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GAUGE AND YUKAWA UNIFICATION IN MODELS WITH GAUGE-MEDIATED SUPERSYMMETRY BREAKING

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Abstract

We examine gauge and Yukawa coupling unification in models with gauge-mediated supersymmetry breaking. We work consistently to two-loop order, and include all weak, messenger and unification-scale threshold corrections. We find that successful unification requires unification-scale threshold corrections that are in conflict with the minimal SU(5) model, but are consistent with the modified missing doublet SU(5) model for small $\tan\beta$, and large $\tan\beta$ with $\mu > 0$.

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The apparent unification of the gauge couplings in the minimal supersymmetric standard model (MSSM) [1] has sparked much interest in supersymmetric extensions to the standard model. In their present form, most phenomenologically viable models have two sectors: a hidden sector, in which supersymmetry is broken, and a visible sector, which contains the standard-model particles and their supersymmetric partners. Supersymmetry breaking is transmitted to the visible sector by gravitational interactions (as in supergravity-inspired models) or by standard-model gauge interactions (as in models with gauge-mediated dynamical supersymmetry breaking).

Models with gauge-mediated supersymmetry breaking are usually constructed to preserve gauge coupling unification to one-loop order. In this letter we will report on a closer look at unification in gauge-mediated models. We will present the results of a complete two-loop analysis for gauge and Yukawa coupling unification. Our computation takes all one-loop thresholds into account, including those at the weak, messenger and unification scales. The thresholds include finite terms which turn out to be very important for our precision analysis.

We will present our results in terms of the model-independent unification-scale threshold corrections ϵ_g and ϵ_b [2]. These parameters describe conditions that must be satisfied by any viable unification model. We will illustrate the range of these parameters for the minimal [3] and (modified) missing-doublet [4, 5] SU(5) models. We will see that present precision measurements exclude the minimal model, but are consistent with gauge and Yukawa unification in the modified missing-doublet case.

In the simplest models of gauge-mediated supersymmetry breaking [6], the messenger sector contains a set of vector-like fields which couple only to a standard-model singlet spurion through trilinear terms in the superpotential. The vector-like messenger fields are chosen to transform in $5+\overline{5}$ or $10+\overline{10}$ representations of SU(5). Requiring the gauge couplings to remain perturbative restricts attention to at most four $5+\overline{5}$ or one $10+\overline{10}$ plus one $5+\overline{5}$ pair of fields. (An additional $5+\overline{5}$ pair can be accommodated if the messenger particles are sufficiently heavy.)

We assume that the lowest (S) and highest (F) components of the spurion acquire vevs through their interactions with the hidden sector. These interactions remove the mass degeneracy of the messenger superfields and transmit supersymmetry breaking from the hidden to the visible sector through loop diagrams which contain spurion insertions. At the messenger scale, gaugino and soft scalar masses are induced by one-loop and two-loop diagrams, respectively. The flavor-blind nature of the gauge interactions ensures that flavor-changing neutral currents are suppressed. To this order, the soft supersymmetry-breaking A-parameter is not generated.

The supersymmetric Higgs mass parameter μ and the soft supersymmetry-breaking B-parameter violate a Peccei-Quinn symmetry and cannot be generated by standard-model gauge interactions. We will assume that they are generated by some minimal mechanism. The region where $B=0$ is theoretically appealing [7] because it gives rise to a large ratio of vevs ($\tan\beta$) without fine tuning. In this region, all CP-violating phases are generated only radiatively, so CP violation is naturally small.

Our approach is as follows. We start with the Fermi constant, G_F, the electromagnetic coupling, α_{em}, the Z-boson mass, M_Z, the $\overline{\text{MS}}$ strong coupling constant, $\alpha_s(M_Z)$, and the top-, bottom-quark and tau-lepton pole masses, m_t, m_b and m_τ (for details, see [8]). We then assume a supersymmetric spectrum and use the full one-loop corrections to calculate the $\overline{\rm DR}$ couplings g_1, g_2, g_3, λ_t, λ_b and λ_τ for a given value of $\tan\beta$. We run these couplings to the messenger scale, M, using the two-loop MSSM renormalization group equations. At M we fix the gaugino and soft scalar masses [9]. We then run the soft parameters back to the squark mass scale, where we impose electroweak symmetry breaking and calculate the supersymmetric spectrum. We iterate the procedure several times to achieve a consistent solution.

Our calculations of the one-loop threshold corrections include the finite and logarithmic terms. The finite corrections, which are often neglected in the literature, allow a precise determination of the gauge couplings g_1 and g_2 at the scale M_Z [2]. The finite corrections to the bottom and tau Yukawa couplings also play an important role in our analysis.

Once we determine the gauge and Yukawa couplings at the messenger scale, we extrapolate them to the unification scale, M_{GUT}, which we define to be the scale where g_1 and g_2 meet. We use the usual two-loop beta functions to compute the evolution of the gauge and Yukawa couplings. We also include the messenger contributions, those listed in Ref. [10], and [11]

$$\mu\frac{dg_i}{d\mu} = -\frac{g_i^3}{(16\pi^2)^2}\Big(\sum_f D_{if}y_f^2\Big) + \ldots \tag{1}$$

$$\mu\frac{d\lambda_a}{d\mu} = (n_5 + 3n_{10})\frac{\lambda_a}{(16\pi^2)^2}\Big(\sum_{i=1}^{3} C_{ai}g_i^4\Big) + \ldots \tag{2}$$

The sum over f runs over *all* messenger multiplets, n_5 and n_{10} are the number of $5+\overline{5}$ and $10+\overline{10}$ messenger fields, and

$$D_{if} = \begin{bmatrix} \frac{4}{5} & \frac{6}{5} & \frac{2}{5} & \frac{16}{5} & \frac{12}{5} \\ 0 & 2 & 6 & 0 & 0 \\ 2 & 0 & 4 & 2 & 0 \end{bmatrix}, \qquad f = d,\ \ell,\ q,\ u,\ e, \tag{3}$$

$$C_{ai} = \begin{bmatrix} \frac{13}{15} & 3 & \frac{16}{3} \\ \frac{7}{15} & 3 & \frac{16}{3} \\ \frac{9}{5} & 3 & 0 \end{bmatrix}, \qquad a = t, b, \tau\ . \tag{4}$$

At M_{GUT} we set the messenger Yukawas to a common value, y_m. We run the messenger Yukawas back to the messenger scale according to their one-loop evolution equations,

$$\mu\frac{dy_f}{d\mu} = \frac{y_f}{16\pi^2}\Big(2y_f^2 + T - 4\sum_{i=1}^{3} g_i^2 C_i(f)\Big)\ , \tag{5}$$

where $T = n_5(3y_d^2 + 2y_\ell^2) + n_{10}(6y_q^2 + 3y_u^2 + y_e^2)$ and the C_i's are the quadratic Casimirs, $3Y^2/5$, $3/4$, and $4/3$ for fundamental representations. (The messenger-Yukawa evolution equations can receive additional model-dependent contributions from the hidden-sector particles. The extra terms do not affect the messenger mass splittings, so we can ignore them in our analysis. Note that the one-loop equations suffice because the messenger-sector Yukawas enter our calculation only through the messenger threshold corrections.)

From the set of $y_f(M)$, we determine the messenger-particle mass spectrum and compute the messenger-scale threshold corrections to the gauge couplings,

$$\Delta\alpha_i^{-1}(M) = \sum_f \frac{D_{if}}{8\pi}\left[\ln\frac{M_f^2}{M^2} + \frac{1}{6}\ln\left(1 - \frac{\Lambda^2}{M_f^2}\right)\right] , \tag{6}$$

where $\Lambda = F/S$ is the supersymmetry-breaking scale and M_f is the messenger fermion mass. The second term in the brackets is small for $\Lambda/M_f \ll 1$, in which case there is a near degeneracy among the masses in the vector-like supermultiplets. Note that there are no messenger-scale Yukawa thresholds to this order.

We iterate this procedure to find a consistent solution in the region between M and M_{GUT}. At M_{GUT} we define the threshold corrections for the gauge and Yukawa couplings, ϵ_g and ϵ_b, as follows,

$$\begin{aligned} g_3(M_{GUT}) &= g_1(M_{GUT})(1+\epsilon_g), \\ \lambda_b(M_{GUT}) &= \lambda_\tau(M_{GUT})(1+\epsilon_b) . \end{aligned} \tag{7}$$

The parameters ϵ_g and ϵ_b describe the unification-scale threshold corrections that are necessary to achieve unification in any particular model. In what follows, we will indicate the allowed ranges of ϵ_g and ϵ_b for two of the simplest unification models, the minimal and the modified missing-doublet SU(5) models.

In the minimal SU(5) model, the unification-scale gauge threshold correction is [12],

$$\epsilon_g = \frac{3g_{GUT}^2}{40\pi^2}\ln\left(\frac{M_H}{M_{GUT}}\right) , \tag{8}$$

where M_H is the mass of the color-triplet Higgs multiplet that mediates nucleon decay. Generally, M_H is bounded from below by the proton decay limits [13], which imply $M_H \gtrsim M_{GUT}$, so $\epsilon_g \gtrsim 0$.

The missing-doublet model is an alternative SU(5) theory in which the heavy color-triplet Higgs particles are split naturally from the light Higgs doublets [4]. This requires large SU(5) representations, such as the 75 and $50 + \overline{50}$, so the SU(5) coupling g_5 diverges below the Planck scale. The modified missing-doublet (MMD) model solves this problem for $n_5 \leq 1$ by lifting the mass of the $50 + \overline{50}$ to the Planck scale and suppressing the nucleon decay rate through an extra Peccei-Quinn symmetry [5]. In this way the modified missing doublet model can accommodate two color-triplet Higgs particles with masses between $10^{13} - 10^{15}$ GeV.

In the modified missing-doublet model, the unification-scale gauge threshold can be written as [5, 14]

$$\epsilon_g = \frac{3g_{GUT}^2}{40\pi^2}\left\{\ln\left(\frac{M_H^{\text{eff}}}{M_{GUT}}\right) - 9.72\right\} , \tag{9}$$

where M_H^{eff} is the effective mass that enters the proton decay amplitude, so the previous lower bounds on M_H apply here as well. In the MMD case, the effective mass is also bounded from above, $M_H^{\text{eff}} \lesssim 10^{20}$ GeV [5].

The Yukawa threshold in minimal SU(5) can be written as follows [15],

$$\epsilon_b = \frac{1}{16\pi^2}\left\{4g_{GUT}^2\left[\ln\left(\frac{M_V}{M_{GUT}}\right) - \frac{1}{2}\right] - \lambda_t^2(M_{GUT})\left[\ln\left(\frac{M_H}{M_{GUT}}\right) - \frac{1}{2}\right]\right\}, \qquad (10)$$

where M_V is the mass of a superheavy SU(5) gauge boson. For the minimal SU(5) model, the most stringent lower limit on M_V comes from requiring that the $5+\overline{5}$ Higgs coupling remain perturbative to the Planck scale. This implies $M_V \gtrsim 0.5 M_H$ [13]. We take the upper limit on M_V to be the Planck scale, $M_V \leq 10^{19}$ GeV.

For the modified missing-doublet model, the Yukawa threshold has the same form as eq. (10), with the color-triplet Higgs mass, M_H, replaced by the effective mass, $M_H^{\rm eff}$. In this case, the lower limit on M_V comes from proton decay experiments, which imply $M_V/g_{\rm GUT} \gtrsim 3.8 \times 10^{15}$ GeV [16]. As before, we impose $M_V \leq 10^{19}$ GeV. Hence, both models have the same upper limit on ϵ_b, but the lower limit in the MMD model is lower, by virtue of the fact that M_V can be smaller and M_H larger.

In what follows, we present our results for gauge-mediated models. In particular, we calculate ϵ_g, ϵ_b, α_{GUT} and M_{GUT} as functions of the input parameters, which we take to be $\tan\beta$, the numbers n_5 and n_{10}, the supersymmetry-breaking scale Λ, the messenger scale M, and the messenger Yukawa at the unification scale, y_m. To examine bottom-tau unification, we fix the sign of μ to be positive.

We find the range of α_{GUT} and M_{GUT} by scanning over the parameter space, with $m_t = 175$ GeV, $m_b = 4.9$ GeV, $\Lambda < 300$ GeV, $1.03 \leq M/\Lambda < 10^4$, $0.03 \leq y_m \leq 3.0$ and $\tan\beta$ in the allowed range. For the case $n_5 = 1$, we determine $\alpha_{GUT} \simeq (0.044 - 0.054)$ and $M_{GUT} \simeq (1.5 - 5.0) \times 10^{16}$ GeV. For $n_5 = n_{10} = 1$, we find $\alpha_{GUT} \simeq (0.062 - 0.28)$ and $M_{GUT} \simeq (1.2 - 7.0) \times 10^{16}$ GeV.

In Fig. 1 we plot ϵ_g and ϵ_b for $n_5 = 1$, $M/\Lambda = 2$, $m_b = 4.9$ GeV and $y_m = 1$, versus Λ and $\tan\beta$, respectively. In (a) we choose $\tan\beta = 20$, while in (b) we take $\Lambda = 100$ TeV. In each case the short-dashed (long-dashed) lines correspond to $\alpha_s(M_Z) = .124$ (.112). The black bands correspond to $\alpha_s(M_Z) = 0.118$ with m_t varying from 170 to 180 GeV. The uncertainty in ϵ_b from varying $m_b = 4.9 \pm 0.3$ GeV is almost the same as that from changing $\alpha_s(M_Z) = 0.118 \pm 0.006$.

In Fig. 1(a) we also show the allowed values for ϵ_g in the minimal and modified missing-doublet SU(5) models. The region of allowed ϵ_g in the modified missing-doublet model almost completely overlaps the region with $\alpha_s(M_Z) = 0.118 \pm 0.006$. In contrast, we see that minimal SU(5) is inconsistent with $\alpha_s(M_Z)$ by more than 2σ.

For $n_5 = 1$ we find that the messenger sector corrections decrease ϵ_g. The change is induced by the messenger thresholds and the differences in the two-loop gauge coupling evolution. Both of these effects are of approximately equal importance.

From Fig. 1(a) we see that raising the supersymmetry-breaking scale Λ decreases the size of the gauge-coupling unification-scale threshold. This is because the superpartner masses scale with Λ, and larger masses decrease the size of the required thresholds [2].

Fig. 1(b) illustrates the well-known fact that bottom-tau unification can only be achieved for very small ($\lesssim 1.8$) or rather large ($\gtrsim 35$) $\tan\beta$. (Very large values of $\tan\beta$ are excluded by the requirement of proper electroweak symmetry breaking.) The figure

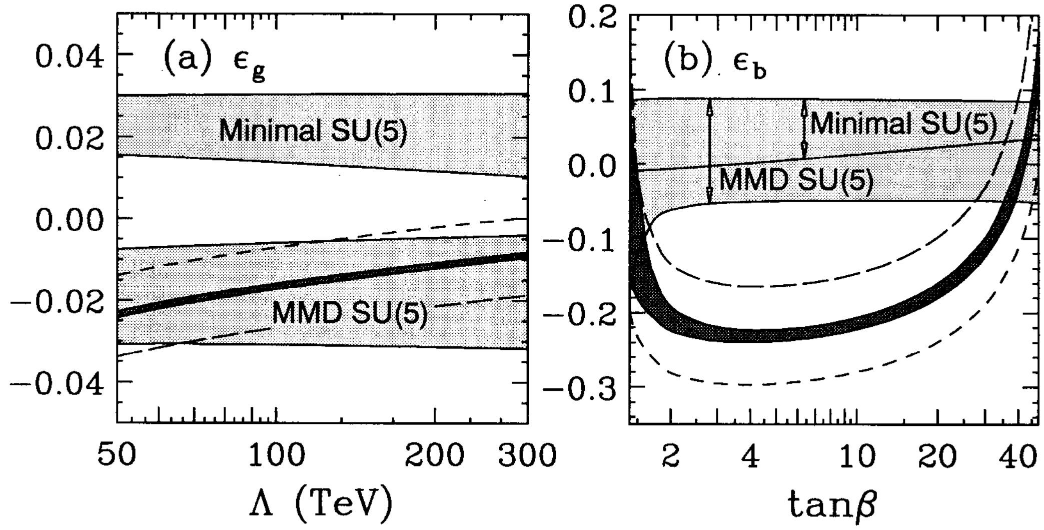

Figure 1: The unification-scale threshold corrections with $n_5 = 1$, $\mu > 0$, $M/\Lambda = 2$, and $y_m = 1$. (a) The gauge coupling unification-scale threshold correction ϵ_g versus Λ, for $\tan\beta = 20$, and $\alpha_s(M_Z) = 0.118$ (black band), 0.124 (short-dashed) and 0.112 (long-dashed). (b) The Yukawa coupling unification-scale threshold correction, ϵ_b, versus $\tan\beta$, for $\Lambda = 100$ TeV and the same values for $\alpha_s(M_Z)$ as in (a). In each case, the black band is obtained by varying the top mass from 170 to 180 GeV. The shaded regions indicate the allowed range for (a) ϵ_g and (b) ϵ_b in the minimal and modified missing-doublet SU(5) models.

also shows the allowed bands for ϵ_b in the minimal and modified missing-doublet SU(5) models.

As above, we can compare this plot to the case with no messengers. There, one typically finds that the bottom and tau Yukawa couplings meet much earlier than the scale M_{GUT}, so a rather large and negative threshold correction ϵ_b is required. For the case at hand, the extra messenger multiplets change the Yukawa evolution equations at two loops. More importantly, however, they also increase the gauge couplings, which feed into the Yukawa evolution equations and cause the bottom and tau couplings to meet even earlier. This makes ϵ_b even more negative.

Fortunately, at large $\tan\beta$ there are significant *finite* threshold corrections to the bottom (and to a smaller extent, tau) Yukawa couplings [8]. These corrections, which are proportional to $\mu\tan\beta$, are sufficiently important to permit bottom-tau unification at large $\tan\beta$ for $\mu > 0$. (The case $\mu < 0$ is completely excluded at large $\tan\beta$.) These finite corrections were omitted in the analysis of Ref. [10], which came to a different conclusion.

For $n_5 = 1$, the value of ϵ_g is not significantly affected by changes in $\tan\beta$ or M/Λ. At the smallest values of $\tan\beta$, ϵ_g increases by about 0.5%, while for $M/\Lambda = 10^4$, ϵ_g increases by about 0.2%. The parameter ϵ_b is more sensitive to changes in M/Λ. For

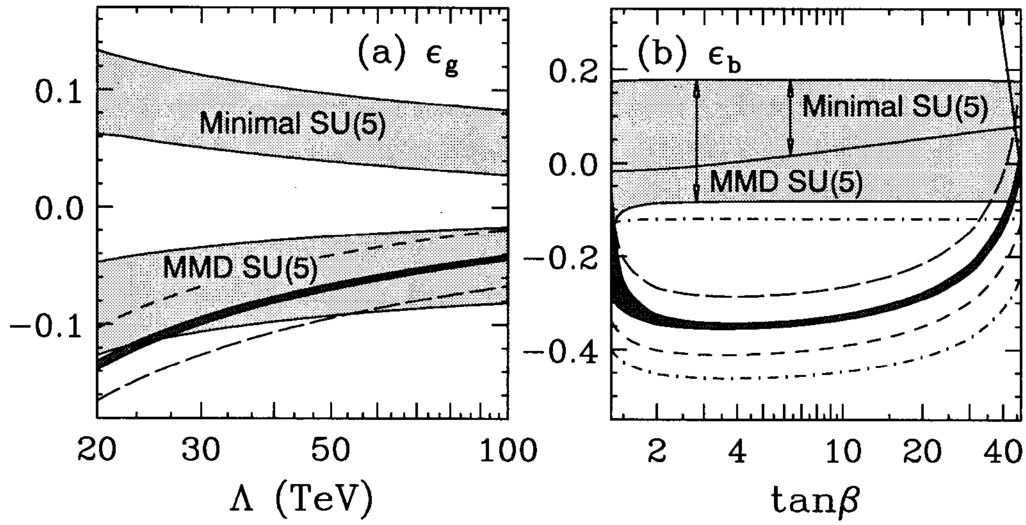

Figure 2: The same as Fig. 1, except that $n_5 = n_{10} = 1$, and in (b) Λ =50 TeV and $M/\Lambda = 100$. The dot-dashed lines indicate ϵ_b and its lower limit in the MMD model for $M/\Lambda = 2$. The line in the upper right-hand corner of (b) describes the top-quark threshold, ϵ_t, for $M/\Lambda = 100$.

$M/\Lambda = 10^4$, the ϵ_b curve is 2.5 to 3% higher at intermediate $\tan\beta$, and rises to +20% at $\tan\beta \simeq 40$.

In Fig. 2 we plot ϵ_g and ϵ_b for the case of $n_5 = n_{10} = 1$, versus Λ and $\tan\beta$ respectively. The other parameters are fixed as in Fig. 1, except that in Fig. 2(b), $\Lambda = 50$ TeV (to keep the scalar masses unchanged) and $M/\Lambda = 100$. (Two $M/\Lambda = 2$ curves are shown in dotted lines.) Figure 2(a) shows that everything shifts because of the larger g_{GUT}, but the overlap between the band from the MMD model and the allowed region for $\alpha_s(M_Z)$ is still almost complete. In this case, increasing M/Λ to 10^4 significantly changes Fig. 2(a). The central value of ϵ_g runs from −4% for $\Lambda = 20$ TeV to −1.5% for $\Lambda = 100$ TeV. The band for the MMD model is such that the required value of ϵ_g lies entirely within the band. (Note, however, that $n_5 = n_{10} = 1$ gives rise to nonperturbative couplings above M_{GUT} in the MMD case.)

The change in Fig. 2(b) as compared to Fig. 1(b) is more dramatic. Because the gauge couplings are even larger than in the previous case, bottom-tau unification turns out to be barely possible for $M/\Lambda = 2$ (dot-dashed lines). Note, however, that there is still a significant region for unification in the missing doublet model with $M/\Lambda = 100$. In the figure we also show the necessary threshold, ϵ_t, for top-tau Yukawa unification. We see from the figure that the top, bottom and tau couplings unify at the largest values of $\tan\beta$ (in the region where $B \simeq 0$). Such a unification is expected in SO(10) models. However, the thresholds in any particular SO(10) model must be calculated to make sure the model is consistent with data.

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