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Laser cooling of electron beams for linear colliders[1]

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Abstract

A novel method of electron beam cooling is considered which can be used for linear colliders. The electron beam is cooled during collision with focused powerful laser pulse. With reasonable laser parameters (laser flash energy about 10 J) one can decrease transverse beam emittances by a factor about 10 per one stage. The ultimate transverse emittances are much below of that given by other methods. Depolarization of a beam during the cooling is about 5–15 % for one stage. This method is especially useful for photon colliders and open new possibilities for e^+e^- colliders and x-ray FEL based on high energy linacs.

It is well known that due to the synchrotron radiation problem in e^+e^- storage rings the energy region beyond LEP-II can only be explored by a linear collider (LC). Such colliders for the center–of–mass energy 0.5–2 TeV are developed now in the main accelerator centers[1]. Beside e^+e^- collision at linear colliders one can 'convert' electron to high energy photon using Compton backscattering of laser light and to obtain $\gamma\gamma$ and γe collisions with energies and luminosities close to that in e^+e^- collisions[2]-[7]. This possibility is included now in projects of the linear colliders.

To obtain high luminosity, beams in linear colliders should be very tiny. At the interaction point (IP) in the current LC designs, beams with transverse sizes as low as $\sigma_x/\sigma_y \sim 200/4$ nm are planned. Beams for e^+e^- collisions should be flat in order to reduce beamstrahlung energy losses during beam collision. For $\gamma\gamma$ collision beamstrahlung radiation is absent and to obtain high luminosity one can use beams with smaller σ_x [5, 6].

The transverse beam sizes are determined by emittances ϵ_x , and ϵ_y . The beam sizes at the interaction point (IP) are $\sigma_i = \sqrt{\epsilon_i \beta_i}$, where β_i is the beta function at the IP.

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With the beam energy increasing the emittance of the bunch decreases: $\epsilon_i = \epsilon_{ni}/\gamma$, where $\gamma = E/mc^2$, ϵ_{ni} is the *normalized* emittance.

The beams with a small ϵ_{ni} are usually prepared in damping rings which naturally produce bunches with $\epsilon_{ny} \ll \epsilon_{nx}$. The equilibriun emittances in a damping ring are determined by the quantum nature of the synchrotron radiation and by the intrabeam scattering. They meet the requirements of current projects of e^+e^-linear colliders but close to a technical limit. Any further noticeable reduction of emittances in damping rings is problematic.

Laser RF photoguns can also produce beams with low emittances[10]. There are hopes to build an RF photo-gun which can be used at the superconducting linear collider TESLA without a damping ring, but there are no hopes to have such source for NLC[1] linear collider, which has smaller a number of particles in each bunch and needs much smaller emittances.

In photon colliders $\gamma\gamma$ luminosity in the high energy peak of luminosity distribution ($W_{\gamma\gamma}\ /2E_0 \geq 0.65$) accounts for about 10% of the e^-e^- geometrical luminosity, and to get the $\gamma\gamma$ luminosity the same (or larger) as in the e^+e^- collisions one has to get at IP electron beams with at least one order smaller cross section[6]. It is very difficult to prepare such beams with the help of damping rings and or photoguns.

In this paper a new method of electron beam cooling is discussed which allows further reduction of the transverse emittances after damping rings or guns by 1–3 orders.

The laser cooling of electron beams is based on the same principle as the damping of electron beams in storage rings. During a collision with laser photons (in the case of the strong field it is more correct to consider interaction of an electron with an electromagnetic wave) the transverse distribution of the electrons (σ_i) remains almost unchanged, the angular spread (σ_i') at certain conditions also changes only a little, so that the emittance $\epsilon_i = \sigma_i\sigma_i'$ remains almost unchanged. At the same time the energy of electrons decreases from E_0 to E. This means that the transverse normalized emittances have decreased: $\epsilon_n = \gamma\epsilon = \epsilon_{n0}(E/E_0)$.

One can reaccelerate the electron beam up to the initial energy and repeat the procedure. Then after n stages of cooling $\epsilon_n/\epsilon_{n0} = (E/E_0)^n$ (if ϵ_n is far from its limit).

To speak seriously about this method we have to consider first the following problems:

- Requirements of laser parameters (these parameters should be attainable)
- Energy spread of the beam after cooling (at the final energy of a linear collider it is necessary to have $\sigma_E/E \sim 0.1\%$; also with a large energy spread it is difficult to repeat cooling many times due to the problem of beam focusing)
- Limit on the final normalized emittance due to nonzero emission angles of scattered photons (it is desirable to have this limit lower than that obtained with storage rings and photoguns)
- Depolarization of electron beams (polarization is very important for linear colliders)

Let us consider the enumerated problems one after another. In the cooling region a laser photon with the energy ω_0 collides almost head–on with an electron with the energy E. The kinematics is determined by two parameters x and ξ [3, 5, 6]. The first one

$$x = \frac{4E\omega_0}{m^2c^4} \simeq 0.0153 \left[\frac{E}{GeV}\right] \left[\frac{\omega_0}{eV}\right] = 0.019 \left[\frac{E}{GeV}\right] \left[\frac{\mu m}{\lambda_0}\right], \tag{1}$$

it determines the maximum energy of the scattered photons

$$\omega_m = \frac{x}{x+1}E \sim 4\gamma^2\omega_0 \;\; (x \ll 1). \tag{2}$$

If the electron beam is cooled at the initial energy $E_0 = 5$ GeV (after damping ring and bunch compression) and $\lambda = 0.5$ μm (neodimium-glass laser) then $x_0 \simeq 0.2$.

The second parameter

$$\xi = \frac{eF\hbar}{m\omega_0 c}, \tag{3}$$

where F_0 is the field strength (E_0, B_0), in the electromagnetic wave, ω_0 is the photon energy. At $\xi \ll 1$ an electron interacts with one photon from the field (Compton scattering), while at $\xi \gg 1$ an electron feels a collective field (synchrotron radiation, SR). We will see that in the considered method ξ^2 varies in the region 1–10, therefore we will obtain formulae both for Compton scattering and synchrotron radiation cases. At the low ξ the electromagnetic wave can be treated as an undulator and at high ξ as a wiggler which is well known in accelerator physics.

In the cooling region near the laser focus the r.m.s radius of the laser beam depends on the distance z to the focus (along the beam) in the following way[3]:

$$r_\gamma = a_\gamma\sqrt{1 + z^2/\beta_\gamma^2}, \tag{4}$$

where $\beta_\gamma = 2\pi a_\gamma^2/\lambda$, a_γ is the r.m.s. focal spot radius, λ is the laser wave length. The equation for β_γ is valid for a Gaussian shape of the beam in the diffraction limit of focusing. The density of laser photons

$$n_\gamma = \frac{A}{\pi r_\gamma^2 \omega_0} \exp(-r^2/r_\gamma^2) F_\gamma(z + ct), \qquad \int F_\gamma(z)dz = 1, \tag{5}$$

where A is the laser flash energy.

In the case of strong field ($\xi \gg 1$) it is more appropriate to speak in terms of strength of the electromagnetic field which is $\bar{B}^2/4\pi = n_\gamma\omega_0$, $B = B_0 cos(\omega_0 t/\hbar - kz)$.

In the case of the Compton scattering the average energy losses in one collision $\bar{\omega} \simeq 0.5xE = 2\gamma^2\omega_0$ and the energy losses per unit length $\Delta E = 2\bar{\omega} n_\gamma \sigma_T dx$, where $\sigma_T = (8\pi/3)r_e^2$, $r_e = e^2/mc^2$ is the classical radius of the electron. Factor 2 is due to the relative motion of electrons and laser photons. Substituting $\bar{\omega}, n_\gamma, \sigma_T$, we get $dE/dx = (4/3)r_e^2\gamma^2 B_0^2$. This is the same as for classical synchrotron radiation in a wiggler magnet with maximum field $B_W = 2B_0$, factor 2 because magnetic and electrical forces in the electromagnetic wave have the same directions.

Assuming $F_\gamma = 1/l_\gamma$ and $\beta_\gamma \ll l_\gamma \simeq l_e$ we can obtain the ratio of the electron energies before and after the laser target that coincides in the our theory with the ratio of the normalized emittances

$$\frac{\epsilon_{n0}}{\epsilon_n} \simeq \frac{E_0}{E} = 1 + \frac{r_e^2}{3m^2c^4}\int B_0^2 dz = 1 + \frac{64\pi^2 r_e^2 \gamma_0}{3mc^2 \lambda l_e} A \tag{6}$$

$$A[J] = \frac{25\lambda[\mu\text{m}\,] l_e[\text{mm}\,]}{E_0[\text{GeV}\,]}\left(\frac{E_0}{E} - 1\right). \tag{7}$$

This is valid for the "classical" case of radiation when $x \ll 1$ and the critical energy of the synchrotron radiation is much less than the electron energy. In this case this equation is correct both for small and large values of ξ^2,

For example: at $\lambda = 0.5$ μm, $l_e = 0.2$ mm , $E_0 = 5$ GeV, $E_0/E = 10$ the laser flash energy $A = 4.5$ J

The eqs (6,7) are valid for $\beta_\gamma \ll l_\gamma \sim l_e$ and give the minimal flash energy for the certain E_0/E ratio. For further estimation of the photon density at the laser focus we will assume $\beta_\gamma \sim 0.25 l_e$. In this case the required flash energy is still close to the minimum one, but the field strength is not so high as for very small β_γ (we will see that the minimum emittances and depolarization decrease with decreasing of the field strength). With such focusing and with the laser bunch somewhat longer than that of electron bunch one should take for practical estimations $A \sim 2A_{min}$.

Now we can estimate the parameter ξ for $\beta_\gamma = l_e/4$. From (4), (5) and (6) it follows $B_0^2/(8\pi) = \omega_0 n_\gamma = A/(\pi a_\gamma^2 l_e) = 8A/(\lambda l_e^2)$. Substituting B_0 to (3) we get

$$\xi^2 = \frac{16 r_e \lambda A}{\pi l_e^2 mc^2} = \frac{3\lambda^2}{4\pi^3 r_e l_e \gamma_0}\left(\frac{E_0}{E} - 1\right) = 4.3 \frac{\lambda^2[\mu\text{m}\,]}{l_e[\text{mm}\,] E_0[\text{GeV}\,]}\left(\frac{E_0}{E} - 1\right) \tag{8}$$

For example:
$\lambda = 0.5$ μm, $l_e = 0.2$ mm (NLC project), $E_0 = 5$ GeV , $E_0/E = 10 \Rightarrow \xi^2 = 9.7$;
$\lambda = 0.25$ μm, $l_e = 1$ mm (TESLA project), $E_0 = 5$ GeV, $E_0/E = 10 \Rightarrow \xi^2 = 0.48$.
So both "undulator" and "wiggler" cases are possible.

Formulae (7) for the flash energy and (8) for ξ^2 were obtained for the optimum focusing to get minimal A for a laser beam with diffraction divergency. Later we will see that to have lower limit on emittance and smaller depolarization it is necessary to have a low ξ^2. With a usual optical focusing system one can reduce ξ^2 (at the fixed cooling factor (E_0/E)) only by increasing l_γ (and proportionaly β_γ) with simultaneous increasing of the laser flash energy. As follows from (6) and (8)

$$A \propto \frac{\lambda^3}{\gamma_0^2 \xi^2}\left(\frac{E_0}{E} - 1\right)^2 \tag{9}$$

Is it possible to reduce ξ^2 keeping all other parameters (including flash energy) constant? Yes, if to find a way to stretch the focus depth without changing the radius of this area. In this case the collision probability (or $\int B^2 dz$) remains the same but maximum value of ξ^2 will be smaller. With a usual lens (focusing mirror) this is impossible, but it seems that this problem can be solved using the nonmonochromaticity of laser light together with a chirped pulse technique (explanation is below).

Let us put somewhere on the way of the laser beam a lens with dispersion, then the rays with the different wave lengths will be focused at different distances from the final focusing mirror. However this is not sufficient for our task. We want to have the scheme where the short laser bunch collides on its way sequentially with n light pulses of approximately the same length $l_\gamma \sim l_e$ and focused with $2\beta_\gamma \sim l_e$. Furthermore, each laser subbunch should come to its focal point exactly in the moment when the electron bunch cross this area. However, if the short laser bunch is focused by a dispersive focusing system, then the rays which are focused closer to the focusing mirror will come to their focus earlier, while the electron bunch moving towards the mirror should first collide with the rays which are focused further from the mirror (just opposite to our desire). This problem is solved in a natural way using a *chirped pulse technique* ("chirped pulse" means a pulse with time-frequency correlation)[8, 9]. Namely this technique is used now for obtaining very powerful short laser pulses. Chirped pulse are obtained from a short (with large bandwidth) laser pulses using a grating pair. After passing the grating pair the short "white" pulse becomes long and chirped. The long pulse is amplified without problem of nonlinear effects in some media and then is compressed by the similar method to the short bunch. This technique is developed now very well, and stretching (compression) by a factor 10000 has been demonstrated. Using this wonderful technique one can prepare the chirped laser bunch of the necessary length which (after despersive element) can be focused onto the electron beam in many focal points (stretched focal area) with necessary time delays.

Note that the bandwidths of many solid state lasers are sufficient for obtaining the required chromatic abberations and stretching of the focal distance. In ref.[9] the authors have considered a laser scheme where one short laser bunch is splited by the (dispersive) grating into twenty separate lines and after amplification all twenty bunches are joined together with the help of the other grating. It is obvious that one can prefocus separate beams by the mirrors with the somewhat different focal lengthes, so that the joined beam will be focused by the final (nondispersive) lens to many focal points.

The number n (the length of focal region/$2\beta_\gamma$, where $\beta_\gamma \sim 0.5 l_e$) depends on the stretching factor we want to get. There is one principal restriction on n: along cooling length $L \approx n \cdot l_e$ the transverse size of the electron beam should be smaller than the laser spot size $a_\gamma \simeq \sqrt{\lambda\beta_\gamma/2\pi} \sim \sqrt{\lambda l_e/4\pi}$. In our examples we will use $n \sim 10$ for stretching the cooling region from 100 μm to 1 mm . This is not the maximal limit. Using larger n will only improve the quality of the cooled electron beam (especially polarization).

The detailed consideration of the optical system is beyond the scope of the present paper.

Let us consider the energy spread of electrons after cooling which arises due to a quantum-statistical nature of the radiation. The increase of the energy spread after losses of energy ΔE

$$\Delta(\sigma_E^2) = \int_0^\infty \varepsilon^2 \dot{n}(\varepsilon) d\varepsilon = -aE^2 \Delta E, \tag{10}$$

where $a = 8\omega_0/3m^2c^4 = 2x_0/3E_0$ or $a = 55\hbar eB_0/(8\pi\sqrt{3}m^3c^5)$ for the Compton and SR[11] cases, respectively.

There is the second effect which leads to a decreasing of the energy spread. It is due to the fact that $dE/dx \propto E^2$ and electron with higher(lower) energy than average one loses more(less) than on average. This results in the damping: $d(\sigma_E^2)/\sigma_E^2 = 4dE/E$ (here dE has the negative sign). We obtained the equation for the energy spread

$$d\sigma_E^2 = -aE^2 dE + \frac{4dE}{E}\sigma_E^2. \tag{11}$$

The solution of this equation is

$$\frac{\sigma_E^2}{E^2} = \frac{\sigma_{E_0}^2 E^2}{E_0^4} + aE_0 \frac{E}{E_0}\left(1 - \frac{E}{E_0}\right) \sim \frac{\sigma_{E_0}^2 E^2}{E_0^4} + \frac{2}{3}x_0(1 + \frac{55\sqrt{3}}{64\pi}\xi)\frac{E}{E_0}\left(1 - \frac{E}{E_0}\right) \tag{12}$$

Here the result for the Compton scattering and SR are joined together. The value of ξ is given by eq.(8). In this formula the term with ξ is approximate because we have neglected the variation of the field along the axis and ξ^2 in (6)is taken at the point with the maximum field. This accuracy is sufficient for our first estimation. For example: at $\lambda = 0.5\mu$m , $E_0 = 5$ GeV ($x_0 = 0.19$) and $E_0/E = 10$ only the first Compton term gives $\sigma_E/E \sim 0.11$ and with second term ($\xi^2 = 9.7$, see the example above) $\sigma_E/E \sim 0.17$.

What σ_E/E is acceptable? In the last example $\sigma_E/E \sim 0.17$ at E = 0.5 GeV. This means that the final collider energy E = 250 GeV we will have $\sigma_E/E \sim 0.034\%$, that is better than necessary (about 0.1 %).

If we want to have two stages of cooling then after reacceleration to the initial energy $E_0 = 5$ GeV the energy spread $\sigma_E/E \sim 1.7\%$. For this value already there may be problems with focusing. Although, at 5 GeV the focusing distance can be made shorter than at 250 GeV (for which there are schemes correcting the chromatic abberations upto 1.5%) that decreases contribution of the chromatic abberation by the same factor.

What are the resources if a smaller energy spread is necessary? After reacceleration to the initial energy $\sigma_E/E_0 = (\sigma_E/E)(E/E_0)$, where σ_E/E is given by (12). Using (12), (1), (8) and (9) one can find that the first(Compton) term $\sigma_E/E_0 \propto (E_0/\lambda)^{1/2}(E/E_0)^{3/2}$ and the second (SR) $\propto (E_0/l_e)^{1/4}(E/E_0)^{5/4}$ for $l_\gamma \sim l_e$ and minimal A; and $\propto \lambda^{1/4}(E_0/E)/A^{1/4}$ for free A and $l_\gamma > l_e$.

Another way is to stretch the cooling region (as it was discussed above) by a factor n. However this dependence is weak: $\sigma_E/E_0 \propto 1/n^{1/4}$ (only the second term).

Resume: the energy spread in the one stage cooling scheme is not a problem; for the multistage (2–3) cooling system one has to use the special focusing system with chromatic corrections in front of each next stage.

Mimumum normalized emittance is determined by the quantum nature of the radiation. Let us start with the case of pure Compton scattering at $\xi^2 \ll 1$ and $x_0 \ll 1$. In this case the scattered photons have the uniform energy distribution: $dp = d\omega/\omega_m$, where $\omega_m = 4\omega_0\gamma^2$. The angle of the electron after scattering[3] $\theta_1^2 = (\omega_m\omega - \omega^2)/(\gamma^2 E^2)$. After averaging over the energy spectrum we get the average θ_1^2 in one collision: $\langle\theta_1^2\rangle = 8\omega_0^2/(3m^2c^4)$. After many collisions the r.m.s. angular spread in i=x,y projection

$$\Delta\langle\theta_i^2\rangle = 0.5\Delta\langle\theta^2\rangle = 0.5n\langle\theta_1^2\rangle = -0.5(\Delta E/\bar{\omega})\langle\theta_1^2\rangle = -\omega_0\Delta E/3E^2. \tag{13}$$

The normalized emittance $\epsilon_{ni}{}^2 = (E^2/m^2c^4)\langle r_i^2\rangle\langle\theta_i^2\rangle$ does not change when $\Delta\langle\theta_i^2\rangle/\langle\theta_i^2\rangle = -2\Delta E/E$. Substituting $\langle\theta_i^2\rangle$ from (13) and taking into account that $\langle\theta_i^2\rangle \equiv \epsilon_{ni}\ /\gamma\beta_i$ we get the equilibrium emittance due to the Compton scattering

$$\epsilon_{ni,min} \approx 0.5\gamma E\beta\Delta\langle\theta_i^2\rangle/\Delta E = \frac{\omega_0}{6mc^2}\beta_i = \frac{\pi}{3}\frac{\lambda_C}{\lambda}\beta_i = \frac{4\cdot 10^{-8}\beta_i[mm]}{\lambda[\mu\mathrm{m}\ ]}\ cm\cdot rad, \tag{14}$$

where $\lambda_C = \hbar/mc$

For example: $\lambda = 0.5\ \mu$m , $\beta = l_e/2 =\ 0.1$ mm $(NLC) \Rightarrow \epsilon_{n,min} = 0.8\cdot 10^{-8}$ cm. For comparison in the NLC project the damping rings have $\epsilon_{nx}\ = 3\cdot 10^{-4}\ cm\cdot rad,\ \epsilon_{ny}\ = 3\cdot 10^{-6}\ cm\cdot rad.$

Let us consider now the case $\xi \gg 1$ when the electron moves as in the wiggler. Assume that the wiggler is planar and deflects the electron in the horizontal plane. If the electron with the energy E emits the photon with the energy ω along its trajectory the emittance changes as follows[11]

$$\delta\epsilon_x = \frac{1}{2}\frac{\omega^2}{E^2}H(s);\quad H(s) = \beta_x\eta_x'^2 + 2\alpha_x\eta_x\eta_x' + \gamma_x\eta_x^2, \tag{15}$$

where $\alpha_x\ =\ -\beta_x'/2,\ \gamma_x\ =\ (1+\alpha_x^2)/\beta_x,\ \beta_x$ is the horizontal beta-function, η_x is the dispersion function, s is the coordinate along the trajectory. For $\beta_x = const$ the second term H is equal to zero, the second term in the wiggler with $\lambda_w \ll \beta$ is small, so with a good approximation $H(s){=}\beta\eta'^2$. In the sinusoidal wiggler field $B(z) = B_w cos\ k_w z,\ k_w = 2\pi/\lambda_w$. As well as $\eta'' = 1/\rho$ one can find that $\eta' = (eB_w/k_wE)\ sin\ k_w z$. In fact η_x' is the angle of the trajectory with respect to the axis z. Increase of the horizontal emittance after emission of many photons

$$\Delta\epsilon_x = \int H(s)\left(\frac{\omega}{E}\right)^2 \dot{n}(\omega)d\omega dt = \frac{55}{48\sqrt{3}}\frac{r_e\hbar c}{(mc^2)^6}E^5\langle\frac{H}{\rho^3}\rangle_w dz, \tag{16}$$

where $\langle H/\rho^3\rangle_w\ =\ 8\beta_x\lambda_w^2(eB_w)^5/(140E^5\pi^3)$ for the wiggler. Averaged over the wiggler period energy losses $\Delta E = r_e^2B_w^2E^2dz/(3m^2c^4)$. The normalized emittance $\epsilon_n = \gamma\epsilon$ does not change when $Ed\epsilon\ + \epsilon\ dE = 0$. Using the obtained equations and replacing B_w by $2B_0$, λ_w by $\lambda/2$ we obtain the equlibrium normalized emittance in the linear polarized electromagnatic wave for $\xi \gg 1$

$$\epsilon_{nx} = \frac{11e^3\hbar c\lambda^2 B_0^3\beta_x}{24\sqrt{3}\pi^3(mc^2)^4} = \frac{11}{3\sqrt{3}}\frac{\lambda_C}{\lambda}\beta_x\xi^3 \approx \frac{8\cdot 10^{-8}\beta_x[\mathrm{mm}\,]\xi^3}{\lambda[\mu\mathrm{m}\,]}\ \mathrm{cm{\cdot}rad} \tag{17}$$

Using eq.(8) for ξ^2 at the minimal laser flash energy we can get scaling for a multistage cooling system with a cooling factor E_0/E in one stage: $\epsilon_{nx}\ \propto \beta_x\lambda^2(E_0/E)^{3/2}/(l_e\gamma_0)^{3/2}$ when $l_\gamma \sim l_e$(minimal A) and $\epsilon_{nx}\ \propto \beta_x\lambda^{7/2}(E_0/E)^3/(\gamma_0^3A^{3/2})$ for free A and $l_\gamma > l_e$(for $\beta_x = const$).

Using the method of stretching the laser focus depth (by a factor n) one can further reduce the horizontal normalized emittance:$\epsilon_{nx}\ \propto 1/n^{1/2}$(if $\beta_x \propto n$).

Example: $\lambda = 0.5\ \mu$m, $l_e = 0.2$ mm, $E_0 = 5$ GeV, $E_0/E = 10$, $\beta_x = 0.1$ mm $\Rightarrow \xi^2 = 9.7$ (see the example to eq(8)) and $\epsilon_{nx}\ = 5\cdot 10^{-7}$ cm $\cdot$rad (in NLC $\epsilon_{nx} = 3{\cdot}10^{-4}$ cm$\cdot$rad). Stretching of cooling region with $n = 10$ further decreases the horizontal emittance by a factor 3.2.

Comparing with the Compton case (14) we see that in the strong electromagnetic field the horizontal emittance is larger by a factor ξ^3. The origin of this factor is clear: $\epsilon_{nx} \propto \eta_x'^2\omega_{crit.}$, where $\eta_x' \sim \xi\theta_{compt.}$ and $\omega_{crit.} \sim \xi\omega_{compt.}$.

Let us now estimate roughly the minimum vertical normalized emittance. At $\xi \gg 1$ it is expected to be $\epsilon_{ny} \ll \epsilon_{nx}$. Assuming that all photons are emitted at the angle $\theta_y = 1/(\sqrt{2}\gamma)$ with the $\omega = \omega_c$ similarly to the Compton case one get

$$\Delta\langle\theta_y^2\rangle = \frac{\omega_c\Delta E}{2\gamma^2E^2} = -\frac{3}{4}\frac{e\hbar\bar{B}_w\Delta E}{E^2mc}. \tag{18}$$

Using the first part of eq.(14) we get minimum vertical normalized emittance for $\xi \gg 1$

$$\epsilon_{ny_{min}} \sim \frac{3}{8}\frac{\hbar e\bar{B}_w}{m^2c^3}\beta_y = \frac{3}{2\pi}\frac{\hbar e\bar{B}_0}{m^2c^3}\beta_y = 3\left(\frac{\lambda_C}{\lambda}\right)\beta_y\xi \approx \frac{1.2\cdot 10^{-7}\beta_y[\mathrm{mm}\,]\xi}{\lambda[\mu\mathrm{m}\,]}\ \mathrm{cm{\cdot}rad} \tag{19}$$

For the previous example (NLC beams) we have obtained $\epsilon_{ny_{min}} \sim 7.5\cdot 10^{-8}$ cm$\cdot$rad (for comparison in the NLC project $\epsilon_{ny}\ =\ 3\cdot 10^{-6}$ cm$\cdot$rad. The scaling: $\epsilon_{ny}\ \propto \beta_y(E_0/E)^{1/2}/(l_e\gamma_0)^{1/2}$ when $l_\gamma\ \sim\ l_e$(minimal A) and $\epsilon_{ny}\ \propto\ \beta_y\lambda^{1/2}(E_0/E)/(\gamma_0A^{1/2})$ for free A and $l_\gamma > l_e$

For arbitrary ξ the minimum emittances can be estimated as the sum of (14) and (17) for ϵ_{nx} and sum of (14) and (19) for ϵ_{ny}

$$\epsilon_{nxmin} \approx \frac{\pi}{3}\frac{\lambda_C}{\lambda}\beta_x(1+2\xi^3); \quad \epsilon_{ny_{min}} \sim \frac{\pi}{3}\frac{\lambda_C}{\lambda}\beta_y(1+3\xi); \tag{20}$$

Finally let us consider the problem of the depolarization. For the Compton scattering the probability of spin flip in one collision is $w = (3/40)x^2$ for $x \ll 1$ (it follows from formulae of ref.[13]). The avarage energy losses in one collision are $\bar{\omega} = 0.5xE$. The decrease of polarization degree after many collisions $dp = 2wdE/\bar{\omega} = (3/10)x(dE/E) = (3/10)x_0(dE/E_0)$. After integration we obtain the relative decrease of longitudinal polarization ζ during one stage of the cooling (at $E_0/E \gg 1$)

$$\frac{\Delta\zeta}{\zeta} = \frac{3}{10}x_0 \quad \propto E_0/\lambda, \tag{21}$$

where x_0 is given by (1). For the parameters we use as the example ($\lambda = 0.5\ \mu$m and $E_0 = 5$ GeV) $x_0 = 0.19$ and $\Delta\zeta/\zeta = 5.7\%$, that is acceptable but on the limit. The way to decrease this number is clear: decrease E_0/λ. Note that this is valid only for $\xi^2 \ll 1$ which is not the case in the cooling region.

In the case of strong field ($\xi \gg 1$) the spin flip probability per unit time is the same as in the uniform magnetic field[12]

$$w = \frac{35\sqrt{3}r_e^3\gamma^2 ce\bar{B}^3}{144\alpha(mc^2)^2}, \tag{22}$$

where for the wiggler $\bar{B}^3 = (4/3\pi)B_w^3$. Using (22) one can find the relative drop of polarization degree during the cooling by the electromagnetic wave

$$\frac{\Delta\zeta}{\zeta} = \int \frac{35\sqrt{3}er_eB_0}{9\pi\alpha(mc^2)^2}dE \sim \frac{70\sqrt{3}}{9}\left(\frac{\lambda_C}{\lambda}\right)\gamma_0\xi = \frac{35\sqrt{3}}{36\pi}x_0\xi \tag{23}$$

For the general case the depolarization can be estimated as the sum of equations (21) and (23)

$$\Delta\zeta/\zeta = 0.3x_0(1 + 1.8\xi) \tag{24}$$

For the example we used everywhere above: $\lambda = 0.5\ \mu$m, $l_e = l_\gamma = 0.2$ mm, $\beta_\gamma = 0.25\ l_\gamma$, $E_0 = 5$ GeV, we have $\xi^2 = 9.7$ and $x_0 = 0.19$ that gives $\Delta\zeta/\zeta = 0.057 + 0.32 = 0.38$. This is not acceptable for linear colliders. This example shows that the depolarization effect imposes the most demanding requirements on parameters of the cooling system. Main contribution to depolarization gives the second term. One way to decrease ξ consists of increasing the cooling region length making l_γ and β_γ larger than l_e. In this method the required flash energy increases and attainable ξ depends on available laser flash energy. From (24) and (9) we can get scaling for the second term $\Delta\zeta/\zeta \propto \lambda^{1/2}(E_0/E-1)/A^{1/2}$. Another method is stretching of the focus depth using the dispersive focusing and the chirped pulse technique, this does not require increasing laser flash energy. Stretching by a factor n reduces the second term as $1/\sqrt{n}$. After stretching the cooling region by a factor n=10 we get $\Delta\zeta/\zeta = 0.057 + 0.1 \sim 15\%$.

Resume. One of possible sets of parameters for the laser cooling is the following: initial beam energy $E_0 = 4.5$ GeV, bunch length $l_e = 0.2$ mm, laser wave length $\lambda = 0.5\ \mu$m , flash energy $A \sim 5 - 10$ J, focusing system with stretching factor n=10, final electron bunch has the energy 0.45 GeV with the energy spread $\sigma_E/E \sim 13\%$, the normalize emittance is reduced by a factor 10 both in x and y directions, limit on final emittance $\epsilon_{nx} \sim \epsilon_{ny} \sim 2 \cdot 10^{-7}$ cm·rad at $\beta_i = 1$ mm, depolarization $\Delta\zeta/\zeta \sim 15\%$. The maximum emittance at the entrance (the electron beam radius is two times smaller than the laser spot size) is about 10^{-3}cm·rad (the increase of this number is possible after some optimization connected may be with some increase of laser flash energy).

The two stage system with the same parameters gives 100 times reduction of emittances (with the same restrictions) and $\Delta\zeta/\zeta \sim 30\%$. If the focus depth stretching technique works and laser flash energy above 10 J is not a problem we can thing about further reduction of the contribution of the second term. The limit due to the first Compton term for the considered example is $\Delta\zeta/\zeta \sim 5\%$ for one stage cooling and 10% for two colling stages.

Note that we can use the same laser pulse many times. According to (6) $\Delta E/E = \Delta A/A$ and 25% attenuation of laser power leads only to additional 7% r.m.s. energy spread, which together with 13% gives $\sigma_E/E \sim 15\%$ at E = 0.45 GeV (0.07% at 100 GeV). It seems possible for NLC and JLC to use one laser bunch about 10 times. For TESLA and DLC projects this number can be larger (30 ?) due to larger distance between bunches. On the other hand in the TESLA and DLC beams are longer ($l_e = 1$ mm) $A_{min} \approx 25\ J$ (in principle to have smaller flash energy one can compress the electron bunch before cooling as much as it is possible and after cooling to stretch it). The total light power for 10^4 electron bunches per second will be about 10–20 kW (may be less). Diode pumped neodimium glass lasers ($\lambda = 1.06\mu$m) have the efficiency about 10% from plug[9]. Frequency doubling has 75% efficiency, so the total power consumption of laser system will be 150–300 kW , that is neglegible in comparison with 100 MW of total linear collider power consumption.

The considered scheme of laser cooling of electron beams seems very promising for linear colliders and allows to reach ultimate luminosities. Especially it is useful for photon colliders, where collision effects allow considerable increase of the luminosity. Also this method can be used for X-ray FEL based on linear colliders.

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