bar{\nu}$, the top contribution to ε, $B \to \mu^+\mu^-$ or $B \to X_s\gamma$), a NLO QCD calculation allows one to make a meaningful distinction between the running mass $\bar{m}_t(m_t) \equiv m_t$ and m_t^{pole}. As we have seen the difference of $\approx 8\,GeV$ between both definitions already exceeds the current experimental error of $6\,GeV$.

3 CP Violation in $K^0 \to \pi\pi - \varepsilon$, ε'

3.1 Preliminaries

CP violation was originally discovered in $K_L \to \pi^+\pi^-$ decays. Among the few cases of CP violation in K_L decays observed since then, the $\pi\pi$ modes are still the best studied examples of CP non-conservation and continue to be under active investigation. The physical neutral kaon states are K_L and K_S and the two-pion final states they decay to can be $\pi^+\pi^-$ or $\pi^0\pi^0$. If CP was a good symmetry, K_L would be CP odd and could not decay into two pions. As a measure of CP violation one introduces therefore the amplitude ratios

$$\eta_{+-} = \frac{\langle \pi^+\pi^- |\mathcal{T}|K_L\rangle}{\langle \pi^+\pi^- |\mathcal{T}|K_S\rangle} \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 |\mathcal{T}|K_L\rangle}{\langle \pi^0\pi^0 |\mathcal{T}|K_S\rangle} \tag{2}$$

If CP violation is entirely due to mixing (indirect CPV), then $\eta_{+-} = \eta_{00}$. Any difference between η_{+-} and η_{00} is thus a measure of direct CP violation. To very good approximation one may write

$$\eta_{+-} = \varepsilon + \varepsilon' \qquad \eta_{00} = \varepsilon - 2\varepsilon' \tag{3}$$

where the observable quantities ε and ε' parametrize indirect and direct CP violation, respectively. ε'/ε is known to be real up to a phase of a few degrees. It can thus be measured from the double ratio of rates

$$|\eta_{+-}/\eta_{00}|^2 \doteq 1 + 6\mathrm{Re}\varepsilon'/\varepsilon \tag{4}$$

Using standard phase conventions the theoretical expressions for ε and ε'/ε can be written to very good approximation as

$$\varepsilon = e^{i\pi/4}\frac{\mathrm{Im}M_{12}}{\sqrt{2}\Delta M_K} \tag{5}$$

$$\frac{\varepsilon'}{\varepsilon} = \frac{\omega}{\sqrt{2}|\varepsilon|}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right) \tag{6}$$

where M_{12} is the off-diagonal element in the $K^0 - \bar{K}^0$ mass matrix and ΔM_K the $K_L - K_S$ mass difference. $A_{0,2}$ are transition amplitudes defined in terms of the strong interaction eigenstates K^0 and $\pi\pi$ states with definite isospin ($I = 0, 2$), $\langle I = 0, 2|\mathcal{T}|K^0\rangle \equiv A_{0,2}\exp(i\delta_{0,2})$. $\delta_{0,2}$ are strong interaction phases and complexities in $A_{0,2}$ arise only from CKM parameters. The smallness of $\omega \equiv \mathrm{Re}A_2/\mathrm{Re}A_0 \approx 1/22$ reflects the famous $\Delta I = 1/2$ rule.

In all current theoretical analyses of ε'/ε, the values of ω, $|\varepsilon|$ and $\mathrm{Re}A_{0,2}$ in (6) are taken from experiment. $\mathrm{Im}A_{0,2}$, which depend on the interesting short-distance physics (top-loops, CKM

Table 2: *NLO results for η_i with $\Lambda_{\overline{MS}}^{(4)} = (325 \pm 110)\,MeV$, $m_c(m_c) = (1.3 \pm 0.05)\,GeV$, $m_t(m_t) = (170 \pm 15)\,GeV$. The third column shows the uncertainty due to the errors in $\Lambda_{\overline{MS}}$ and quark masses. The fourth column indicates the residual renormalization scale uncertainty at NLO in the product of η_i with the corresponding mass dependent function from eq. (9). These products are scale independent up to the order considered in perturbation theory. The central values of the QCD factors at LO are also given for comparison.*

	NLO(central)	$\Lambda_{\overline{MS}}, m_q$	scale dep.	NLO ref.	LO(central)
η_1	1.38	$\pm 35\%$	$\pm 15\%$	[12]	1.12
η_2	0.574	$\pm 0.6\%$	$\pm 0.4\%$	[13]	0.61
η_3	0.47	$\pm 3\%$	$\pm 7\%$	[14]	0.35

phase) are then calculated using an effective Hamiltonian approach (OPE) as described in section 2.

Experimentally ε is known very precisely, whereas the situation with ε'/ε is still somewhat unclear. The current values are

$$|\varepsilon| = (2.282 \pm 0.019) \cdot 10^{-3} \tag{7}$$

$$\mathrm{Re}\frac{\varepsilon'}{\varepsilon} = \begin{cases} (23 \pm 7) \cdot 10^{-4} & \text{NA31[10]} \\ (7.4 \pm 5.9) \cdot 10^{-4} & \text{E731[11]} \end{cases} \tag{8}$$

3.2 Theoretical Status of ε

The parameter ε is determined by the imaginary part of M_{12} which in turn is generated by the usual $\Delta S = 2$ box-diagrams. The low energy effective Hamiltonian contains only a single operator $(\bar{d}s)_{V-A}(\bar{d}s)_{V-A}$ in this case and one obtains

$$\varepsilon = e^{i\frac{\pi}{4}}\frac{G_F^2 M_W^2 f_K^2}{12\pi^2}\frac{m_K}{\sqrt{2}\Delta M_K}B_K \cdot \mathrm{Im}\left[\lambda_c^{*2}S_0(x_c)\eta_1 + \lambda_t^{*2}S_0(x_t)\eta_2 + 2\lambda_c^*\lambda_t^*S_0(x_c,x_t)\eta_3\right] \tag{9}$$

Here $\lambda_i = V_{is}^*V_{id}$, $f_K = 160\,MeV$ is the kaon decay constant and the bag parameter B_K is defined by

$$B_K = B_K(\mu)[\alpha_s^{(3)}(\mu)]^{-2/9}\left[1 + \frac{\alpha_s^{(3)}(\mu)}{4\pi}J_3\right] \tag{10}$$

$$\langle K^0|(\bar{d}s)_{V-A}(\bar{d}s)_{V-A}|\bar{K}^0\rangle \equiv \frac{8}{3}B_K(\mu)f_K^2 m_K^2 \tag{11}$$

The index (3) in eq. (10) refers to the number of flavors in the effective theory and $J_3 = 307/162$ (in the NDR scheme).

The Wilson coefficient multiplying B_K in (9) consists of a charm contribution, a top contribution and a mixed top-charm contribution. It depends on the quark masses, $x_i \equiv m_i^2/M_W^2$, through the functions S_0. The η_i are the corresponding short-distance QCD correction factors (which depend only slightly on quark masses). Detailed definitions can be found in [8]. Numerical values for η_1, η_2 and η_3 are summarized in Table 2.

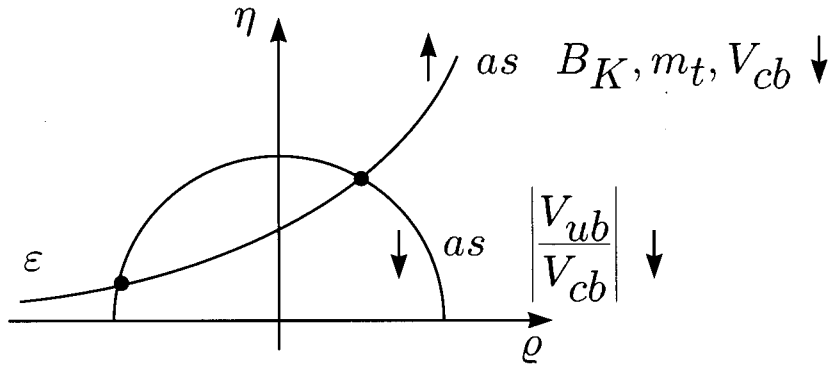Concerning these results the following remarks should be made.

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Figure 2: *The ε-constraint on the unitarity triangle.*

- ε is dominated by the top contribution ($\sim 70\%$). It is therefore rather satisfying that the related short distance part $\eta_2 S_0(x_t)$ is theoretically extremely well under control, as can be seen in Table 2. Note in particular the very small scale ambiguity at NLO, $\pm 0.4\%$ (for $100\,GeV \leq \mu_t \leq 300\,GeV$). This intrinsic theoretical uncertainty is much reduced compared to the leading order result where it would be as large as $\pm 9\%$.

- The η_i factors and the hadronic matrix element are not physical quantities by themselves. When quoting numbers it is therefore essential that mutually consistent definitions are employed. The factors η_i described here are to be used in conjunction with the so-called scheme- (and scale-) invariant bag parameter B_K introduced in (10). The last factor on the rhs of (10) enters only at NLO. As a numerical example, if the (scale and scheme dependent) parameter $B_K(\mu)$ is given in the NDR scheme at $\mu = 2GeV$, then (10) becomes $B_K = B_K(NDR, 2\,GeV) \cdot 1.31 \cdot 1.05$.

- The quantity B_K has to be calculated by non-perturbative methods. Large N_C expansion techniques for instance find values $B_K = 0.75 \pm 0.15$ [15, 16, 17]. The results obtained in other approaches are reviewed in [8]. Ultimately a first principles calculation should be possible within lattice gauge theory. Ref. [6] quotes an estimate of $B_K(NDR, 2\,GeV) = 0.66 \pm 0.02 \pm 0.11$ in full QCD. The first error is the uncertainty of the quenched calculation. It is quite small already and illustrates the progress achieved in controlling systematic uncertainties in lattice QCD [6, 7]. The second error represents the uncertainties in estimating the effects of quenching and non-degenerate quark masses.

Phenomenologically ε is used to determine the CKM phase δ. The relevant input parameters are B_K, m_t, V_{cb} and $|V_{ub}/V_{cb}|$. For fixed B_K, m_t and V_{cb}, the measured $|\varepsilon|$ determines a hyperbola in the ϱ–η plane of Wolfenstein parameters (Figure 2). Intersecting the hyperbola with the circle defined by $|V_{ub}/V_{cb}|$ determines the unitarity triangle (up to a two-fold ambiguity). As any one of the four input parameters becomes too small (with the others held fixed), the SM picture becomes inconsistent. Using this fact lower bounds on these parameters can be derived [18]. The large value that has been established for the top-quark mass in fact helps to maintain the consistency of the SM.

3.3 Theoretical Status of ε'/ε

The expression (6) for ε'/ε may also be written as

$$\frac{\varepsilon'}{\varepsilon} = -\frac{\omega}{\sqrt{2}|\varepsilon|\mathrm{Re}A_0}\left(\mathrm{Im}A_0 - \frac{1}{\omega}\mathrm{Im}A_2\right) \qquad (12)$$

$\mathrm{Im}A_{0,2}$ are calculated from the general low energy effective Hamiltonian for $\Delta S = 1$ transitions. Including electroweak penguins this Hamiltonian involves ten different operators and one has

$$\mathrm{Im}A_{0,2} = -\mathrm{Im}\lambda_t \frac{G_F}{\sqrt{2}}\sum_{i=3}^{10} y_i(\mu)\langle Q_i\rangle_{0,2} \qquad (13)$$

Here y_i are Wilson coefficients and $\langle Q_i\rangle_{0,2} \equiv \langle\pi\pi(I = 0, 2)|Q_i|K^0\rangle$, $\lambda_t = V_{ts}^*V_{td}$.
For the purpose of illustration we keep only the numerically dominant contributions and write

$$\frac{\varepsilon'}{\varepsilon} = \frac{\omega G_F}{2|\varepsilon|\mathrm{Re}A_0}\mathrm{Im}\lambda_t\left(y_6\langle Q_6\rangle_0 - \frac{1}{\omega}y_8\langle Q_8\rangle_2 + \ldots\right) \qquad (14)$$

Q_6 originates from gluonic penguin diagrams and Q_8 from electroweak contributions. The matrix elements of Q_6 and Q_8 can be parametrized by bag parameters B_6 and B_8 as

$$\langle Q_6\rangle_0 = -4\sqrt{\frac{3}{2}}\left[\frac{m_K}{m_s(\mu) + m_d(\mu)}\right]^2 m_K^2(f_K - f_\pi) \cdot B_6 \sim \left(\frac{m_K}{m_s}\right)^2 B_6 \qquad (15)$$

$$\langle Q_8\rangle_2 \simeq \sqrt{3}\left[\frac{m_K}{m_s(\mu) + m_d(\mu)}\right]^2 m_K^2 f_\pi \cdot B_8 \sim \left(\frac{m_K}{m_s}\right)^2 B_8 \qquad (16)$$

$B_6 = B_8 = 1$ corresponds to the factorization assumption for the matrix elements, which holds in the large N_C limit of QCD.
$y_6\langle Q_6\rangle_0$ and $y_8\langle Q_8\rangle_2$ are positive numbers. The value for ε'/ε in (14) is thus characterized by a cancellation of competing contributions. Since the second contribution is an electroweak effect, suppressed by $\sim \alpha/\alpha_s$ compared to the leading gluonic penguin $\sim \langle Q_6\rangle_0$, it could appear at first sight that it should be altogether negligible for ε'/ε. However, a number of circumstances actually conspire to systematically enhance the electroweak effect so as to render it a very important contribution:

- Unlike Q_6, which is a pure $\Delta I = 1/2$ operator, Q_8 can give rise to the $\pi\pi(I = 2)$ final state and thus yield a non-vanishing $\mathrm{Im}A_2$ in the first place.

- The $\mathcal{O}(\alpha/\alpha_s)$ suppression is largely compensated by the factor $1/\omega \approx 22$ in (14), reflecting the $\Delta I = 1/2$ rule.

- By contrast to $\langle Q_6\rangle_0$, $\langle Q_8\rangle_2$ is not chirally suppressed ($\langle Q_6\rangle_0$ vanishes in the chiral limit, where $f_K \to f_\pi$). As a consequence the matrix element of Q_8 is somewhat enhanced relative to the matrix element of Q_6.

- $-y_8\langle Q_8\rangle_2$ gives a negative contribution to ε'/ε that strongly grows with m_t [19, 20]. For the realistic top mass value it is quite substantial.

Table 3: *Estimates of B_6 and B_8 and calculations of ε'/ε. (g) refers to the assumption of a Gaussian distribution of errors in the input parameters, (s) to the more conservative 'scanning' of parameters over their full allowed ranges.*

B_6	B_8	$B_{6,8}$ ref.	ε'/ε ref.	$(\varepsilon'/\varepsilon)/10^{-4}$
1.0 ± 0.2	1.0 ± 0.2	large N_C [23]	[24]	$[-1.2, 16]$ (s) $[0.2, 7.0]$ (g)
1.0 ± 0.2	1.0 ± 0.2	lattice [25, 26, 27, 28]	[29]	$[0.6, 5.6]$ (g)
1.0 ± 0.4	2.2 ± 1.5	chiral quark model [30]	[31]	$[-50, 14]$ (s)
~ 1.3	~ 0.7	[32]	[33]	$[5.8, 14.0]$

Table 4: *Results for the running strange quark mass (in the $\overline{MS}$ scheme). The lattice results correspond to the quenched approximation. The numbers in brackets are estimates for the unquenched case.*

$m_s(2\,GeV)/MeV$	
145 ± 20	QCD sum rules [34, 35, 36]
127 ± 18	lattice (Rome) [37]
$90 \pm 20\ (55 - 70)$	lattice (Los Alamos) [38]
$95 \pm 16\ (54 - 92)$	lattice (Fermilab) [39]

The Wilson coefficients y_i have been calculated at NLO [21, 22]. The short-distance part is therefore quite well under control. The remaining problem is then the computation of matrix elements, in particular B_6 and B_8. The cancellation between these contributions enhances the relative sensitivity of ε'/ε to the anyhow uncertain hadronic parameters which makes a precise calculation of ε'/ε impossible at present. The results found in various recent analyses are collected in Table 3.

Recently, the issue of the strange quark mass has received increased attention due to new lattice results reporting lower than anticipated values. As we have seen in (15), (16) the matrix elements of Q_6 and Q_8 are expected to behave as $1/m_s^2$, up to B-factors. This result is based on the factorization ansatz, which holds in the large N_C limit of QCD, and reflects the particular, scalar-current type structure of Q_6 and Q_8. The phenomenological predictions thus show a marked dependence on the strange quark mass used in the analysis. Generally ε'/ε will increase with decreasing m_s. The estimates for ε'/ε in Table 3 are based on strange quark masses in the ball park of $m_s(2GeV) = 130\,MeV$. Table 4 collects a few recent determinations of m_s from QCD sum rules and from lattice calculations.

Using the low m_s values indicated by the very recent Los Alamos and Fermilab lattice results Buras et al. [24] find

$$0 \leq \varepsilon'/\varepsilon \leq 43 \cdot 10^{-4} \text{(scanning)} \tag{17}$$

$$-2.1 \cdot 10^{-4} \leq \varepsilon'/\varepsilon \leq 18.7 \cdot 10^{-4} \text{(Gaussian)} \tag{18}$$

for $m_s(2\,GeV) = (86 \pm 17)\,MeV$. This is compatible with both experimental results (8), within the rather large uncertainties. Using $m_s(2\,GeV)$ around $130\,MeV$, on the other hand,
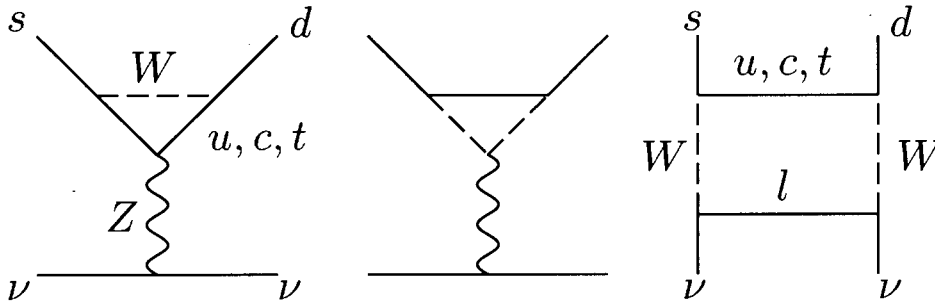
Figure 3: *Leading order electroweak diagrams contributing to $K \to \pi\nu\bar\nu$ in the standard model.*

the results are consistent with E731, but somewhat low compared to NA31 (see the first line of Table 3).

In conclusion, the SM prediction for ε'/ε suffers from large hadronic uncertainties, reinforced by substantial cancellations between the $I = 0$ and $I = 2$ contributions. Despite this problem, the characteristic pattern of CP violation observed in $K \to \pi\pi$ decays, namely $\varepsilon = \mathcal{O}(10^{-3})$ and $\varepsilon' = \mathcal{O}(10^{-6})$ (or below), is well accounted for by the standard theory, which can be considered a non-trivial success of the model.

On the experimental side a clarification of the current situation is to be expected by the upcoming new round of ε'/ε experiments conducted at Fermilab (E832), CERN (NA48) and Frascati (KLOE). The goal is a measurement of ε'/ε at the 10^{-4} level. The demonstration that $\varepsilon' \neq 0$ would constitute a qualitatively new feature of CP violation and as such be of great importance. However, due to the large uncertainties in the theoretical calculation, a quantitative use of this result for the extraction of CKM parameters will unfortunately be severely limited. For this purpose one has to turn to theoretically cleaner observables. As we will see in the next section, rare K decays in fact offer very promising opportunities in this direction.

4 · The Rare Decays $K^+ \to \pi^+\nu\bar\nu$ and $K_L \to \pi^0\nu\bar\nu$

The decays $K \to \pi\nu\bar\nu$ proceed through flavor changing neutral current effects. These arise in the standard model only at second (one-loop) order in the electroweak interaction (Z-penguin and W-box diagrams, Figure 3) and are additionally GIM suppressed. The branching fractions are thus very small, at the level of 10^{-10}, which makes these modes rather challenging to detect. However, $K \to \pi\nu\bar\nu$ have long been known to be reliably calculable, in contrast to most other decay modes of interest. A measurement of $K^+ \to \pi^+\nu\bar\nu$ and $K_L \to \pi^0\nu\bar\nu$ will therefore be an extremely useful test of flavor physics. Over the recent years important refinements have been added to the theoretical treatment of $K \to \pi\nu\bar\nu$. These have helped to precisely quantify the meaning of the term 'clean' in this context and have reinforced the unique potential of these observables. Let us briefly summarize the main aspects of why $K \to \pi\nu\bar\nu$ is theoretically so favorable and what recent developments have contributed to emphasize this point.

Table 5: *Compilation of important properties and results for $K \to \pi\nu\bar{\nu}$.*

	$K^+ \to \pi^+\nu\bar{\nu}$	$K_L \to \pi^0\nu\bar{\nu}$
	CP conserving	CP violating
CKM	V_{td}	$\mathrm{Im}V_{ts}^*V_{td} \sim J_{CP} \sim \eta$
contributions	top and charm	only top
scale uncert. (BR)	$\pm20\%$ (LO) $\to \pm5\%$ (NLO)	$\pm10\%$ (LO) $\to \pm1\%$ (NLO)
BR (SM)	$(0.9 \pm 0.3) \cdot 10^{-10}$	$(2.8 \pm 1.7) \cdot 10^{-11}$
exp. limit	$< 2.4 \cdot 10^{-9}$ BNL 787 [48]	$< 5.8 \cdot 10^{-5}$ FNAL 799 [49]

- First, $K \to \pi\nu\bar{\nu}$ is semileptonic. The relevant hadronic matrix elements $\langle \pi|(\bar{s}d)_{V-A}|K\rangle$ are just matrix elements of a current operator between hadronic states, which are already considerably simpler objects than the matrix elements of four-quark operators encountered in many other observables ($K - \bar{K}$ mixing, ε'/ε). Moreover, they are related to the matrix element

$$\langle \pi^0|(\bar{s}u)_{V-A}|K^+\rangle \tag{19}$$

by isospin symmetry. The latter quantity can be extracted from the well measured leading semileptonic decay $K^+ \to \pi^0 l\nu$. Although isospin is a fairly good symmetry, it is still broken by the small up- down quark mass difference and by electromagnetic effects. These manifest themselves in differences of the neutral versus charged kaon (pion) masses (affecting phase space), corrections to the isospin limit in the formfactors and electromagnetic radiative effects. Marciano and Parsa [40] have analyzed these corrections and found an overall reduction in the branching ratio by 10% for $K^+ \to \pi^+\nu\bar{\nu}$ and 5.6% for $K_L \to \pi^0\nu\bar{\nu}$.

- Long distance contributions are systematically suppressed as $\mathcal{O}(\Lambda_{QCD}^2/m_c^2)$ compared to the charm contribution (which is part of the short distance amplitude). This feature is related to the hard ($\sim m_c^2$) GIM suppression pattern shown by the Z-penguin and W-box diagrams, and the absence of virtual photon amplitudes. Long distance contributions have been examined quantitatively [41, 42, 43, 44, 45] and shown to be numerically negligible (below $\approx 5\%$ of the charm amplitude).

- The preceeding discussion implies that $K \to \pi\nu\bar{\nu}$ are short distance dominated (by top- and charm-loops in general). The relevant short distance QCD effects can be treated in perturbation theory and have been calculated at next-to-leading order [46, 47]. This allowed to substantially reduce (for K^+) or even practically eliminate (K_L) the leading order scale ambiguities, which are the dominant uncertainties in the leading order result.

In Table 5 we have summarized some of the main features of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. The neutral mode proceeds through CP violation in the standard model. This is due to the definite CP properties of K^0, π^0 and the hadronic transition current $(\bar{s}d)_{V-A}$. The violation of CP symmetry in $K_L \to \pi^0\nu\bar{\nu}$ arises through interference between $K^0 - \bar{K}^0$ mixing and the decay amplitude. This mechanism is sometimes refered to as mixing-induced CP violation. Now, in the standard model, the mixing-induced CP violation in $K_L \to \pi^0\nu\bar{\nu}$ is by orders of magnitude larger than the one in $K_L \to \pi^+\pi^-$, for instance. Any difference in the magnitude
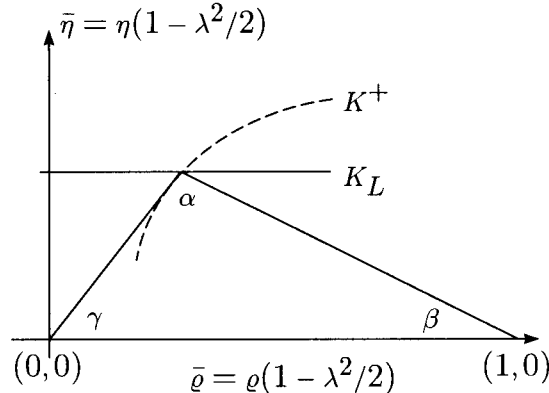
Figure 4: *Unitarity triangle from* $K \to \pi\nu\bar{\nu}$.

of mixing induced CP violation between two K_L decay modes is a signal of direct CP violation. In this sense, the standard model decay $K_L \to \pi^0\nu\bar{\nu}$ is a signal of almost pure direct CP violation, revealing an effect that can not be explained by CP violation in the $K - \bar{K}$ mass matrix alone.

While already $K^+ \to \pi^+\nu\bar{\nu}$ can be reliably calculated, the situation is even better for $K_L \to \pi^0\nu\bar{\nu}$. Since only the imaginary part of the amplitude (in standard phase conventions) contributes, the charm sector, in $K^+ \to \pi^+\nu\bar{\nu}$ the dominant source of uncertainty, is completely negligible for $K_L \to \pi^0\nu\bar{\nu}$ (0.1% effect on the branching ratio). Long distance contributions ($\lesssim 0.1\%$) and also the indirect CP violation effect ($\lesssim 1\%$) are likewise negligible. In summary, the total theoretical uncertainties, from perturbation theory in the top sector and in the isospin breaking corrections, are safely below $2 - 3\%$ for $B(K_L \to \pi^0\nu\bar{\nu})$. This makes this decay mode truly unique and very promising for phenomenological applications. (Note that the range given as the standard model prediction in Table 5 arises from our, at present, limited knowledge of standard model parameters (CKM), and not from intrinsic uncertainties in calculating $B(K_L \to \pi^0\nu\bar{\nu})$).

With a measurement of $B(K^+ \to \pi^+\nu\bar{\nu})$ and $B(K_L \to \pi^0\nu\bar{\nu})$ available very interesting phenomenological studies could be performed. For instance, $B(K^+ \to \pi^+\nu\bar{\nu})$ and $B(K_L \to \pi^0\nu\bar{\nu})$ together determine the unitarity triangle (Wolfenstein parameters ϱ and η) completely (Figure 4). The expected accuracy with $\pm 10\%$ branching ratio measurements is comparable to the one that can be achieved by CP violation studies at B factories before the LHC era [50]. The quantity $B(K_L \to \pi^0\nu\bar{\nu})$ by itself offers probably the best precision in determining $\mathrm{Im}V_{ts}^* V_{td}$ or, equivalently, the Jarlskog parameter

$$J_{CP} = \mathrm{Im}(V_{ts}^* V_{td} V_{us} V_{ud}^*) = \lambda \left(1 - \frac{\lambda^2}{2}\right) \mathrm{Im}\lambda_t \qquad (20)$$

The prospects here are even better than for B physics at the LHC. As an example, let us assume the following results will be available from B physics experiments

$$\sin 2\alpha = 0.40 \pm 0.04 \quad \sin 2\beta = 0.70 \pm 0.02 \quad V_{cb} = 0.040 \pm 0.002 \qquad (21)$$

The small errors quoted for $\sin 2\alpha$ and $\sin 2\beta$ from CP violation in B decays require precision measurements at the LHC. In the case of $\sin 2\alpha$ we have to assume in addition that the

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theoretical problem of 'penguin-contamination' can be resolved. These results would then imply $\mathrm{Im}\lambda_t = (1.37 \pm 0.14) \cdot 10^{-4}$. On the other hand, a $\pm 10\%$ measurement $B(K_L \to \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.3) \cdot 10^{-11}$ together with $m_t(m_t) = (170 \pm 3) GeV$ would give $\mathrm{Im}\lambda_t = (1.37 \pm 0.07) \cdot 10^{-4}$. If we are optimistic and take $B(K_L \to \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.15) \cdot 10^{-11}$, $m_t(m_t) = (170 \pm 1) GeV$, we get $\mathrm{Im}\lambda_t = (1.37 \pm 0.04) \cdot 10^{-4}$, a truly remarkable accuracy. The prospects for precision tests of the standard model flavor sector will be correspondingly good.

The charged mode $K^+ \to \pi^+ \nu \bar{\nu}$ is being currently pursued by Brookhaven experiment E787. The latest published result [48] gives an upper limit which is about a factor 25 above the standard model range. Several improvements have been implemented since then and the SM sensitivity is expected to be reached in the near future [51]. For details see the contribution of S. Kettell (these proceedings). Recently an experiment has been proposed to measure $K^+ \to \pi^+ \nu \bar{\nu}$ at the Fermilab Main Injector [52]. Concerning $K_L \to \pi^0 \nu \bar{\nu}$, a proposal exists at Brookhaven (BNL E926) to measure this decay at the AGS with a sensitivity of $\mathcal{O}(10^{-12})$ (see [51]). There are furthermore plans to pursue this mode with comparable sensitivity at Fermilab [53] and KEK [54]. It will be very exciting to follow the development and outcome of these ambitious projects. The 'holy grail of kaon physics' could finally come within reach.

5 $\quad K_L \to \pi^0 e^+ e^-$ and $K_L \to \mu^+ \mu^-$

5.1 $\quad K_L \to \pi^0 e^+ e^-$

This decay mode has obvious similarities with $K_L \to \pi^0 \nu \bar{\nu}$ and the apparent experimental advantage of charged leptons, rather than neutrinos, in the final state. However there are a number of quite serious difficulties associated with this very fact. Unlike neutrinos the electron couples to photons. As a consequence the amplitude, which was essentially purely short distance in $K_L \to \pi^0 \nu \bar{\nu}$, becomes sensitive to poorly calculable long distance physics (photon penguin). Simultaneously the importance of indirect CP violation ($\sim \varepsilon$) is strongly enhanced and furthermore a long distance dominated, CP conserving amplitude with two-photon intermediate state can contribute significantly. Treating $K_L \to \pi^0 e^+ e^-$ theoretically one is thus faced with the need to disentangle three different contributions of roughly the same order of magnitude.

- Direct CP violation: This part is short distance in character, theoretically clean and has been analyzed at next-to-leading order in QCD [55]. Taken by itself this mechanism leads to a $K_L \to \pi^0 e^+ e^-$ branching ratio of $(4.5 \pm 2.6) \cdot 10^{-12}$ within the standard model.

- Indirect CP violation: This amplitude is determined through $\sim \varepsilon \cdot A(K_S \to \pi^0 e^+ e^-)$. The K_S amplitude is dominated by long distance physics and has been investigated in chiral perturbation theory [56, 57, 58]. Due to unknown counterterms that enter this analysis a reliable prediction is not possible at present. The situation would improve with a measurement of $B(K_S \to \pi^0 e^+ e^-)$, which could become possible at DAΦNE. Present day estimates for $B(K_L \to \pi^0 e^+ e^-)$ due to indirect CP violation alone give typically values of $(1 - 5) \cdot 10^{-12}$.

- The CP conserving two-photon contribution is again long-distance dominated. It has been analyzed by various authors [58, 59, 60]. The estimates are typically a few 10^{-12}.

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Improvements in this sector might be possible by further studying the related decay $K_L \to \pi^0 \gamma\gamma$ whose branching ratio has already been measured to be $(1.7 \pm 0.3) \cdot 10^{-6}$.

Originally it had been hoped for that the direct CP violating contribution is dominant. Unfortunately this could so far not be unambiguously established and requires further study. Besides the theoretical problems, $K_L \to \pi^0 e^+ e^-$ is also very hard from an experimental point of view. The expected branching ratio is even smaller than for $K_L \to \pi^0 \nu\bar{\nu}$. Furthermore a serious irreducible physics background from the radiative mode $K_L \to e^+ e^- \gamma\gamma$ has been identified, which poses additional difficulties [61]. A background subtraction seems necessary, which is possible with enough events. Additional information could in principle also be gained by studying the electron energy asymmetry [58, 60] or the time evolution [58, 62, 63].

5.2 $K_L \to \mu^+ \mu^-$

$K_L \to \mu^+ \mu^-$ receives a short distance contribution from Z-penguin and W-box graphs similar to $K \to \pi\nu\bar{\nu}$. This part of the amplitude is sensitive to the Wolfenstein parameter ϱ. In addition $K_L \to \mu^+ \mu^-$ proceeds through a long distance contribution with two-photon intermediate state, which actually dominates the decay completely. The long distance amplitude consists of a dispersive (A_{dis}) and an absorbtive contribution (A_{abs}). The branching fraction can thus be written

$$B(K_L \to \mu^+ \mu^-) = |A_{SD} + A_{dis}|^2 + |A_{abs}|^2 \tag{22}$$

Using $B(K_L \to \gamma\gamma)$ it is possible to extract $|A_{abs}|^2 = (6.8 \pm 0.3) \cdot 10^{-9}$ [61]. A_{dis} on the other hand can not be calculated accurately at present and the estimates are strongly model dependent [64, 65, 66, 67, 68]. This is rather unfortunate, in particular since $B(K_L \to \mu^+ \mu^-)$, unlike most other rare decays, has already been measured, and this with very good precision

$$B(K_L \to \mu^+ \mu^-) = \begin{cases} (6.9 \pm 0.4) \cdot 10^{-9} & \text{BNL 791 [69]} \\ (7.9 \pm 0.7) \cdot 10^{-9} & \text{KEK 137 [70]} \end{cases} \tag{23}$$

For comparison we note that $B(K_L \to \mu^+ \mu^-)_{SD} = (1.3 \pm 0.6) \cdot 10^{-9}$ is the expected branching ratio in the standard model based on the short-distance contribution alone. Due to the fact that A_{dis} is largely unknown, $K_L \to \mu^+ \mu^-$ is at present not a very useful constraint on CKM parameters.

6 The Radiative Rare Decay $B \to X_s \gamma$

The radiative decay $B \to X_s \gamma$ is justifiably one of the highlights in the field of flavor changing neutral currents. First of all, its rate is of order $G_F^2 \alpha$, while most other FCNC processes are only $\sim G_F^2 \alpha^2$. This leads to a relatively sizable branching fraction of $\mathcal{O}(10^{-4})$. The decay is accessible to experiment already today and its branching ratio has been measured at CLEO [71]

$$B(B \to X_s \gamma) = (2.32 \pm 0.67) \cdot 10^{-4} \tag{24}$$

At the same time the inclusive transition $B \to X_s \gamma$ can be systematically treated by standard theoretical techniques such as heavy quark expansion and renormalization group improved

perturbation theory. It is sensitive to short distance physics and provides therefore a good test of flavordynamics. Extensions of the standard model, for instance the Two-Higgs-Doublet Model [72, 73, 74, 75, 76], models with three Higgs-doublets [77], minimal SUSY [78, 79, 80] or left-right symmetric models [81] receive important constraints from $B \to X_s\gamma$.

In the following we shall briefly sketch the theoretical status of $B \to X_s\gamma$ in the standard model.

The basic structure of the $b \to s\gamma$ transition is quite interesting from a theoretical point of view. Schematically one has, in the leading log approximation [82]

$$B(B \to X_s\gamma) \sim \left| F(m_t) + \sum_n \sim \left(\alpha_s \ln \frac{M_W}{\mu} \right)^n \right|^2 \tag{25}$$

$F(m_t)$ is a function describing the top quark mass dependence. The large logarithmic QCD corrections $\sim \alpha_s \ln(M_W/\mu)$, $\mu = \mathcal{O}(m_b)$, are resummed to all orders. Their contribution is formally $\mathcal{O}(1)$, of the same order as $F(m_t)$. Technically these effects, although of leading order, are generated from two-loop contributions, whereas usually leading logarithmic effects arise at the one-loop level. This peculiarity is due to the radiative nature of the FCNC in $b \to s\gamma$. Numerically one finds $B(B \to X_s\gamma) \approx 1.2 \cdot 10^{-4}$ neglecting all QCD effects, but $\approx 2.8 \cdot 10^{-4}$ including the tower of leading logarithmic corrections. This illustrates the decisive impact of short distance QCD effects on the prediction of $B(B \to X_s\gamma)$. With this feature $B \to X_s\gamma$ is the prototype example for the importance of perturbative QCD corrections in weak decays. A somewhat unwelcome side effect of the predominance of QCD contributions is the rather strong scale (μ) ambiguity of the result at leading order [83, 76], implying an uncertainty of $\pm 25\%$ in the braching fraction (for $m_b/2 \leq \mu \leq m_b$). This is the dominant uncertainty in the leading order prediction of $B(B \to X_s\gamma)$. Several other, somewhat less prominent sources of error exist.

- Long distance contributions arise from intermediate $(c\bar{c})$ bound states coupling to the on-shell photon. Their impact on the branching ratio is expected to be of the order $\lesssim 10\%$ [84, 85, 86].

- The theoretical prediction of $B(B \to X_s\gamma)$ is normalized to $B(B \to X_c l\nu)$, which depends on m_c/m_b. The corresponding error is about 6%.

- The ratio $|V_{ts}^* V_{tb}/V_{cb}|^2$ entering $B(B \to X_s\gamma)/B(B \to X_c l\nu)$ is quite well constrained to 0.95 ± 0.03 from CKM unitarity and using input from ε_K and $B - \bar{B}$ mixing.

- The uncertainty from the error in $\alpha_s(M_Z)$ is $\lesssim 10\%$. The errors due to the experimental values for $B(B \to X_c l\nu)$ and m_t are small.

- Non-perturbative contributions to $B(B \to X_s\gamma)$ from subleading terms $(\sim 1/m_b^2)$ in the heavy quark expansion have also been analyzed [87]. They are likewise negligible.

The essential step for further improvement is therefore a complete and consistent NLO calculation. For an overview of the various parts of such an analysis and detailed references see [8]. The last two major ingredients in this very complex calculation have recently been performed and results were reported at the 28th International Conference on High Energy

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Physics (ICHEP 96) in Warsaw. NLO QCD corrections to the matrix elements have been addressed by Greub, Hurth and Wyler [88] and the three-loop contribution to the NLO renormalization group evolution has been worked out by Chetyrkin, Misiak and Münz [89]. The *preliminary* result reads

$$B(B \to X_s\gamma) = (3.3 \pm 0.5) \cdot 10^{-4} \qquad \text{(NLO, preliminary)} \qquad (26)$$

The error represents the total uncertainty, including the one from residual scale dependence. The latter has decreased as expected, from $\pm 25\%$ to about $\pm 6\%$ after incorporating the NLO corrections. Eq. (26) can be compared with the leading order result $B(B \to X_s\gamma)_{LO} = (2.8 \pm 0.8) \cdot 10^{-4}$ and with the experimental number in (24). Although the central value of (26) is apparently higher than the experimental $2.32 \cdot 10^{-4}$, it is still premature to draw definitive conclusions.

Exclusive channels, such as $B \to K^*\gamma$, have also been studied [90, 91, 92, 93], but are more difficult from a theoretical point of view.

7 The Rare Decays $B_s \to \mu^+\mu^-$ and $B \to X_s\nu\bar\nu$

These decays are both theoretically very clean since they are entirely dominated by virtual top contributions which proceed at very short distances. The relevant Feynman graphs are Z-penguin and W-box diagrams similar to those for $K \to \pi\nu\bar\nu$. Next-to-leading order QCD corrections essentially eliminate the leading order scale uncertainty of $\pm 10\%$ to merely $\pm 1\%$ in the branching ratios [46].

The branching ratio for $B_s \to \mu^+\mu^-$ is proportional to $|V_{ts}|^2$ and $f_{B_s}^2$. Detailed expressions can be found in [8]. The standard model expectation is $B(B_s \to \mu^+\mu^-) = (3.6 \pm 1.8) \cdot 10^{-9}$, based on $f_{B_s} = (210 \pm 30) \, MeV$. The current experimental upper limit on the branching ratio is $8.4 \cdot 10^{-6}$ [94].

For the related mode $B_d \to \mu^+\mu^-$ the theoretical prediction is about an order of magnitude lower than for $B_s \to \mu^+\mu^-$ and an upper limit of $1.6 \cdot 10^{-6}$ has been set by CDF [94]. The decays could become accessible at the LHC. Their ratio

$$\frac{B(B_d \to \mu^+\mu^-)}{B(B_s \to \mu^+\mu^-)} = \frac{\tau(B_d)}{\tau(B_s)}\frac{m_{B_d}}{m_{B_s}} \cdot \frac{f_{B_d}^2}{f_{B_s}^2}\left|\frac{V_{td}}{V_{ts}}\right|^2 \qquad (27)$$

is a measure of $|V_{td}/V_{ts}|$, once $SU(3)$ breaking effects in f_{B_d}/f_{B_s} are properly taken into account. Results for other final states, $B_{d,s} \to e^+e^-$ or $\tau^+\tau^-$ are summarized in [8].

The inclusive decay $B \to X_s\nu\bar\nu$ is similar to $B_s \to \mu^+\mu^-$. The disadvantage is a more challenging experimental signature. Advantages of $B \to X_s\nu\bar\nu$ over $B_s \to \mu^+\mu^-$, on the other hand, are the absence of the strong helicity suppression, resulting in a much larger branching fraction, and the inclusive nature of the decay, which allows a reliable calculation of the matrix element with heavy quark expansion and perturbative QCD. The ratio of $B(B \to X_s\nu\bar\nu)/B(B \to X_d\nu\bar\nu)$ is a clean measure of $|V_{td}/V_{ts}|$.

The decay $B \to X_s\nu\bar\nu$ received renewed interest after a proposal to extract an upper limit on its branching fraction from available data by Grossman et al. [95]. Subsequently this led to an upper bound of $7.7 \cdot 10^{-4}$ by the ALEPH collaboration [96], already quite close to the

standard model range $B(B \to X_s \nu \bar{\nu}) = (3.8 \pm 0.8) \cdot 10^{-5}$. The result constrains scenarios of new physics [95]. In view of the experimental situation and the theoretically clean character $B \to X_s \nu \bar{\nu}$ clearly deserves further attention.

8   Other Opportunities

There are several other possibilities to investigate flavor physics by studying rare decay modes.

In the field of B decays the inclusive mode $B \to X_s l^+ l^-$ ($l = e, \mu, \tau$), for instance, has been widely discussed in the literature. The next-to-leading order QCD corrections are known [97, 98]. The decay branching ratio, dilepton invariant mass spectrum, forward-backward charge asymmetry and lepton polarization could be useful probes of the standard model and its extensions [99, 100, 101, 102, 103].

A particular class of rare kaon decays are the modes $K_L \to \mu e$, $K^+ \to \pi^+ \mu e$ and $K_L \to \pi^0 \mu e$, which violate lepton flavor and are altogether forbidden in the standard model. Current limits for their branching ratios are at the level of $\sim 10^{-10}$. They will be improved by future experiments (see the talks by S. Pislak, W. Molzon and E. Ramberg, these proceedings) down to the 10^{-12} level, corresponding to a sensitivity to scales of typically a few hundred TeV. This might be a way to probe, albeit indirectly, high energy scales not accessible by any other method.

Besides K and B physics, also D mesons might yield interesting clues on flavordynamics. Here standard model effects are generally very small and long-distance contributions usually play an essential role. Still the charmed meson sector could provide a window for new physics. This topic has been reviewed by Burdman [104], where more details and references can be found. A general reference for new physics in FCNC processes is Hewett et al. [105].

9   Summary

We have reviewed the present status of CP violation in kaon decays and discussed selected rare decays of both K and B mesons. To conclude we summarize some of the main issues.

- The field of CP violation and rare decays is an important probe of flavordynamics.

- Short distance QCD corrections have by now been calculated at next-to-leading order for almost all cases of practical interest.

- So far the parameter ε in the neutral kaon system is still the only signal of CP violation observed in the laboratory. Important phenomenological constraints can be derived from this measurement.

- The situation of whether ε'/ε is zero or not will soon be clarified experimentally with an accuracy of 10^{-4}. This could establish an important, qualitatively new aspect of CP violation. The quantitative use of this result for the extraction of CKM parameters, however, is severely limited by large hadronic uncertainties.

- Precise extractions of CKM quantities along with accurate standard model tests will be possible with theoretically clean observables. A prime candidate is the 'golden reaction' $K_L \to \pi^0 \nu \bar{\nu}$, which is in particular an ideal measure of the Jarlskog parameter J_{CP}.

- Complemetary information from as many other sources as possible is needed and could be provided for by CP violation studies with B decays and various rare decays like $K^+ \to \pi^+ \nu \bar{\nu}$, $B \to X_s \gamma$, $B \to X_s \nu \bar{\nu}$ or $B \to X_s \mu^+ \mu^-$.

In our presentation we have largely focussed on such decays that can be calculated reliably. In this spirit one may group the various observables, roughly, into classes according to their theoretical 'cleanliness':

- Class 1 ('gold plated'): $K_L \to \pi^0 \nu \bar{\nu}$; $K^+ \to \pi^+ \nu \bar{\nu}$, $B \to X_{s,d} \nu \bar{\nu}$

- Class 2 (very clean): $B(B_d \to l^+ l^-)/B(B_s \to l^+ l^-)$; $\Delta M_{B_d}/\Delta M_{B_s}$

- Class 3 (moderate uncertainties and/or improvements possible): ε, $B \to X_s \gamma$; $K_L \to \pi^0 e^+ e^-$, $B \to X_s l^+ l^-$

- Class 4 (large hadronic uncertainties): ε'/ε, $K_L \to \mu^+ \mu^-$

The quantities that have already been measured, ε, $B \to X_s \gamma$, $K_L \to \mu^+ \mu^-$, or that are about to be observed (ε'/ε, $K^+ \to \pi^+ \nu \bar{\nu}$) are seen to cluster mainly in the lower part of this list. Let us hope that future experimental developments will eventually map out the full range of possibilities, including the unique instances where unambiguous and clear theoretical predictions can be made.

Acknowledgements

References

[1] M. Neubert, Phys. Lett. **B338**, 84 (1994).

[2] M. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. **D51**, 2217 (1995).

[3] P. Ball, M. Beneke and V.M. Braun, Phys. Rev. **D52**, 3929 (1995).

[4] J.P. Alexander *et al.* (CLEO), CLNS 96/1419, CLEO 96-9 (1996).

[5] P. Tipton, talk presented at the International Conference on High Energy Physics, Warsaw (1996).

[6] S.R. Sharpe, Talk given at Lattice 96, St. Louis, MO, hep-lat/9609029.

[7] S. Aoki *et al.*, Talk given at Lattice 96, St. Louis, MO, hep-lat/9608134.

[8] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[9] A.J. Buras, M. Jamin and M.E. Lautenbacher, to appear.

[10] G.D. Barr *et al.*, Phys. Lett. **B317**, 233 (1993).

[11] L.K. Gibbons *et al.*, Phys. Rev. Lett. **70**, 1203 (1993).

[12] S. Herrlich and U. Nierste, Nucl. Phys. **B419**, 292 (1994).

[13] A.J. Buras, M. Jamin and P.H. Weisz, Nucl. Phys. **B347**, 491 (1990).

[14] S. Herrlich and U. Nierste, Nucl. Phys. **B476**, 27 (1996).

[15] W.A. Bardeen, A.J. Buras and J.-M. Gérard, Phys. Lett. **B211**, 343 (1988).

[16] J.-M. Gérard, Acta Physica Polonica **B21**, 257 (1990).

[17] J. Bijnens and J. Prades, Nucl. Phys. **B444**, 523 (1995).

[18] A.J. Buras, Phys. Lett. **B317**, 449 (1993).

[19] J.M. Flynn and L. Randall, Phys. Lett. **B224**, 221 (1989); erratum ibid. **B235**, 412 (1990).

[20] G. Buchalla, A.J. Buras and M.K. Harlander, Nucl. Phys. **B337**, 313 (1990).

[21] A.J. Buras *et al.*, Nucl. Phys. **B400**, 37 (1993); A.J. Buras, M. Jamin and M.E. Lautenbacher, Nucl. Phys. **B400**, 75 (1993).

[22] M. Ciuchini *et al.*, Nucl. Phys. **B415**, 403 (1994).

[23] A.J. Buras and J.-M. Gérard, Phys. Lett. **B192**, 156 (1987).

[24] A.J. Buras, M. Jamin and M.E. Lautenbacher, hep-ph/9608365.

[25] E. Franco *et al.*, Nucl. Phys. **B317**, 63 (1989).

[26] C. Bernard and A. Soni, Nucl. Phys. **B (Proc. Suppl.) 9**, 155 (1989).

[27] G.W. Kilcup, Nucl. Phys. **B (Proc. Suppl.) 20**, 417 (1991).

[28] S.R. Sharpe, Nucl. Phys. **B (Proc. Suppl.) 20**, 429 (1991).

[29] M. Ciuchini *et al.*, Z. Phys. **C68**, 239 (1995).

[30] S. Bertolini, J.O. Eeg and M. Fabbrichesi, Nucl. Phys. **B449**, 197 (1995).

[31] S. Bertolini, J.O. Eeg and M. Fabbrichesi, Nucl. Phys. **B476**, 225 (1996).

[32] J. Fröhlich *et al.*, University of Dortmund preprint, DO-TH-91-02 (1991).

[33] J. Heinrich *et al.*, Phys. Lett. **B279**, 140 (1992); E.A. Paschos, DO-TH 96/01, talk presented at the 27th Lepton-Photon Symposium, Beijing, China (1995).

[34] M. Jamin and M. Münz, Z. Phys. **C66**, 633 (1995).

[35] K.G. Chetyrkin *et al.*, Phys. Rev. **D51**, 5090 (1995).

[36] S. Narison, Phys. Lett. **B358**, 113 (1995).

[37] C.R. Allton *et al.*, Nucl. Phys. **B431**, 667 (1994).

[38] R. Gupta and T. Bhattacharya, hep-lat/9605039.

[39] B.J. Gough *et al.*, hep-lat/9610223.

[40] W. Marciano and Z. Parsa, Phys. Rev. **D53**, R1 (1996).

[41] D. Rein and L.M. Sehgal, Phys. Rev. **D39**, 3325 (1989).

[42] J.S. Hagelin and L.S. Littenberg, Prog. Part. Nucl. Phys. **23**, 1 (1989).

[43] M. Lu and M. Wise, Phys. Lett. **B324**, 461 (1994).

[44] C.Q. Geng, I.J. Hsu and Y.C. Lin, Phys. Lett. **B355**, 569 (1995).

[45] S. Fajfer, HU-SEFT-R-1996-05, hep-ph/9602322.

[46] G. Buchalla and A.J. Buras, Nucl. Phys. **B400**, 225 (1993).

[47] G. Buchalla and A.J. Buras, Nucl. Phys. **B412**, 106 (1994).

[48] S. Adler et al., Phys. Rev. Lett. **76**, 1421 (1996).

[49] M. Weaver et al., Phys. Rev. Lett. **72**, 3758 (1994).

[50] G. Buchalla and A.J. Buras, Phys. Rev. **D54**, 6782 (1996).

[51] L. Littenberg and J. Sandweiss, eds., AGS-2000, Experiments for the 21st Century, BNL 52512

[52] P. Cooper, M. Crisler, B. Tschirhart and J. Ritchie (CKM collaboration), EOI for measuring $B(K^+ \to \pi^+\nu\bar{\nu})$ at the Main Injector, Fermilab EOI 14, 1996

[53] K. Arisaka et al., KAMI conceptual design report, FNAL, June 1991

[54] T. Inagaki, T. Sato and T. Shinkawa, Experiment to search for the decay $K_L \to \pi^0\nu\bar{\nu}$ at KEK 12 GeV proton synchrotron, 30 Nov. 1991

[55] A.J. Buras *et al.*, Nucl. Phys. **B423**, 349 (1994).

[56] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B303**, 665 (1988).

[57] C. Bruno and J. Prades, Z. Phys. **C57**, 585 (1993).

19

[58] J.F. Donoghue and F. Gabbiani, Phys. Rev. **D51**, 2187 (1995).

[59] A.G. Cohen, G. Ecker and A. Pich, Phys. Lett. **B304**, 347 (1993).

[60] P. Heiliger and L. Sehgal, Phys. Rev. **D47**, 4920 (1993).

[61] L.S. Littenberg and G. Valencia, Ann. Rev. Nucl. Part. Sci. **43**, 729 (1993)

[62] L.S. Littenberg, in *Proceedings of the Workshop on CP Violation at a Kaon Factory*, ed. J.N. Ng, TRIUMF, Vancouver, Canada (1989), p. 19.

[63] G.O. Köhler and E.A. Paschos, Phys. Rev. **D52**, 175 (1995).

[64] L. Bergström, E. Massó and P. Singer, Phys. Lett. **B249**, 141 (1990).

[65] C.Q. Geng and J.N. Ng, Phys. Rev. **D41**, 2351 (1990).

[66] G. Bélanger and C.Q. Geng, Phys. Rev. **D43**, 140 (1991).

[67] P. Ko, Phys. Rev. **D45**, 174 (1992).

[68] J.O. Eeg, K. Kumericki and I. Picek, hep-ph/9605337

[69] A.P. Heinson *et al.*, Phys. Rev. **D51**, 985 (1995).

[70] T. Akagi *et al.*, Phys. Rev. **D51**, 2061 (1995).

[71] M.S. Alam *et al.*, Phys. Rev. Lett. **74**, 2885 (1995).

[72] B. Grinstein, R. Springer and M.B. Wise, Nucl. Phys. **B339**, 269 (1990).

[73] J.L. Hewett, Phys. Rev. Lett. **70**, 1045 (1993).

[74] V. Barger, M.S. Berger and R.J.N. Phillips, Phys. Rev. Lett. **70**, 1368 (1993).

[75] T. Hayashi, M. Matsuda and M. Tanimoto, Prog. Theo. Phys. **89**, 1047 (1993).

[76] A.J. Buras *et al.*, Nucl. Phys. **B424**, 374 (1994).

[77] Y. Grossman and Y. Nir, Phys. Lett. **B313**, 126 (1993).

[78] S. Bertolini *et al.*, Nucl. Phys. **B353**, 591 (1991).

[79] R. Barbieri and G.F. Giudice, Phys. Lett. **B309**, 86 (1993).

[80] F. Borzumati, Z. Phys. **C63**, 291 (1994).

[81] P. Cho and M. Misiak, Phys. Rev. **D49**, 5894 (1994).

[82] M. Ciuchini *et al.*, Nucl. Phys. **B421**, 41 (1994).

[83] A. Ali and C. Greub, Z. Phys. **C60**, 433 (1993).

[84] N.G. Deshpande, X.-G. He and J. Trampetić, Phys. Lett. **B367**, 362 (1996).

[85] J.M. Soares, Phys. Rev. **D53**, 241 (1996).

[86] G. Eilam *et al.*, Phys. Rev. **D53**, 3629 (1996).

[87] I.I. Bigi *et al.*, in *B-Decays* (2nd edition), ed. S.L. Stone, World Scientific, Singapore (1994), p. 132.

[88] C. Greub, T. Hurth and D. Wyler, Phys. Rev. **D54**, 3350 (1996).

[89] K.G. Chetyrkin, M. Misiak and M. Münz, in preparation.

[90] P. Ball, hep-ph/9308244.

[91] A. Ali, V.M. Braun and H. Simma, Z. Phys. **C63**, 437 (1994).

[92] S. Narison, Phys. Lett. **B327**, 354 (1994).

[93] R.N. Faustov, V.O. Galkin and A.Yu. Mishurov, Nucl. Phys. **B (Proc. Suppl.) 49**, 144 (1996).

[94] J. Kroll *et al.*, Fermilab preprint (1995), FERMILAB-CONF-95/229-E

[95] Y. Grossman, Z. Ligeti and E. Nardi, Nucl. Phys. **B465**, 369 (1996); erratum ibid. **B480**, 753 (1996).

[96] ALEPH Collaboration, PA10-019, 1996, Contribution to the International Conference on High Energy Physics, Warsaw, Poland.

[97] A.J. Buras and M. Münz, Phys. Rev. **D52**, 186 (1995).

[98] M. Misiak, Nucl. Phys. **B393**, 23 (1993); erratum ibid. **B439**, 461 (1995).

[99] A. Ali, G.F. Giudice and T. Mannel, Z. Phys. **C67**, 417 (1995).

[100] J.L. Hewett, Phys. Rev. **D53**, 4964 (1996).

[101] F. Krüger and L.M. Sehgal, Phys. Lett. **B380**, 199 (1996).

[102] P. Cho, M. Misiak and D. Wyler, Phys. Rev. **D54**, 3329 (1996).

[103] J.L. Hewett and J.D. Wells, hep-ph/9610323.

[104] G. Burdman, hep-ph/9508349.

[105] J.L. Hewett, T. Takeuchi and S. Thomas, hep-ph/9603391