SLAC-PUB-7378
December 1996

Plasma Possibilities in the NLC*

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Abstract

Basic idea and preliminary analysis are presented of a possibility to compensate the energy spread and to collimate beam in the next generation of linear colliders (NLC) using tunneling ionization of a gas by the beam.

Contributed to ICFA Workshop, Nonlinear and Collective Phenomena in Beam Physics,
Arcidosso, Italy
September 2-September 6, 1996

*Work supported by Department of Energy contract DE–AC03–76SF00515.

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Substantial progress in plasma based methods of acceleration opens new option for optimization of the design of future colliders. In this paper, we discuss two possible uses of plasma in the Next Linear Collider (NLC). First, we discuss the use of a short wavelength plasma accelerator to reduce the correlated energy spread along the bunch at the end of the NLC linac. Second, we consider plasma generated by the bunch in a gas chamber by tunneling ionization as the mean to simplify the beam collimation.

The optimal BNS energy spread $\delta E/E$ of a bunch is of the order of 1% rms in the NLC [1] and is induced by the longitudinal wakefields. A substantial component of this correlated energy variation is linear along the bunch, see Fig. 1. Presently, it is removed by shifting the rf phase to 30° off crest along the last quarter of the linac. This both reduces the effectiveness of the BNS damping and decreases the net acceleration. Instead, this energy spread could be compensated if the bunch with the total length $l_B = 4\sigma_l$, where typical rms length $\sigma \simeq 100 - 150\mu$, passing through a section with preliminary ionized plasma with the density of the order of $n_g = 10^{15}$ cm^{-3} excites plasma wave with the wave length of the order of $2l_B$. The amplitude of the accelerating field in such a wave would increase from the head of the bunch to the tail producing desirable energy compensation. Another section, with higher plasma density and shorter plasma wave length, may be used to compensate remaining nonlinear variation of the energy spread. With a gradient of $1\,GeV/m$, a plasma length l_g of five meters is needed to achieve the energy compensation. This length is restricted by the Coulomb scattering. NLC collimation is designed with the restriction $\Delta N < 10^4$ on the fraction of the electrons ΔN scattered to the angle $\Theta > k\sigma'$, $k = 35$ for the vertical plane. This sets the limit

$$l_g < 10^{-6} k^2 \frac{\gamma^2 \sigma_\perp^2}{2\pi r_e^2 n_g Z^2 \beta_\perp^2}. \qquad (1)$$

Taking the NLC parameters: $\sigma_y = 1\mu$, beta function $\beta_y = 35$ m, and $\gamma = 1.0 \times 10^6$, and the density $n_g = 10^{15}$ cm^{-3} we get $l_g < 1.6(k^2/Z^2)$ cm. Therefore, the light gases ($Z \simeq 1$) are preferable. In this case, for the design value of $k = 35$, the length l_g would be limited to $l_g < 20.3$ m which should not present a problem.

Experiments with the goal to demonstrate accelerating gradients of the order of $1\,GeV/m$ in a meter long channel of preliminary ionized plasma, are proposed or in progress today [2] and, if succeed, would make such scheme feasible.

Another problem which may have a solution based on progress achieved in our understanding of beam-plasma interaction is the beam collimation. In the present design, a substantial part of the total length of the machine is dedicated to collimation of the halo particles, which is necessary to reduce the background in the detector. In this paper, we study collimation based on strongly nonlinear focusing produced by plasma generated in neutral gas by the beam itself. Collimation based on nonlinear optics was studied before [3]. There are two primary limitations with this approach: first, it requires very strong nonlinear magnets and, second, the alignment tolerances on the magnets is severe (fractions of a micron). A plasma generated by tunneling ionization has the advantage that it can produce a strong nonlinear field which is self-aligning; the field is centered at the beam location.

As it is shown in the following, the tunneling ionization produces focusing on the beam which is almost linear for the core particles and is strongly nonlinear for the halo particles. We want to utilize the nonlinearity of the kick to induce a beta-mismatch, that is periodic modulation of amplitudes of betatron oscillations downstream in the optical line. A collimator can be placed at the location where the displacement of the halo particles is maximum while the core particles remain almost unperturbed. There are several possibilities to maximize displacement of the halo particles while minimizing, at the same time, perturbation of the core particles using properly designed optical lines. We can consider, for example, two gas chambers or a gas chamber and a regular focusing quad with equal linear focusing strengths separated by an optical line with betatron phase advance 90^o. As it is well known, the two kicks in the linear approximation in this case compensate each other retaining the ellipse in the phase plane intact. This approximation is good for most of the particles within a core of a bunch. There is some nonlinearity of the focusing for particles at very small amplitudes $r << \sigma_r$ which may produce the beta-mismatch downstream in the optical line. However, the amplitude modulation due to the mismatch is proportional to the initial amplitude and, therefore, is small for such particles.

The situation is different for the halo particles where nonlinearity and the initial amplitudes are large. Numerical simulations are needed to choose the optimum optics and to give the detail answer on the transformation of the phase plane both for the core and halo particles. In particular, the kick is different for the halo particles located very close to the head of the bunch and all other halo particles. We should remember however, that most of the halo particles are due to the wake fields [1] and, hence, tend to be located at the tail of the bunch.

The tunneling ionization is the same both for the electron and positron beams. However, dynamics of the ionized electrons is quite different and focusing due to average field of oscillating electrons remains for the positron beam. We leave this case for computer simulations, and restrict consideration here with an electron beam.

The flat NLC bunch [1] with the transverse rms dimensions $\sigma_x = 10\mu$, $\sigma_y = 1.0\mu$ in the regular accelerating sections, bunch length $\sigma_z = 150\mu$ and the bunch population $N_B = 1.1 \times 10^{10}$ particles per bunch has very high charge density. Assuming a Gaussian beam, the density in the bunch center

$$n_b = (2\pi)^{-3/2} N_B / (\sigma_x \sigma_y \sigma_z) \qquad (2)$$

is $n_b = 4.6 \times 10^{17}$ cm^{-3}. Such a bunch generates electric field E_b comparable with atomic field, of the order of V/A^o, and produce almost instantaneous ionization of the residual gas. The probability of the tunneling ionization [4] per unit time in the quasi-static limit is

$$W = \frac{6\alpha_0^2 c}{\lambda_c} \xi e^{-\xi} = 2.52 \times 10^{17} \xi e^{-\xi} \ [s^{-1}], \qquad (3)$$

where

$$\xi = \frac{4}{3} \frac{\alpha_0}{\lambda_c} \frac{U}{eE_b} = 0.255 \times 10^9 \frac{U}{eE_b} \ \mathrm{cm}^{-1}. \qquad (4)$$

Here λ_c is the Compton wave length, α_0 is the fine structure constant, and U is ionization potential. Eq. (4) is valid if ionization time is small compared to the time variation of the field E_b, $W\sigma_z/c >> 1$. The probability is maximum at $\xi = 1$ and, in this case, ionization length is $l_i = c/W = 3.23 \times 10^{-7}$ cm. Ionization potential for typical gases is of the order of 15 eV (U = 13.6 eV for H, U = 14.53 eV for N), however there are gases with much lower U such as vapors of Cs_2 with an ionization potential U = 3.6 eV. For a flat beam $\sigma_y << \sigma_x$, the field of a bunch increases from zero at the bunch center to the maximum at $y \simeq 2\sigma_y$, $x = 0$, remains almost constant up to $y \simeq \sigma_x$, and then rolls off inversely proportional to the distance y. The maximum field is $eE_b^y = (2\pi)^{3/2} n_b e^2 \sigma_y$. For a round beam with the density

$$n(r) = \frac{N_B}{2\pi\sigma_r^2} e^{-r^2/2\sigma_r^2} \rho(z), \quad \rho(z) = \frac{1}{\sqrt{2\pi\sigma_z^2}} e^{-z^2/2\sigma_z^2} \tag{5}$$

the field is

$$eE_b^r(r,z) = \frac{N_B e^2}{\sigma_r} f(r)\rho(z), \quad f(r) = \frac{2\sigma_r}{r}[1 - e^{-r^2/2\sigma_r^2}]. \tag{6}$$

It is maximum at $r/\sigma_r \simeq 1.58$ where the factor $f(r) \simeq 0.90$. Defining σ_r by the relation $\sigma_r^2 = \sigma_x \sigma_y$, we get for the ratio of the maximum field for a flat and the round beam $E_y/E_r = 2.77\sqrt{\sigma_y/\sigma_x}$. Hence, for the NLC parameters, the maximum filed of a round bunch is 10% higher than the field of a flat bunch. For this reason, we consider a locally round beam with the rms $\sigma_r = \sqrt{\sigma_x \sigma_y} = 3.16\mu$. The maximum field Eq. (6) in this case is $eE_r = 0.12 \times 10^9$ eV/cm and, for Cs_2 gas, the parameter $\xi = 7.7$. The ionization length is longer than at $\xi = 1$ only by two order of magnitude and is equal 0.34μ. The function $f(r)$ decreases e-times at $r/\sigma_r = 1.12 \pm 1.03$. Ionization remains efficient for all core particles except for very small $r/\sigma_r < 0.03$, and completely suppressed for the halo particles, $l_i > 10^6$ cm at $r/\sigma_r = 10$ for Cs_2.

Coulomb scattering gives the upper limit on the density of the gas: l_g = 0.3 cm of gas with density $n_g = 10^{15}$ cm^{-3} produces the same scattering as 10 km of the accelerator with pressure 10^{-8} Torr. Although low ionization potential of Cs_2 makes Cs vapors very attractive, the fraction of electrons Coulomb scattered in this gas to the angles $\Theta > k\sigma'$

$$\frac{\Delta N}{N} = \frac{2\pi r_e^2 n_g Z^2}{k^2 \epsilon_\perp^N} \frac{\beta_\perp l_g}{\gamma} \tag{7}$$

being proportional to Z^2 will limit the plasma length l_g. Probability of collision ionisation in a short section of plasma is negligible small.

Ionization itself does not produce electric field before the ionized electrons are repelled from the bunch. For plasma with density n_p small compared to the bunch density, $n_p << n_b$, the field of the ions and ionized electrons is small compared to the field of the bunch (so-called "blow-out regime"). Let us choose $t = 0$ at the moment when the bunch centroid enters the section with gas located at $s = 0$. An electron may be freed at the location $s > 0$, radius $r(0)$, at the moment t_0 by a slice of the bunch located at the distance $z = s - ct_0$ from the bunch centroid with probability $W(r,z)$, Eq. (3). The trajectory of the electron $R(t - t_0, r')$ with the initial condition $R(0, r') = r'$ is described by equation

$$\frac{d^2}{dt^2}\frac{R}{\sigma_r} = \omega_b^2 \frac{2\sigma_r}{R}(1 - e^{-R^2/2\sigma_r^2}) e^{-(s-ct)^2/2\sigma^2}, \tag{8}$$

where ω_b is the bunch plasma frequency,

$$\frac{\omega_b^2}{c^2} = \frac{N_B r_e}{\sqrt{2\pi}\sigma_r^2\sigma_z}, \tag{9}$$

and r_e is the classical electron radius. For small $R << \sigma_r$,

$$R(t-t_0, r', z) = r' \cosh\left[\frac{\omega_b}{c}\int_0^{c(t-t_0)} dz' e^{-(z'-z)^2/4\sigma_z^2}\right]. \tag{10}$$

$R(t)$ grows exponentially fast, $R = r'\cosh[\omega_b(t-t_0)]$, for $0 < c(t-t_0) << \sigma_z$ with the characteristic time c/ω_b. For the NLC parameters given above, this time is of the order of ten microns, $c/\omega_b = 3.5 \times \sigma_r$. At larger $R > \sigma_r$ electrons moves, basically, as free particle with velocity $dR/dt \simeq c$. Determination of the charge density at large r requires numerical analysis which may include effect of the magnetic field on the rapidly moving electrons. Here we note only that, in the blow-out regime, plasma oscillations may have period $2\pi/\omega_p$, where $(\omega_p/c)^2 = 4\pi n_g r_e$, and do not play essential role for a single bunch collimation provided that $\omega_p l_B/c < \pi/2$. For the length of a bunch $l_B = 4\sigma_z$, this gives an upper limit on the gas density $n_g < 2.0 \times 10^{14}$ cm^{-3}. As noted, Coulomb scattering may also require reduction of pressure.

To consider all bunches in the bunch train independently, the plasma oscillations excited by a bunch have to be damped out and possibly full recombination should take place. The plasma oscillations are believed [2] to damp out in 10-20 plasma oscillations. For a train of bunches with the NLC bunch spacing $\tau_B = 1.4$ ns, we requiring $20 * (2\pi/\omega_p) < \tau_B$ ns, which gives $n_g > 2.5 \times 10^{12}$ cm^{-3}. Unfortunately, the recombination takes longer time, and it is possible only due to the three-body recombination. It sets a limit on the acceptable gas density of the order of $n_g > 2.0 \times 10^{14}$cm^{-3}. Therefore, with full recombination, there is a narrow window for the gas density at the present bunch spacing.

Additional restriction may come from the motion of ions which get substantial kick from the parent bunch. Hopefully, overfocusing of the ions by the electron beam disperses ions during the time between bunches and makes the lower gas density possible. It is also not clear if the requirement of the full recombination is needed, and computer simulations may clarify the situation.

The expelled electrons generate charge density $n(r,s,t)$ and (in this geometry, radial) electric field $E_r^p(r,s,t)$ given by equations

$$\frac{\partial n}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r j_r = 0, \quad \frac{1}{r}\frac{\partial}{\partial r}E_r^p = 4\pi n e. \tag{11}$$

The radial current density $j(r,s,t)$ is given by the trajectory R,

$$j_r(r,s,t) = \frac{e}{2\pi r}\frac{\partial}{\partial t}\int_0^r n_g 2\pi r' dr' \Theta[R(t-t_0, r', z) - r] W(r', z) dt_0. \tag{12}$$

where $\Theta(x)$ is a step function, $\Theta(x) = 1$ for $x > 0$ and $\Theta(x) = 0$ for $x < 0$, n_g is the density of the residual gas, and $z = s - ct_0$.

Eqs. (11-12) give

$$eE_r^p(r,s,t) = -\frac{4\pi e^2 n_g}{cr}\int_0^r r'dr'dz'W(r',z'), \tag{13}$$

where the integration is performed over the region $r'\cosh[\omega_b(t-t_0)] > r$, $ct_0 = s - z'$. Electric field E_r^p gives a radial focusing kick to a particle in the slice z of a bunch , $\Delta p_r/p = \int eE_r^p(r, s = ct+z, t)dt$, or

$$\frac{\Delta p_r(r,z)}{p} = -\frac{4\pi n_g l_g r_e}{\gamma rc}\int_0^r r'dr'dz'W(r',z'), \tag{14}$$

with integration over the region $r'\cosh[\omega_b(z-z')/c] > r$. Here γ is relativistic factor, and l_g is the length of the gas chamber.

For $\sigma_z >> \sigma_r$, the integral over dz' gives, approximately, factor $l_B \simeq 4\sigma_z$, except for particles in the head of the bunch. In this approximation, the kick is given by the integral

$$\frac{\Delta p_r(r,z)}{p} = -\frac{\kappa}{y}\int_0^y xdx\xi(x)e^{-\xi(x)}. \tag{15}$$

Here $y = r/\sigma_r$,

$$\kappa = 2.52\times 10^{17}\frac{4\pi n_g l_g l_B r_e \sigma_r}{\gamma c}\frac{1}{s}, \tag{16}$$

and $\xi(x) = \Lambda x/(1-e^{-x^2/2})$, where

$$\Lambda = 0.123\times 10^9\frac{U}{m\omega_b^2\sigma_r}\,\frac{1}{\text{cm}}. \tag{17}$$

For the parameters given above, $\Lambda = 3.55$, $\kappa = 0.56\times 10^{-2}$ for $n_g = 10^{13}$ cm^{-3}, $l_g = 1$ cm, and $\gamma = 10^6$.

The kick $(\Delta p_r/p)/\kappa$ is shown in Fig. 2 as function of r/σ_r for several values of Λ. For small *Lambda*, $\Lambda << 1$, the kick grows almost linearly with r/σ_r up to large $r/\sigma_r \simeq 10-20$. For large *Lambda*, $\Lambda \simeq 3-5$, the kick is almost zero for the core particles, $r/\sigma_r < 0.8$, grows sharply and almost linearly up to $r/\sigma_r \simeq 2-4$, and then rolls off inversely with r/σ_r.

The linear part of the kick, $\Delta p_r/p = \zeta\kappa r/\sigma_r$, with the slope ζ is equivalent to a kick of a focusing quad with the focusing length $1/F = \zeta\kappa/\sigma_r$. Calculations gives $\zeta = 1.6\times 10^{-4}$ at $\Lambda = 3.5$ defining $F = 3.52$ m. This has to be compared with $F = 25$ m for a FODO lattice with the average $\beta_\perp = 50$ m. Hence, we can go to much lower gas density and pressure for a single bunch.

The main potential problem here is that some core particles may be kicked to large amplitudes. A point of concern might be particles located at very small transverse amplitudes or within few microns (of the order of c/ω_b) close to the head of the bunch where ionized electrons have no time to escape. Although the fraction of such particles is of the order of a percent of the total bunch population, the absolute number of such particles nevertheless can be as large as 10^8, much larger than acceptable level of $10^4 - 10^5$ halo particles per bunch. We hope that this is not the case because, as was mentioned above, the nonlinearity for the core particles is small and, additional to that, for particles having small initial amplitudes, the mismatch amplitude modulation is small.

In summary, this scheme of collimation may be advantageous making collimation much more compact than in the present design. However, constraints from the Coulomb scattering, three-body recombination time and suppression of the effect of plasma oscillations may close the window of acceptable gas density. More numerical simulations of the particle dynamics, especially of particles in the head of the bunch, are needed to evaluate perspectives of such method of collimation.

We thank T. Katsouleas for the useful discussion. Authors would like to thank organizers of the Workshop for the warm and inspiring atmosphere at Arcidosso which lead to the ideas described in this paper.

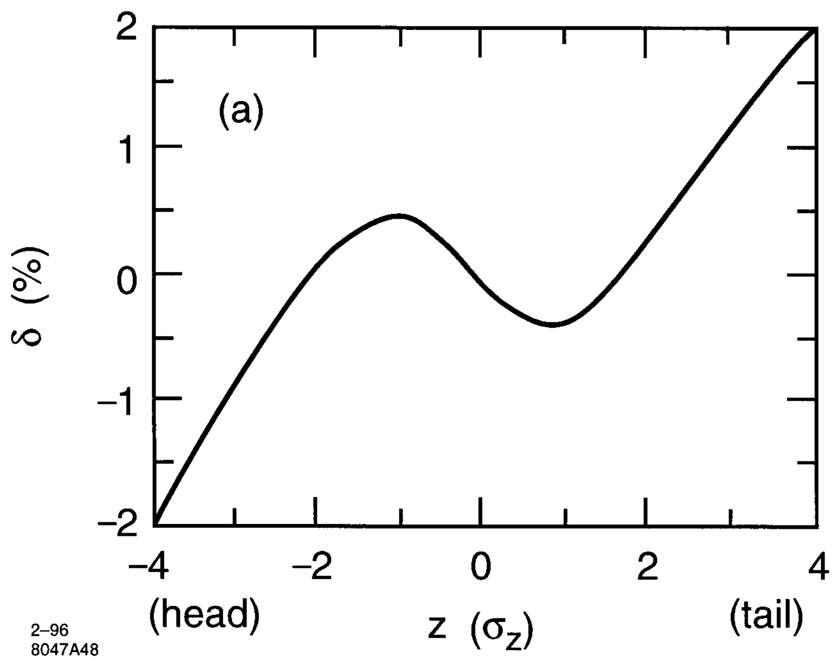

Figure 1: Energy spread at the end of the NLC linac after compensation by placing the bunch 30^o degree ahead of the rf crest for the last 20% of the linac. Without compensation the head has higher energy than the tail with a roughly linear correlation and 1% rms

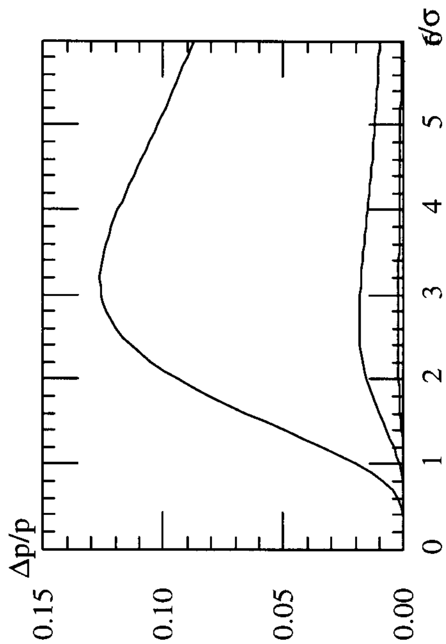

Figure 2: Dependence of the kick on the offset. $\Lambda = 1.5$ (upper) and $\Lambda = 2.5, 3.5, 4.5$ (lower) curves

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