

## $K_L \rightarrow \pi^0 \nu \bar{\nu}$ Beyond the Standard Model

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### Abstract

We analyze the decay  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  in a model independent way. If lepton flavor is conserved the final state is (to a good approximation) purely CP even. In that case this decay mode goes mainly through CP violating interference between mixing and decay. Consequently, a theoretically clean relation between the measured rate and electroweak parameters holds in any given model. Specifically,  $\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu})/\Gamma(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \sin^2 \theta$  (up to known isospin corrections), where  $\theta$  is the relative CP violating phase between the  $K - \bar{K}$  mixing amplitude and the  $s \rightarrow d \nu \bar{\nu}$  decay amplitude. The experimental bound on  $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  provides a model independent upper bound:  $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 1.1 \times 10^{-8}$ . In models with lepton flavor violation, the final state is not necessarily a CP eigenstate. Then CP conserving contributions can dominate the decay rate.

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In the Standard Model  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  is dominantly a CP violating decay [1]. The main contributions come from penguin and box diagrams with an intermediate top quark and can be calculated with very little theoretical uncertainty [2,3]. It then provides a clean measurement of the Wolfenstein CP violating parameter  $\eta$  or, equivalently, of the Jarlskog measure of CP violation  $J$  and, together with  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , of the angle  $\beta$  of the unitarity triangle [3]. The Standard Model predictions are  $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (9.1 \pm 3.2) \times 10^{-11}$  and  $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (2.8 \pm 1.7) \times 10^{-11}$  [4]. Such rates are within the reach of near future experiments [4]. The Standard Model contributions to the amplitude are fourth order in the weak coupling and proportional to small CKM matrix elements. Consequently, this decay can be sensitive to New Physics effects [5].

In this paper we study the  $K \rightarrow \pi \nu \bar{\nu}$  decay in a model independent way. We are mainly interested in the question of what can be learned in general if a rate for  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  much larger than the Standard Model prediction is observed. We find that the information from a measurement of the rate is particularly clean and simple to interpret if lepton flavor is conserved. In this case the  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  decay is dominated by CP violation in the interference between mixing and decay. The theoretical calculation of the decay rate is then free of hadronic uncertainties and allows the clean determination of CP violating parameters even in the presence of New Physics. Knowledge of neither magnitudes of decay amplitudes nor strong phases is required. Models with  $Z$ -mediated flavor changing neutral currents serve as an example of these points. In models with lepton flavor violation, the final  $\pi^0 \nu \bar{\nu}$  state is not necessarily a CP eigenstate. We show that in this case the CP conserving contributions can be significant and even dominant. The results are still informative but more complicated to interpret, as they depend on both CP violating and lepton flavor violating parameters. We give an explicit example of models with leptoquarks (or, equivalently, supersymmetry without  $R$ -parity).

Our notation follows refs. [6,7]. We define the decay amplitudes  $A$  and  $\bar{A}$ ,

$$A = \langle \pi^0 \nu \bar{\nu} | H | K^0 \rangle, \quad \bar{A} = \langle \pi^0 \nu \bar{\nu} | H | \bar{K}^0 \rangle. \quad (1)$$

If the final  $\pi^0\nu\bar{\nu}$  is a CP eigenstate then in the CP limit  $|\bar{A}/A| = 1$ ; if it is not then  $\bar{A}$  and  $A$  are not related by a CP transformation. We further define the components of interaction eigenstates in mass eigenstates,  $p$  and  $q$ :

$$K_{L,S} = p|K^0\rangle \mp q|\bar{K}^0\rangle. \quad (2)$$

Note that  $|q/p|$  is measured by the CP asymmetry in  $K_L \rightarrow \pi\ell\nu$  and is very close to unity:  $1 - |q/p| = 2\text{Re } \varepsilon$ . Finally, we define a quantity  $\lambda$ ,

$$\lambda \equiv \frac{q\bar{A}}{pA}. \quad (3)$$

The decay amplitudes of  $K_L$  and  $K_S$  into a final  $\pi^0\nu\bar{\nu}$  state are then

$$\langle\pi^0\nu\bar{\nu}|H|K_{L,S}\rangle = pA \mp q\bar{A}, \quad (4)$$

and the ratio between the corresponding decay rates is

$$\frac{\Gamma(K_L \rightarrow \pi^0\nu\bar{\nu})}{\Gamma(K_S \rightarrow \pi^0\nu\bar{\nu})} = \frac{1 + |\lambda|^2 - 2\text{Re}\lambda}{1 + |\lambda|^2 + 2\text{Re}\lambda}. \quad (5)$$

We first assume that the final state is purely CP even. This is the case to a good approximation when lepton flavor is conserved. In general, a three body final state does not have a definite CP parity. However, for purely left-handed neutrinos (which is presumably the case if neutrinos are massless), the lowest dimension term in the effective Hamiltonian relevant to  $K_L \rightarrow \pi^0\nu\bar{\nu}$  decay is  $K(\partial_\mu\pi)(\bar{\nu}_{iL}\gamma^\mu\nu_{iL})$ . Using the CP transformation properties of the leptonic current, we find that this interaction ‘forces’ the  $\nu_i\bar{\nu}_i$  system into a state of well-defined CP, namely CP even. As far as Lorentz and CP transformation properties are concerned, we can then think of the final  $\pi\nu\bar{\nu}$  state as a two body  $\pi Z^*$  state which, when produced by  $K_L$  decay (namely, carrying total angular momentum  $J = 0$ ), is CP even [8,9]. Higher dimension operators can induce CP conserving contributions. For example,  $K(\partial_\nu\partial_\mu\pi)(\bar{\nu}_{iL}\gamma^\mu\vec{\partial}^\nu\nu_{iL})$  will lead to an amplitude that is proportional to  $p_\pi \cdot (p_\nu - p_{\bar{\nu}})$  and, consequently, to a CP odd final state. However, these contributions are  $\mathcal{O}(m_K^2/m_W^2) \sim 10^{-4}$  compared to the leading CP violating ones and can be safely neglected. (In the Standard

Model this operator arises from the box diagram when external momenta are not neglected.) With massive neutrinos, new CP conserving operators arise, *e.g.*  $K\pi(\bar{\nu}_i\nu_i)$ . The final state is now equivalent (in the Lorentz and CP properties) to a two body  $\pi H^*$  state (where  $H$  is a scalar), which is CP odd. However, this amplitude is proportional to the neutrino mass and again negligible. We conclude then that, for any model where lepton flavor is conserved, the CP conserving transition amplitude for  $K_L \rightarrow \pi^0\nu\bar{\nu}$  is highly suppressed and can be neglected.

If the final state  $\pi^0\nu\bar{\nu}$  is CP even, then  $K_L \rightarrow \pi^0\nu\bar{\nu}$  vanishes in the CP limit. This can be seen directly from eq. (5): if CP is a good symmetry then  $|q/p| = 1$ ,  $|\bar{A}/A| = 1$  and  $\lambda = 1$ . With CP violation we can still neglect CP violation in the mixing ( $|q/p| \neq 1$ ) and in the decay ( $|\bar{A}/A| \neq 1$ ). As mentioned above, the deviation of  $|q/p|$  from unity is experimentally measured and is  $\mathcal{O}(10^{-3})$ . The deviation of  $|\bar{A}/A|$  from unity is expected to be even smaller: such an effect requires contributions to the decay amplitude which differ in both strong and weak phases [6]. While in the presence of New Physics we could easily have more than a single weak phase involved, we do not expect the various amplitudes to differ in their strong phases. An absorptive phase comes from light intermediate states. In the language of quark subprocesses, only an intermediate up quark could contribute. But there is a hard GIM suppression that makes these contributions negligibly small [10–14,3]. Therefore, it is safe to assume that  $|\lambda| = 1$  to  $\mathcal{O}(10^{-3})$  accuracy. The leading CP violating effect is then  $\text{Im}\lambda \neq 0$ , namely interference between mixing and decay. This puts the ratio of decay rates (5) in the same class as CP asymmetries in various  $B$  decays to final CP eigenstates, *e.g.*  $B \rightarrow \psi K_S$ , where a very clean theoretical analysis is possible [6].

As a result of this cleanliness, the CP violating phase can be extracted almost without any hadronic uncertainty, even if this phase comes from New Physics. Specifically, defining  $\theta$  to be the relative phase between the  $K - \bar{K}$  mixing amplitude and the  $s \rightarrow d\nu\bar{\nu}$  decay amplitude, namely  $\lambda = e^{2i\theta}$ , we get from eq. (5)

$$\frac{\Gamma(K_L \rightarrow \pi^0\nu\bar{\nu})}{\Gamma(K_S \rightarrow \pi^0\nu\bar{\nu})} = \frac{1 - \cos 2\theta}{1 + \cos 2\theta} = \tan^2 \theta. \quad (6)$$

This ratio measures  $\theta$  without any information about the magnitude of the decay amplitudes. In reality it will be impossible to measure  $\Gamma(K_S \rightarrow \pi^0 \nu \bar{\nu})$ . We can use the isospin symmetry relation,  $A(K^0 \rightarrow \pi^0 \nu \bar{\nu})/A(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1/\sqrt{2}$ , to replace the denominator by the charged kaon decay mode:

$$a_{CP} \equiv r_{is} \frac{\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\Gamma(K^+ \rightarrow \pi^+ \nu \bar{\nu})} = \frac{1 - \cos 2\theta}{2} = \sin^2 \theta, \quad (7)$$

where  $r_{is} = 0.954$  is the isospin breaking factor [15]. The ratio (7) may be experimentally measurable, as the relevant branching ratios are  $\mathcal{O}(10^{-10})$  in the Standard Model and even larger in some of its extensions. It will provide us with a very clean measurement of the CP violating phase  $\theta$  which has a clear interpretation in any given model.

In the Standard Model, the penguin and box diagrams mediating the  $s \rightarrow d \nu \bar{\nu}$  transition get contributions from top and charm quarks in the loop. The charm diagrams carry the same phase as the mixing amplitude,  $\arg(V_{cd} V_{cs}^*)$ . The top diagrams depend on  $\arg(V_{td} V_{ts}^*)$ , so that their phase difference from the mixing amplitude is the angle  $\beta$  of the unitarity triangle. Had the top contribution dominated both  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  and  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , we would have  $\theta = \beta$ . However, while the charm contribution to  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  is negligible, it is comparable to the top contribution to  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ . Then we cannot directly relate the experimentally-derived  $\theta$  of eq. (7) to the model parameter  $\beta$ , and a calculation of the charm and top amplitudes is also needed [3]. With New Physics, the magnitude of the decay amplitude is generally not known. The ratio (7) is most useful if both  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  and  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  are dominated by the same combination of mixing angles. The phase of this combination is then directly identified with  $\theta$ , and we need not know any other of the new parameters.

Eq. (7) allows us to set an upper bound on  $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ . Using  $\sin^2 \theta \leq 1$  and  $\tau_{K_L}/\tau_{K^+} = 4.17$ , we have

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 4.4 \times \text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}). \quad (8)$$

Using the 90% CL experimental upper bound [16]

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) < 2.4 \times 10^{-9}, \quad (9)$$

we get

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 1.1 \times 10^{-8}. \quad (10)$$

Actually, eq. (8) assumes only isospin relations and does not even require that the final state is CP even. Therefore, the bound (10) is model independent. This bound is much stronger than the direct experimental upper bound [17]  $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 5.8 \times 10^{-5}$ .

New Physics can modify both the mixing and the decay amplitudes. The contribution to the mixing can be of the same order as the Standard Model one. However,  $\varepsilon = \mathcal{O}(10^{-3})$  implies that any such new contribution to the mixing amplitude carries the same phase as the Standard Model one (to  $\mathcal{O}(10^{-3})$ ). On the other hand, the upper bound (9) which is about 30 times larger than the Standard Model prediction [3] allows New Physics to dominate the decay amplitude (with an arbitrary phase). We conclude that the only relevant new contribution to  $a_{CP}$  can come from the decay amplitude. This is in contrast to the  $B$  system where we expect significant effects of New Physics mainly in the mixing amplitude (see e.g. [18]).

We now give an explicit example of a New Physics model with potentially large effects on  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ . We consider a model with extra quarks in vector-like representations of the standard Model gauge group,

$$d_4(3, 1)_{-1/3} \quad + \quad \bar{d}_4(\bar{3}, 1)_{+1/3}. \quad (11)$$

Such (three pairs of) quark representations appear, for example, in GUTs with an  $E_6$  gauge group. It is well known that the presence of new heavy fermions with non-canonical  $SU(2)$  transformations (left-handed singlets and/or right-handed doublets) mixed with the standard leptons and quarks would give rise to tree level flavor changing neutral currents in  $Z$  interactions [19]. Moreover, these flavor changing  $Z$  couplings can be CP violating [20]. The flavor changing part of the couplings reads

$$\mathcal{L}_{FCNC}^Z = \frac{g}{2 \cos \theta_W} \sum_{i \neq j} \left[ \bar{d}_L^i U_{ij} \gamma^\mu d_L^j \right] Z_\mu. \quad (12)$$

As the flavor changing couplings are very small, the flavor diagonal  $Z$  couplings are still very close to their Standard Model values. Assuming that the  $Z$ -mediated tree diagram dominates  $K \rightarrow \pi\nu\bar{\nu}$ , we get [20,21]

$$\frac{\Gamma(K^+ \rightarrow \pi^+\nu\bar{\nu})}{\Gamma(K^+ \rightarrow \pi^0e^+\nu)} = r_{is}^+ \frac{1}{4} \frac{|U_{ds}|^2}{|V_{us}|^2}, \quad \frac{\Gamma(K_L \rightarrow \pi^0\nu\bar{\nu})}{\Gamma(K^+ \rightarrow \pi^0e^+\nu)} = r_{is}^0 \frac{1}{4} \frac{|\text{Im}U_{ds}|^2}{|V_{us}|^2}. \quad (13)$$

Here  $r_{is}^0 = 0.944$  and  $r_{is}^+ = 0.901$  are the isospin breaking corrections [15] (so that  $r_{is} = r_{is}^+/r_{is}^0$ ). The ratio (7) measures, in this case,  $\sin\theta = \text{Im}U_{ds}/|U_{ds}|$ .

We now show that the experimental bounds on the model parameters indeed still allow large effects in  $K \rightarrow \pi\nu\bar{\nu}$ . From  $K_L \rightarrow \mu^+\mu^-$  we get [20,22] (taking into account uncertainties from long distance contributions [23]),

$$|\text{Re}(U_{ds})| \lesssim 2 \times 10^{-5}. \quad (14)$$

From  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  we get (see (13) and (9))

$$|U_{ds}| \leq 1.0 \times 10^{-4}. \quad (15)$$

The measurement of  $\varepsilon$  implies [20,22]

$$|\text{Re}(U_{ds}) \text{Im}(U_{ds})| \lesssim 1.3 \times 10^{-9}. \quad (16)$$

Then indeed a strong enhancement of the  $K \rightarrow \pi\nu\bar{\nu}$  rates is possible. If  $|\text{Re}(U_{ds})|$  and  $|\text{Im}(U_{ds})|$  are close to their upper bounds, the branching ratios  $\text{BR}(K^+ \rightarrow \pi^+\nu\bar{\nu})$  and  $\text{BR}(K_L \rightarrow \pi^0\nu\bar{\nu})$  are  $\mathcal{O}(10^{-9})$  and  $a_{CP}$  of Eq. (7) is  $\mathcal{O}(1)$ . The measurement of  $\text{BR}(K^+ \rightarrow \pi^+\nu\bar{\nu})$  determines  $|U_{ds}|$ , and the additional measurement of  $\text{BR}(K_L \rightarrow \pi^0\nu\bar{\nu})$  determines  $\arg(U_{ds})$ .

Before turning to the investigation of models with lepton flavor violation, we would like to clarify one more point. It is often stated that a measurement of  $\text{BR}(K_L \rightarrow \pi^0\nu\bar{\nu}) \geq \mathcal{O}(10^{-11})$  will provide a manifestation of direct CP violation. This statement is somewhat confusing because, as explained above,  $K_L \rightarrow \pi^0\nu\bar{\nu}$  at this level is a manifestation of interference between mixing and decay,  $\text{Im}\lambda \neq 0$ , and not of what is usually called direct CP violation,

namely  $|\bar{A}/A| \neq 1$ . Furthermore, CP violation in the interference of mixing and decay has already been observed in  $\text{Im}(\varepsilon) \neq 0$  (see discussion in [6]). What is then meant by the above statement is the following: the measurement of  $\text{Im}(\varepsilon) = \mathcal{O}(10^{-3})$  together with a measurement of  $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \geq \mathcal{O}(10^{-11})$  will show that CP violation cannot be confined to  $\Delta s = 2$  processes (mixing) but necessarily affects  $\Delta s = 1$  processes (decays) as well. More specifically, while one of the two ratios  $A(K \rightarrow \pi\pi)/A(\bar{K} \rightarrow \pi\pi)$  and  $A(K \rightarrow \pi^0 \nu \bar{\nu})/A(\bar{K} \rightarrow \pi^0 \nu \bar{\nu})$  can always be chosen real by convention, it will be impossible to do so for both [6]. This will exclude those superweak scenarios where CP violation appears in the mixing only.

We next explain how, in the presence of lepton flavor violating new physics,  $\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu}) \neq 0$  is allowed even if CP is conserved. The crucial point is that the final state in  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  is not necessarily a CP eigenstate anymore. Specifically,  $K_L \rightarrow \pi^0 \nu_i \bar{\nu}_j$  with  $i \neq j$  is allowed. Then,  $A$  and  $\bar{A}$  of eq. (1) are no longer related by a CP transformation, and we may have

$$\left| \frac{\bar{A}_{ij}}{A_{ij}} \right| \equiv \left| \frac{A(\bar{K}^0 \rightarrow \pi \nu_i \bar{\nu}_j)}{A(K^0 \rightarrow \pi \nu_i \bar{\nu}_j)} \right| \neq 1 \quad \implies \quad |\lambda_{ij}| = \left| \frac{q \bar{A}_{ij}}{p A_{ij}} \right| \neq 1, \quad (17)$$

and the rate  $\Gamma(K_L \rightarrow \pi^0 \nu_i \bar{\nu}_j) \propto (1 + |\lambda_{ij}|^2 - 2\text{Re}\lambda_{ij})$  does not vanish even in the CP limit.

To gain further insight into the consequences of (17), we note that the vanishing of strong phases implies a relation between the transition amplitudes into  $\pi^0 \nu_i \bar{\nu}_j$  and  $\pi^0 \nu_j \bar{\nu}_i$ :

$$A_{ij} = \bar{A}_{ji}^*, \quad \bar{A}_{ij} = A_{ji}^*. \quad (18)$$

Eq. (18) together with  $|q/p| = 1$  give

$$\lambda_{ij} = (\lambda_{ji}^{-1})^* \quad (19)$$

and  $\Gamma(K_L \rightarrow \pi^0 \nu_i \bar{\nu}_j) = \Gamma(K_L \rightarrow \pi^0 \nu_j \bar{\nu}_i)$ . Recalling the isospin relations,

$$A(K^+ \rightarrow \pi^+ \nu_i \bar{\nu}_j) = \sqrt{2} A_{ij}, \quad A(K^+ \rightarrow \pi^+ \nu_j \bar{\nu}_i) = \sqrt{2} A_{ji}, \quad (20)$$

we find



$$a_{ij} \equiv r_{is} \frac{\Gamma(K_L \rightarrow \pi^0 \nu_i \bar{\nu}_j) + \Gamma(K_L \rightarrow \pi^0 \nu_j \bar{\nu}_i)}{\Gamma(K^+ \rightarrow \pi^+ \nu_i \bar{\nu}_j) + \Gamma(K^+ \rightarrow \pi^+ \nu_j \bar{\nu}_i)} = \frac{|1 - \lambda_{ij}|^2}{2(1 + |\lambda_{ij}|^2)}. \quad (21)$$

A few comments are in order with regard to eq. (21):

1. This ratio is always smaller than unity so, as argued above, the bound (10) applies also to this case.
2. Things are particularly simple if there is only a single pair of indices  $(ij)$  for which  $|1 - \lambda_{ij}| = \mathcal{O}(1)$ . Then eq. (21) gives the ratio of total rates,  $a \equiv r_{is} \frac{\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\Gamma(K^+ \rightarrow \pi^+ \nu \bar{\nu})} = a_{ij}$ .
3. This ratio is invariant under  $\lambda_{ij} \rightarrow (\lambda_{ij}^{-1})^*$ , as it should.
4. In the CP limit,  $\lambda_{ij}$  is real and  $a_{ij} = \frac{(1 - \lambda_{ij})^2}{2(1 + \lambda_{ij}^2)}$ . Note, however, that for final states that are not CP eigenstates, the  $\lambda$ 's are real only if both the weak and the strong phases vanish [7]. This is in contrast to final CP eigenstates for which  $\lambda$  is always real in the CP limit.

As an explicit example of lepton flavor violation we consider a model with light leptoquarks (LQ). (This example is of particular interest in the framework of SUSY models without  $R$ -parity where the  $\lambda' L Q \bar{d}$  terms in the superpotential give the same effects, with the  $\tilde{d}$  squark playing the role of the leptoquark.) An iso-singlet scalar leptoquark,  $S_0$ , couples to neutrinos and down quarks [24]:

$$\mathcal{L}_{LQ} = -h_{iq} \bar{q}_L^c \nu_L^i S_0 + h.c., \quad (22)$$

with  $i = e, \mu, \tau$  and  $q = d, s, b$ . Such couplings contribute to  $K \rightarrow \pi \nu \bar{\nu}$  through tree level LQ exchange:

$$A(K \rightarrow \pi \nu_i \bar{\nu}_j) \propto \frac{h_{is} h_{jd}^*}{m_{S_0}^2}. \quad (23)$$

The strongest bounds on  $|h_{is} h_{jd}^*|$  come from the bound on  $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  (Eq. (9)) [24], so obviously LQ exchange can dominate  $K \rightarrow \pi \nu \bar{\nu}$ . Neglecting the Standard Model contribution we get

$$\lambda_{ij} = \frac{q h_{is} h_{jd}^*}{p h_{id} h_{js}^*}. \quad (24)$$

If there is no fine-tuning we expect  $1 - |\lambda_{ij}| = \mathcal{O}(1)$  for  $i \neq j$  ( $|\lambda_{ii}| = 1$  follows directly from (24)). We learn that, in this scenario, the CP conserving effect in the  $i \neq j$  channels is expected to be the same order of, or even dominate over, the CP violating one. For example, assuming hierarchical flavor structure (namely,  $h_{iq}$  is smaller for lighter generations) and CP symmetry (namely,  $h_{iq}$  is real), we find that  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  has only CP conserving contributions, and (barring a fine-tuned relation between  $h_{\mu d} h_{\tau s}^*$  and  $h_{\tau d} h_{\mu s}^*$ ) dominated by  $\pi^0 \nu_\mu \bar{\nu}_\tau$  and  $\pi^0 \nu_\tau \bar{\nu}_\mu$  final states. Note that under the same assumptions  $K^+ \rightarrow \pi^+ \nu_\tau \bar{\nu}_\tau$  is the dominant charged decay mode and the ratio of total rates is small,  $a \ll 1$ . If, however, either  $h_{\tau s}$  or  $h_{\tau d}$  is small (that could be a result of the interplay between horizontal symmetries and holomorphy [25]), then  $a = \mathcal{O}(1)$  even without CP violation.

Let us summarize our main points. In models with lepton flavor conservation,  $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \neq 0$  signifies CP violation. More precisely, it is a manifestation of CP violation in the interference between mixing and decay, which allows a theoretically clean analysis. The ratio  $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})/\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  (see Eq. (7)) provides a clean measurement of a CP violating phase. This phase can be either the Standard Model phase or one coming from New Physics (or a combination of the two). The same ratio gives a model independent bound on  $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  (see Eq. (10)). In general  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  can also have CP conserving contributions. These contributions are negligible in the Standard Model and expected to be very small in all its extensions with lepton flavor conservation. In models with lepton flavor violation, however, CP conserving contributions can be large, and even dominate the decay rate. A measurement of  $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  is then guaranteed to provide us with valuable information. It will either give a new clean measurement of CP violation, or indicate lepton flavor violation.

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